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The Neutrino Hypothesis for Dark Matter in Dwarf Spheroidals

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Abstract. The theoretical expectation of the high mass of $\gtrsim$ 400 eV for the particles constituting the dark matter in dwarf-spheroidals as an artifact of the implicit assumption that the density of particles vanishes at the visible edge. On the contrary if our Galaxy and the dwarf-spheroidals are embedded in a neutrino condensation of the dimensions of the cluster then $m_\nu \simeq 10$ eV can accommodate all the observations.

Key words: galaxies, dark matter — galaxies, dwarf-spheroidals—neutrino, mass

1. Statement of the problem and its resolution

The measurement of a finite mass for the electron-neutrino by the ITEP group using the β-spectrum in ^{3}He decay (Boris *et al.* 1983 quoted by V. A. Lubimov at the *Int. Conf. on High Energy Phys.*, Brighton; see also Lubimov *et a1.*1980 and references therein) has led to a number of very interesting studies (Tremaine & Gunn 1979; Bond, Efstathiou & Silk 1980; Schramm & Steigman 1980; Klinkhamer & Norman 1981; Sato & Takahara 1981; Wasserman 1981; Davis *et al.* 1981; Doroshkevich *et al.* 1981; Melott 1983; Peebles 1984 and references therein) of the basic idea that 'if neutrinos have a rest mass of a few eV/c^2 then they would dominate the gravitational dynamics of large clusters of galaxies and of the universe, and through their mutual gravitational interactions, neutrinos may have triggered the initial condensations that led to the formation of clusters of galaxies' (Cowsik & McClelland 1973). Also, observationally, evidence for the extent and the distribution of nonluminous matter on various scales has been accumulated by systematic observations on clusters of galaxies (Bahcall 1977; Peebles 1979), binary galaxies, rotation curves of spirals (Faber & Gallagher 1979) and recently in a sequence of papers on dwarf spheroidals (Aaronson 1983; Faber & Lin 1983; Lin & Faber 1983). These observations, particularly those mentioned last, have exacerbated the apparent conflict (Bludman 1977) between the neutrino hypothesis and the observed amount of dark matter in various systems. In this paper, we first present the arguments that have lead to the apparent conflict and then point out that the observations are indeed in conformity with the idea that on length-scales comparable to or greater than that of cluster of galaxies ‘neutrinos dominate the gravitational dynamics of the Universe’ (Cowsik & McClelland 1973).

The nature of the disagreement can be understood as follows: the density contributed by neutrinos at any point can be written as

$$\rho_\nu = g m_\nu \int_0^{p_{max}} f(p) \mathrm{d}^3 p. \tag{1}$$

Here, g represents the total number of degrees of freedom for the neutrinos, counting one for each helicity state, each flavour and one for each particle or antiparticle; $f(p)$ is the phase-space density. Davis *et al.* (1981), following the ideas of Tremaine & Gunn (1979), estimate

$$\rho_\nu \lesssim \frac{4\pi g m_\nu}{3}\left(\frac{m_\nu v}{h}\right)^3 \tag{2}$$

where $v = p/m$ is a suitably defined average velocity of the neutrino. Lin & Faber (1983) assume that the neutrino distribution terminates at R, the visible edge of the dwarf spheroidal so that

$$\frac{p_{max}^2}{2m_\nu} = \frac{GMm_\nu}{R}; \qquad \text{or} \qquad v^2 \simeq \frac{2GM}{R}. \tag{3}$$

Substituting this in Equation (2), and assuming $\rho_\nu \simeq M/(4\pi/3)R^3$ consistent with the idea of neutrino domination, one finds

$$m_\nu^8 \gtrsim \frac{3^4 \pi^2}{2^5 g^2} \frac{\hbar^6}{G^3 R^3 M} \tag{4}$$

which matches exactly the formula derived with the assumption of a square-well potential and ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$, giving $g = 8$ by Cowsik & McClelland (1973). Lin & Faber, adopting $R \sim$ 1–3 kpc and $M = 10^7\ M_\odot$, note that Equation (4) yields $m_\nu \gtrsim 400$ eV which exceeds substantially the bounds on neutrino masses (Boris *et al.* 1983 quoted by V. A. Lubimov at the *Int. Conf. on High Energy Phys.,* Brighton; see also Lubimov *et al.* 1980; Dolgov & Zeldovich 1981; Particle data group 1982 and references cited therein).

This contradiction detailed here stems from the implicit assumption that the neutrino distribution terminates at the visible edge of the baryonic matter used in interpreting Equation (4). On the contrary, the formula should merely be interpreted as the typical R and M over which neutrinos of a given mass m_y can form self-gravitating systems, and all the work on the condensation of neutrinos indicate that the typical scales are that of clusters of galaxies with $M \sim 10^{15}\ M_\odot$ and $R \sim 1$ Mpc. If this is so, then the distribution of galaxies in a cluster essentially traces out the cluster's gravitational potential generated predominently by the neutrino condensations, and our Galaxy, the dwarf spheroidals *etc. are* embedded in such a neutrino cloud. Strictly speaking, this does perturb the neutrino density by a small amount (Cowsik 1983; Gilbert 1970; Marochnik 1968), as shown schematically in Fig. 1. What is important here to note is that contributions to the density comes not merely from neutrinos with $v_\nu < 10$ km s^{-1} and orbits confined by the visible edge of the dwarf spheroidals but more importantly from neutrinos with velocities up to ~ 1000 km s^{-1} and with orbits extending up to the visible edge of the cluster. With such high velocities, Equation (2) yields a possible maximum density of neutrinos $\sim 10^{-24}$ g cm^{-3} for an acceptable $m_\nu \sim 10$ eV/c^2. The actual density at the location of any specific dwarf spheroidal would depend upon its proximity to the centre of the neutrino cloud; closer it is to the centre, higher is the density of the background neutrinos. To see that such a density is more than adequate to explain the observations on dwarf spheroidals let us focus attention

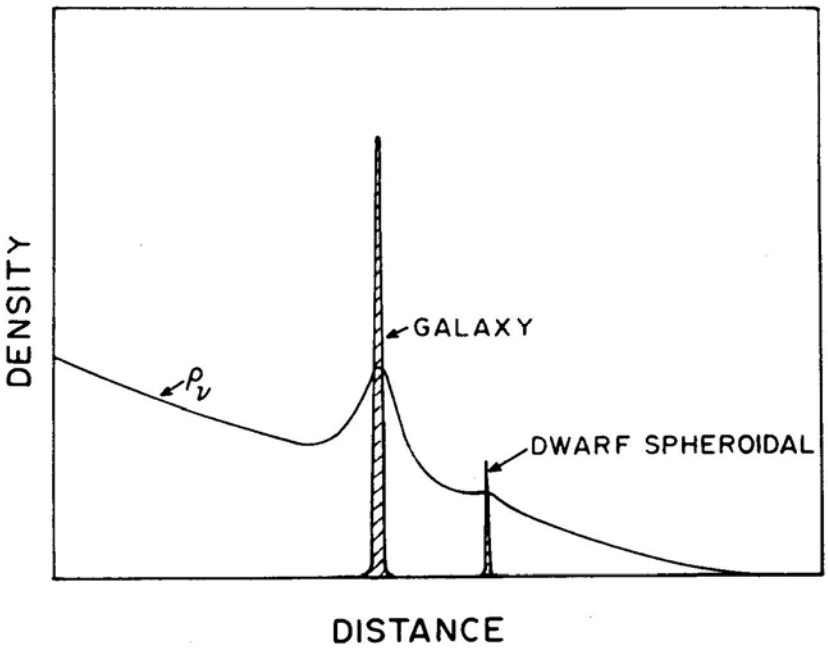

Figure 1. Schematic representation of density perturbations induced by the Galaxy and the dwarf spheroidals in a neutrino cloud of cluster dimensions.

on Leo I which has a central density of visible matter $\rho_\star(0) \simeq 6 \times 10^{-3}\ M_\odot\ \mathrm{pc}^{-3} \simeq 4 \times 10^{-25}\ \mathrm{g\ cm}^{-3}$ and requires for tidal stability a mass $M_t \sim 4 \times 10^7\ M_\odot$ within a radius $R \sim 0.88$ kpc. This latter requirement implies a neutrino density $\rho_\nu \simeq 1.3 \times 10^{-25}\ \mathrm{g\ cm}^{-3}$ quite consistent with the possible maximum density. Also it is interesting to note that the rms velocity of stars needed to yield the observed core radius, r_c 0.53 kpc, is

$$\begin{aligned}\langle V^2 \rangle^{1/2} &= \tfrac{1}{3} r_c [4\pi G(\rho_\star(0) + \rho_\nu)]^{1/2} \\ &\simeq \tfrac{1}{3} r_c [4\pi G \rho_\star(0)]^{1/2} \simeq 5.6\ \mathrm{km\,s^{-1}}\end{aligned} \tag{5}$$

since the central density is dominated by stars. The Keplerian velocity of the stars at the tidal radius r_t 0.88 kpc is given by

$$V_K = (GM_t/r_t)^{1/2} \simeq (\tfrac{4}{3}\pi G \rho_\nu r_t^2)^{1/2} \simeq 15\ \mathrm{km\,s^{-1}} \tag{6}$$

since the mean density up to the limiting radius is dominated by the neutrinos. These numbers are consistent with observations.

The point that is made here is that the neutrinos inside the visible edge of the spheroidal exert gravitational force on the stars, irrespective of whether the neutrino orbit takes them later beyond the edge or not. Indeed, in Equation (1), the upper limit P_{max} should pertain to the cluster in which the various galaxies, dwarf spheroidals, *etc. are* embedded. In such an embedding the dwarf spheroidals are stable against tidal disruption if they are situated in the central regions of the neutrino cloud. The exact criterion for tidal stability is derived in Appendix. Thus we might conclude that the new observations on the dwarf spheroidals indeed support the idea of neutrino domination and that the baryonic condensates like the stars and galaxies serve merely to light up the potential wells generated by the neutrinos. This idea is a viable alternative to the one in which weakly interacting particles of different masses hypothesized in the context of GUT are invoked to explain the invisible matter on different scales (Peebles 1983; Rees 1983; D. W. Sciama 1984, personal communication). The basic point that by increasing the effective radius of the distribution of dark matter one can reduce the mass of particles needed to explain the observations of dwarf spheroidals has been noted independently (Norman & Silk 1984).

2. Discussion

The hypothesis that dwarf spheroidals are embedded in extended clouds of neutrinos, typically of the dimensions of rich clusters of galaxies, immediately leads to several questions and consequences: 1. Would the dwarf spheroidals be stable against tidal disruption when they are so embedded? 2. How is the dynamics of their constituent stars affected, and could one model their profiles of luminosity on such a dynamical basis? 3. If the clouds of neutrinos are extended they would encompass other galaxies, galactic groups and galactic clusters as well; what are the predictions for such embedding? 4. Would the dynamical friction play an important rote? 5. Finally, in the expanding universe how are such clouds formed and how does the formation of such clouds affect the formation of galaxies?

All these questions cannot be answered immediately and we have listed them here merely to indicate that they should be considered carefully. In the appendix to this paper the criterion for the tidal stability of dwarf spheroidals is derived assuming that they are moving in orbits within the cloud of neutrinos; the dwarf spheroidals are stable in the central regions of the neutrino clouds. The dynamical equations for the self-consistent distribution of stars in galaxies embedded in a cloud of neutrinos have been developed; their solutions and their applicability to the luminosity profiles and rotation curves of galaxies will be discussed in forthcoming papers. We have also studied systematically how the dynamical masses of astronomical systems vary with their radii. These studies also support the embedding hypothesis.

The last question listed above as to the detailed dynamics of the formation of structures and growth of fluctuations in the universe is indeed the most difficult one and has been the focus of extensive work by many workers over the last five years. The reader may refer to the recent papers by Bond *et al.* (1984), White, Frank & Davis (1984), Shappiro, Marcell & Melott (1983) and references therein, for the various ideas that are being investigated. It is safe to say that the field is a very complex one and that as yet no consensual view has emerged as to the dynamical evolution of the condensates in the universe. It is for this reason that our first attempts will be primarily focussed on characterizing the state of the various astronomical systems today. Once a dynamically self-consistent and a sufficiently detailed picture of the astronomical system is obtained, the problem of the formation of such systems in an expanding universe may become tractable.

Appendix

Criterion for Tidal Disruption of a Stellar Cluster Embedded in a Cloud of Neutrinos

The equations of motion of a test particle around a star cluster is best studied in a frame of reference rotating about the centre of the cloud with an angular frequency ω which is constant over a circular orbit assumed here for simplicity. The rotating frame is spanned by three Cartesian co-ordinates ζ,η,ζ, along the radius vector from the centre of the cloud, along the tangent to the assumed circular path and along the normal to the orbital plane. respectively. The Hamiltonian for the system is given by

$$H = \{\omega(\eta p_\xi - \xi p_\eta) + (1/2m)(p_\xi^2 + p_\eta^2 + p_\zeta^2)\} + U \quad \text{(A1)}$$

with

$$p_\xi = m(\dot{\xi} - \omega\eta), \quad p_\eta = m(\dot{\eta} + \omega\xi) \quad \text{and} \quad p_\zeta = m\dot{\zeta}. \quad \text{(A2)}$$

The relevent equations of motion are

$$\dot{\xi} = \omega\eta + \frac{p_\xi}{m}, \quad \dot{\eta} = -\omega\xi + \frac{p_\eta}{m}, \quad \dot{\zeta} = \frac{p_\zeta}{m} \quad \text{(A3)}$$

and

$$\dot{p}_\xi = -\left\{-\omega m(\dot{\eta} + \omega\xi) + \frac{\partial U}{\partial \xi}\right\}, \quad \dot{p}_\eta = -\left\{-\omega m(\dot{\xi} - \omega\eta) + \frac{\partial U}{\partial \eta}\right\},$$

$$\dot{p}_\zeta = -\frac{\partial U}{\partial \zeta}. \quad \text{(A4)}$$

Assuming now that the size of the star cluster r is small compared to the distance ξ_c to the cloud centre, Equation (A4) for the radial acceleration of a test particle reads

$$\ddot{r} = -\left\{-\omega(\dot{\eta} + \omega r + \omega\xi_c) + \frac{GM_v(\xi_c + r)}{(\xi_c + r)^2} + \frac{GM_*}{r^2}\right\}. \quad \text{(A5)}$$

Note that $M_v = (4\pi/3)\,\xi^3\bar{\rho}_v$, $dM_v/d\xi = 4\pi\xi^2\,\rho_v$ and $M_* = (4\pi/3)r^3\,\bar{\rho}_*$ with $\bar{\rho}_v$ being the mean density internal to ξ, ρ_v the density at and $\bar{\rho}_*$ the mean density of the stellar cluster. The most favourably bound orbit obtains for $\eta \quad - a\omega_c$, its maximum negative value; here the Coriolis force is directed inward. Expanding Equation (A5) about ξ_c and substituting $\omega = [(4\pi G/3)\,\bar{\rho}_v]^{1/2}$ and $\omega_c = (4\pi G/3)(\rho_v + \bar{\rho}_*)^{1/2}$, we get the condition for tidal stability by demanding $\ddot{r}$ be negative

$$\bar{\rho}_* + \{(\bar{\rho}_* + \rho_v)\bar{\rho}_v\}^{1/2} - 3(\bar{\rho}_v - \rho_v) > 0. \quad \text{(A6)}$$

Now, specializing to the case of dwarf spheroidals embedded in an Emden-sphere of isothermal neutrinos we find that this condition for stability is satisfied up to 5 times its core radius even for very small $\bar{\rho}_*$ The forces along η and ζ are compressive and so do not impose any restrictions. Now, for radial orbits the condition for tidal stability can be shown similarly to be

$$\bar{\rho}_* + 3\rho_v - 2\bar{\rho}_v > 0. \quad \text{(A7)}$$

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