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The General Theory of Relativity: Why "It is Probably the most Beautiful of all Existing Theories"*

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By common consent, the general theory of relativity has a special aesthetic appeal to those who have studied it. I have chosen to quote Landau and Lifschitz from their *Classical Fields* in the title of my talk since their magnificent series of volumes, encompassing the whole range of physics, gives to their assessment a special authenticity. Others besides Landau and Lifschitz have applied the epithet 'beautiful to general relativity. Thus, Pauli, in his well-known article on 'The Theory of Relativity' in the *Encyclopädie der Mathematischen Wissenschaftien* (1921) has written

> This fusion of two previously quite disconnected subjects—metric and gravitation—must be considered as the most beautiful achievement of the general theory of relativity.

And in a similar vein, Dirac has written

> There was difficulty reconciling the Newtonian theory of gravitation with its instantaneous propagation of forces with the requirements of special relativity; and Einstein working on this difficulty was led to a generalization of his relativity—which was probably the greatest scientific discovery that was ever made.

In this lecture, I shall attempt to examine the origins and the reasons for the continuing belief that the general theory of relativity represents a beautiful scientific structure; and in this examination I shall try to be as objective as possible.

I

I shall begin with some remarks on the aesthetic impact which a discovery sometimes makes on the discoverer. That Einstein himself felt this aesthetic impact, when he finally arrived at his field equations, is evident from the concluding remark in his first

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preliminary announcement of his equations:

> Scarcely anyone who fully understands this theory can escape from its magic.

Einstein's reaction to his discovery is not very different from Heisenberg's reaction to his discovery that his matrix representation of the position and the momentum coordinates together with his commutation relation led to the correct energy levels of the simple harmonic oscillator. He has written

> . . . one evening I reached the point where I was ready to determine the individual terms in the energy table [Energy Matrix]. . . . When the first terms seemed to accord with the energy principle, I became rather excited, and I began to make countless arithmetical errors. As a result, it was almost three o'clock in the morning before the final result of my computations lay before me. The energy principle had held for all the terms, and I could no longer doubt the mathematical consistency and coherence of the kind of quantum mechanics to which my calculations pointed. At first, I was deeply alarmed. I had the feeling that, through the surface of atomic phenomena, I was looking at a strangely beautiful interior, and felt almost giddy at the thought that I now had to probe this wealth of mathematical structure nature had so generously spread out before me.

Heisenberg has recalled a meeting with Einstein soon after his discovery of quantum mechanics; and his remarks, at that meeting, as he has recounted them, illuminate the role of aesthetic sensibility in the discerning of great truths about nature:

> If nature leads us to mathematical forms of great simplicity and beauty . . . we cannot help thinking that they are "true," that they reveal a genuine feature of nature. . . . You must have felt this too: the almost frightening simplicity and wholeness of the relationships which nature suddenly spreads out before us and for which none of us was in the least prepared.

It may be argued that the aesthetic sensibility which may have guided Einstein or Heisenberg reflects only on their individuality; and that in any event the aesthetic appeal of a scientific insight is not really relevant to a judgement of its significance. One may indeed contend that the usefulness of any scientific discovery—be it theoretical or experimental—is to be measured only by its consequences. I shall not argue with those who take this pragmatic view. But it is fair to point out that the 'usefulness' of what one does is not always the prime motive for what one chooses to pursue. For example, Freeman Dyson has quoted Hermann Weyl as having said

> My work always tried to unite the true with the beautiful; but when I had to choose one or the other, I usually chose the beautiful.

I shall return later to give some examples of how, in the case of Hermann Weyl at any rate, his choice of the beautiful did eventually turn out to be true as well. But the task I now wish to set myself is to explain, as objectively as I can, why the general theory of relativity has had so strong an aesthetic appeal. In this attempt, I wish to be as serious as one is in literary or art criticisms of the works of Shakespeare or Beethoven.

II

At the outset one encounters a curious paradox. It is that the very beauty of the general theory of relativity is sometimes used as an argument for not pursuing it! Thus, Max Born has written

> It [the general theory of relativity] appeared to me like a great work of art, to be enjoyed and admired from a distance.

I am frankly troubled by Born's remark that the general theory of relativity is to be admired only from a distance. Is one to conclude that the theory does not require study and further development like the other branches of physical science?

I find it equally difficult to interpret a statement such as this of Rutherford:

> The theory of relativity by Einstein, apart from any question of its validity, cannot but be regarded as a magnificent work of art.

Apparently, beauty and truth are not to be confused!

For the present, I shall not concern myself with the question whether there can be beauty without truth. I shall turn instead to consider why a study of the general theory of relativity conduces in one a feeling not dissimilar to one's feelings after seeing a play of Shakespeare or hearing a symphony of Beethoven. But in attempting this task, it is useful to have some definite criteria for beauty in spite of the following view expressed by Dirac and shared by many:

> [Mathematical beauty] cannot be defined any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating.

I shall adopt the following two criteria for beauty. The first is that of Francis Bacon,

> There is no excellent beauty that hath not some strangeness in the proportion!

('Strangeness', in this context, has the meaning 'exceptional to a degree that excites wonderment and surprise.') And the second is that of Heisenberg:

> Beauty is the proper conformity of the parts to one another and to the whole.

III

That the general theory of relativity has some strangeness in the proportion, in the Baconian sense, is manifest. It consists primarily in relating, in juxtaposition, two fundamental concepts which had, till then, been considered as entirely independent: the concepts of space and time, on the one hand, and the concepts of matter and motion on the other. Indeed, as Pauli wrote in 1919, "The geometry of space-time is not given; it is determined by matter and its motion." In the fusion of gravity and metric that followed, Einstein accomplished in 1915 what Riemann had prophesied in 1854, namely, that the metric field must be causally connected with matter and its motion.

Perhaps the greatest strangeness in the proportion consists in our altered view of space-time with metric as the principal notion. As Eddington wrote: “Space is not a lot of points close together; it is a lot of distances interlocked.”

There is another aspect of Einstein’s founding of his general theory of relativity which has contributed to its uniqueness among physical theories. The uniqueness arises in this way.

We can readily concede that Newton’s laws of gravitation require to be modified to allow for the finiteness of the velocity of light and to disallow instantaneous action at a distance. With this concession, it follows that the deviations of the planetary orbits from the Newtonian predictions must be quadratic in v/c where v is a measure of the velocity of the planet in its orbit and c is the velocity of light. In planetary systems, these deviations, even in the most favourable cases, can amount to no more than a few parts in a million. Accordingly, it would have been entirely sufficient if Einstein had sought a theory that would allow for such small deviations from the predictions of the Newtonian theory by a perturbative treatment. That would have been the normal way. But that was not Einstein’s way: he sought, instead, an exact theory. And his only guides in his search for an exact theory were the geometrical base of his special theory of relativity provided by Minkowski and the principle of equivalence embodying the equality of the inertial and the gravitational mass. The empirical equality of the inertial and the gravitational mass, assumed to be exact, is at the base of the Newtonian theory of gravitation; and Newton gave it its supreme place by formulating it in the opening sentences of his *Principia.* But the equality, as Weyl has stated, is an ‘enigmatic fact’; and Einstein wished to eliminate this enigma. The fact that Einstein was able to arrive at a complete physical theory with such slender guides has been described by Weyl as “one of the greatest examples of the power of speculative thought.” There is clearly an element of revelation in the manner of Einstein’s arriving at the basic elements of his theory. One feels, as Weyl has expressed, “it is as if a wall which separated us from Truth has collapsed.”

IV

The general theory of relativity thus stands beside the Newtonian theory of gravitation and motion, as the only examples of a physical theory born whole, as a perfect chrysalis, in the single act of creation of a supreme mind. It is this feature of the general theory of relativity, more than any other, that is normally in one’s mind when one describes the theory as a “great work of art to be admired from a distance.” But for a serious student of relativity, the aesthetic appeal derives even more from discovering that at every level of further understanding, fresh strangenesses in the proportion emerge always in conformity of the parts to one another and to the whole, even as an iridescent butterfly emerges from a chrysalis. I should like to give some illustrations of this feature of the theory. But to the extent they are illustrations, they may reflect my own perspective of the theory. I am sure that others will choose other illustrations.

My first illustration will relate to the solutions which the general theory of relativity provides as a basis for the description of the black holes of nature.

It is now a matter of common knowledge that black holes are objects so condensed that the force of gravity on their surfaces is so strong that even light cannot escape from them. The most elementary physical ideas combined with the most rudimentary facts concerning stars, their sources of energy and their evolution, dictate their occurrence in very large numbers in the astronomical universe. This is not the occasion, and I do not have the time either, to elaborate on these astrophysical matters. I shall turn instead to what the general theory of relativity has to say about them. For this purpose, it is necessary to give a somewhat more precise definition of a black hole than I have given.

A black hole partitions the three-dimensional space into two regions: an inner region which is bounded by a smooth two-dimensional surface called the *event horizon*; and an outer region external to the event horizon which is *asymptotically flat*; and it is required that no point in the inner region can communicate with any point of the outer region. This incommunicability is guaranteed by the impossibility of any light signal, originating in the inner region, crossing the event horizon. The requirement of asymptotic flatness of the outer region is equivalent to the requirement that the black hole is isolated in space, which means only that far away from the event horizon the space-time approaches the customary space-time of terrestrial physics.

In the general theory of relativity we must seek solutions of Einstein's vacuum equations compatible with the two requirements I have stated. It is a startling fact that compatible with these very simple and necessary requirements, the general theory of relativity allows for stationary (*i.e.,* time-independent) black holes exactly a single, unique, two-parameter family of solutions. This is the Kerr family, in which the two parameters are the mass of the black hole and the angular momentum of the black hole. What is even more remarkable, the metric describing these solutions is simple and can be explicitly written down.

I do not know if the full import of what I have said is clear. May I explain.

As I have already stated, there are innumerable black holes in the present astronomical universe. They are macroscopic objects with masses varying from a few solar masses to millions of solar masses. To the extent they may be considered as stationary and isolated, they are all—every one of them—described exactly by the Kerr solution. This is the only instance we have of an exact description of a macroscopic object. Macroscopic objects, as we see them all around us, are governed by a variety of forces derived from a variety of approximations to a variety of physical theories. In contrast, the only elements in the construction of black holes are our notions of space and time. They are thus, almost by definition, the most perfect among all the macroscopic objects we know. And since the general theory of relativity provides a single unique two-parameter family of solutions for their description, they are the simplest objects as well.

As I have said on another occasion, Kerr's discovery of his solution is the only astronomical discovery comparable to the discovery of an elementary particle in physics; but in contrast to elementary particles, the black holes are pristine in their purity.

V

Again we need not be content with the discovery of the Kerr solution. We can study its properties in a variety of ways: by examining, for example, the manner of the interaction of the Kerr black-hole with external perturbations such as the incidence of waves of different sorts. Such studies reveal an analytic richness of the Kerr space-time which one could hardly have expected. This is not the occasion to elaborate on these technical matters. Let it suffice to say that, contrary to every prior expectation, all the Standard equations of mathematical physics can be solved exactly and explicitly in the Kerr space-time. The Hamilton-Jacobi equation governing the motion of test particles, Maxwell's equations governing the propagation of electromagnetic waves, the gravitational equations governing the propagation of gravitational waves, and the Dirac equation governing the motion of electrons, all of them can be separated and solved explicitly in Kerr geometry. And the solutions predict a variety of physical phenomena which black holes must exhibit in their interaction with the outside world.

Let me illustrate by one particular process, discovered by Roger Penrose, which can take place in such interactions. It is that one can extract, under suitable conditions, the rotational energy of a black hole. When this phenomenon was first investigated, one found that such extraction of energy was accompanied by an increase in the surface area of the black hole. Generalizing this result, Hawking was able to prove an 'area theorem' to the effect that any interaction, experienced by a black hole, in which energy is exchanged, must result in an increase in its surface area. This fact suggests that the surface area of a black hole is in some sense analogous to thermodynamic entropy which has also the monotonic property of always increasing. By considering the quantum mechanics of pressure-free gravitational collapse, Hawking soon showed that this is more than an analogy and that one can, without ambiguity, define not only the entropy of a black hole but a surface temperature as well; and also that there is a flux of radiation from the surface of a black hole with a Planck distribution for the temperature that was assigned.

I stated earlier that one of the remarkable features of Einstein's formulation of general relativity was its bringing into a direct relationship the geometry of the space-time with its content of matter and motion. It is "this fusion of two previously quite disconnected notions" that Pauli found as the "most beautiful achievement of the general theory of relativity." We now find in Hawking's synthesis a still grander fusion of geometry, matter, and thermodynamics. There is clearly no lack in the strangeness in the proportion which a further study of relativity does not reveal.

VI

Let me consider one last illustration. It relates to certain singularity theorems proved by Penrose and Hawking. The theorems state, in effect, that the occurrence of singularities in space-times is generic to general relativity. Roughly speaking, what this statement means is that during the course of evolution of material objects, there exist 'points of no return' such that the trespassing of these points will necessarily lead, inexorably, to singularities. This theorem provides in fact the strongest reason for our present belief that our universe started with an initial singularity. The reason is that from the existence of the three-degree microwave-radiation, we can conclude that the universe retained its present homogeneity and isotropy when its radius was some one thousand times smaller than the present. It follows from this result and some additional astronomical facts that the universe was already then (or a little earlier)at a point of no return; and the inference of an initial past singularity cannot be avoided. The problems associated with the conditions just preceding the initial singularity thus become a necessary part of current investigations both in cosmology and in physics.

VII

So far I have considered the aesthetic appeal of the general theory of relativity in the manner of its founding and in the matter of its implications. But Poincaré, who has often emphasized the role of beauty in the motivations for scientific pursuits, has also stated that the "value of a discovery is to be measured by the fruitfulness of its consequences." I shall therefore consider some of the "fruitful consequences" of the general theory of relativity. Since astronomy is the natural home of general relativity, we must seek for its consequences in astronomy. I shall consider two such consequences. Both of them relate to certain crucial respects in which considerations of relativity have altered the astrophysicist's views relative to the stability of stars and stellar systems.

It is well known that in the framework of the Newtonian theory, the condition for the dynamical stability of a star, derives from its modes of radial oscillations and, that for stability the average ratio of the specific heats γ (defined as the ratio of the fractional changes in the pressure and in the density for adiabatic changes) must exceed 4/3. Alternatively, a star will become dynamically unstable if γ, or some average of it, is less than 4/3. This Newtonian condition is changed in the framework of general relativity: a star with an average ratio of specific heats γ, no matter how high, will become unstable if its radius falls below a certain determinate multiple of the Schwarzschild radius, $R_S = 2GM/c^2$ (where M denotes the mass of the star, G is the constant of gravitation, and c is the velocity of light). It is this fact which is responsible for the existence of a maximum mass for stable neutron stars. I may parenthetically point out that this important result is closely related to an early deduction of Karl Schwarzschild that a star in hydrostatic equilibrium must necessarily have a radius exceeding $\frac{9}{8} R_s$: this is the radius at which a star, with a ratio of specific heats tending to infinity, becomes unstable.

This instability of relativistic origin, discovered some twenty years ago, plays a central role in all current discussions pertaining to the onset of instability during the course of evolution of massive stars prior to gravitational collapse.

There is another consequence of general relativity for the stability of neutron stars. The instability to which I now refer was discovered some ten years ago and derives from a dissipative phenomenon which general relativity naturally builds into the theory of non-axisymmetric oscillations of gravitating masses. The dissipation results from the emission of gravitational radiation with accompanying loss of energy and angular momentum. The manner in which this mode of dissipation of energy and angular momentum induces instability is in some ways similar to the manner in which viscous damping sometimes induces instability. It now appears, especially from the work of John Friedman, that this mode of instability sets a limit to the rotation of pulsars and bears on the stability of fast pulsars like the ones that have recently been discovered.

It is clear, then, that there are fruitful consequences of the general theory of relativity for the astronomer's view of the universe. He need not be content with admiring general relativity from a distance.

IX

I now turn to a somewhat more general question concerning the relation of truth to beauty in science.

I made a reference earlier to a statement of Weyl's to the effect that in his work he always tried to unite the true with the beautiful and that, when he had to make a choice he generally chose the beautiful. An example which Weyl gave was his gauge theory of gravitation, developed in his *Raum*, *Zeit*, *und Materie* (Space, Time, and Matter, 1918). Weyl became convinced that his theory was not true as a theory of gravitation; but he nevertheless kept it alive because it was beautiful But much later, it did turn out that Weyl's instinct was right after all: the formalism of gauge invariance was incorporated into quantum electrodynamics. A second example is provided by the two-component relativistic wave-equation of the massless neutrino. Weyl discovered this equation and the physicists ignored it for some thirty years because it violated parity invariance. And again it turned out that Weyl's instinct was right: he had discerned truth by trusting to what he conceived as beautiful.

A similar example is provided by Kerr's discovery of his solution. Kerr was not seeking solutions that would describe black holes. He was seeking instead solutions of Einstein's equation which had a very special algebraic property. But once he had discovered his solution, he could show quite readily that it did indeed describe a black hole. But its uniqueness for representing black holes was established only ten years later by Edward Robinson.

The foregoing examples provide evidence that a theory developed by a scientist with an exceptionally well-developed aesthetic sensibility can turn out to be true even if at the time of its formulation, it did not appear relevant to the physical world.

It is, indeed, an incredible fact that what the human mind, at its deepest and most profound, perceives as beautiful finds its realization in external nature.

What is intelligible is also beautiful.

We may well ask: how does it happen that beauty in the exact sciences becomes recognizable even before it is understood in detail and before it can be rationally demonstrated? In what does this power of illumination consist?

These questions have puzzled many since the earliest times. Thus, Heisenberg has drawn attention, precisely in this connection, to the following thought expressed by Plato in the *Phaedrus*:

> The soul is awestricken and shudders at the sight of the beautiful, for it feels that something is evoked in it, that was not imparted to it from without by the senses, but has always been already laid down there in the deeply unconscious region.

The same thought is expressed in the following aphorism of David Hume:

> Beauty in things exists in the mind which contemplates them.

Kepler was so struck by the harmony of nature as revealed to him by his discovery of the laws of planetary motion that in his *Harmony of the World*, he wrote:

> Now, it might be asked how this faculty of the soul, which does not engage in conceptual thinking and can therefore have no prior knowledge of harmonic relations, should be capable of recognizing what is given in the outward world. . . . To this, I answer that all pure Ideas, or archetypal patterns of harmony, such as we are speaking of, are inherently present in those who are capable of apprehending them. But they are not first received into the mind by a

> conceptual process, being the product, rather, of a sort of instinctive intuition and innate to those individuals.

> More recently, Pauli, elaborating on these ideas of Kepler, has written:

> The bridge, leading from the initially unordered data of experience to the Ideas, consists in certain primeval images pre-existing in the soul—the archetypes of Kepler. These primeval images should not be located in consciousness or related to specific rationally formulizable ideas. It is a question, rather, of forms belonging to the unconscious region of the human soul, images of powerful emotional content, which are not thought, but beheld, as it were, pictorially. The delight one feels, on becoming aware of a new piece of knowledge, arises from the way such pre-existing images fall into congruence with the behaviour of the external objects. . . .

Pauli concludes with

> One should never declare that theses laid down by rational formulation are the only possible presuppositions of human reason.

It is clear that following these thoughts one is dangerously led into the path of the mystical. I shall desist following this path but conclude instead by quoting two ancient mottoes:

The simple is the seal of the true

and

Beauty is the splendour of truth.