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Astrophysical Boosters

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Abstract. Constraints are derived on the acceleration of charges in shocks to highly relativistic energies. When applied to the extended extragalactic radio sources and to the cosmic rays, they cast doubt on the mechanism of 'in-situ acceleration', both for energy, entropy and statistical mechanics reasons.

Key words: radio galaxies—shock acceleration—cosmic rays

1. Sources of high-energy particles

Our cosmic neighbourhood is capable of accelerating protons and/or ions to energies in excess of 10^{20} eV — corresponding to. Lorentz factors γ larger than 10^{11}—and of accelerating electrons to Lorentz factors in excess of 10^6, perhaps even 10^8. The former are observed directly as 'cosmic rays' by particle detectors, or indirectly via scintillation events and air showers. The latter are often inferred from the upper cutoff frequency ν_u in non-thermal spectra:

$$\nu_u = e\, B_\perp\, \gamma^2/\pi m_e c = 5.6 \times 10^2 \gamma^2 B_{-4} \text{ Hz}, \tag{1}$$

where $B_\perp$ is the transverse magnetic field strength, with $B^{-4} := B/10^{-4}$ G. ν_u reaches optical frequencies in the jets of some extragalactic radio sources, like Cen A, M87, 3C273, NGC 1097 and several others, (Schreier, Gorenstein & Feigelson 1982), and also in young pulsars. What engines can achieve such high energies at reasonable efficiencies?

In a microscopic description, a charge e can only gain energy ΔE with respect to an inertial frame by falling through an electric field $\mathbf{E}$:

$$\Delta E = e \int \mathbf{E} \cdot \mathrm{dx} \tag{2}$$

We are not aware of large electrostatic fields in the interplanetary, interstellar or intergalactic medium, and do not expect such to exist because of the high prevailing conductivities. But $\mathbf{E}$ may belong to the outgoing wave radiated by a rotating magnet, as envisaged for pulsars, or E may be the electric field $-\boldsymbol{\beta} \times \mathbf{B}$ seen by a charge when a magnetic field $\mathbf{B}$ is convected across it at velocity $\mathrm{v} = c\, \boldsymbol{\beta}$:

$$\Delta E = e \int (\boldsymbol{\beta} \times \mathbf{B}) \cdot \mathrm{dx} \tag{3}$$

For pulsars, the highest available voltages are probably those across a certain fraction of the speed-of-light cylinder, of radius c/Ω, whence (*cf* Kundt 1981a)

$$E_{\text{PSR}} \lesssim e\, B_s R\, (\Omega R/c)^2 = 10^{17} B_{13} \Omega^2_2 \text{ eV}. \tag{4}$$

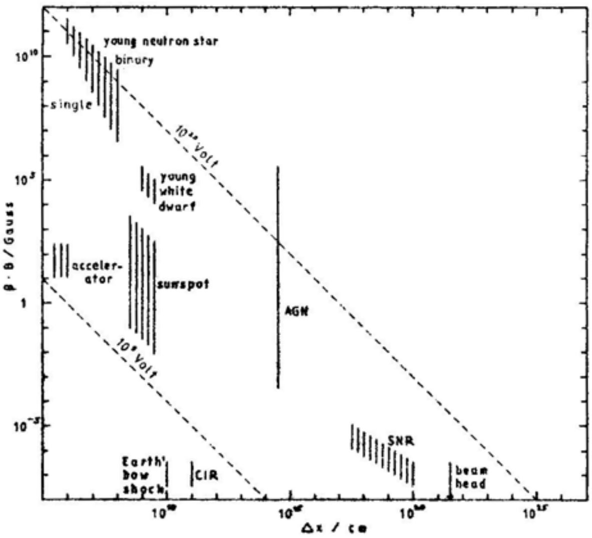

Figure 1. Estimated values of β $B/$ Gauss versus Δx for astrophysical boosters (*cf.* Equations 3–5 and 19). $\beta = \upsilon/c$ ranges from 1 (for wavelike phenomena) through $\Omega R/c$ (for rotators) down to some 10^{-3} (for old SNRs) and is often quite uncertain (*e.g.* for sunspots). The largest uncertainty rests in the distances Δx which particles can traverse during the acceleration event; for instance, the entries 'SNR' and 'beam head' may have to he moved to the left by four and seven scale units respectively (factors 10^4 and 10^7 in Δx). For a repeated number Z of independent accelerations, an entry should be 'raised' by the factor $\sqrt{Z}$.

The broken lines show voltages $\Delta E/e$. A similar diagram is contained in Hillas (1983).

For interplanetary shocks, field strengths $B_\perp$ of $\lesssim 10^{-4}$ G and velocities $\lesssim 10^3$ km s $^{-1}$ lead to maximal energy gains per particle

$$E^{\text{shock}}_{\max} \simeq e\beta B_\perp \Delta x \simeq 10^6\,(\beta B)_{-7}\,(\Delta x)_{10}\ \text{eV}, \tag{5}$$

in agreement with measurements of both electrons and protons near the Earth's bowshock [$(\Delta x)_{10} \lesssim 1$] and: in the corotating interaction regions of the solar wind [$(\Delta x)_{10} \lesssim 10$], (*cf* Kundt 1983a). In both cases, the spectra peak at energies ~ 50 times below the maximum and have exponential tails. Interstellar shocks can be more than ten times faster; they may or may not extend through much larger distances Δx, depending on the internal geometry of a supernova shell (Kundt 1983b). Even in the most optimistic case, however, Equation (5) does not yield energies anywhere near the maximal ones of cosmic rays, not even anywhere near the maximal energy for galactic containment (= 10^{19} eV).

It has been suggested that the highest-energy cosmic rays are of extragalactic origin, perhaps from the nuclei of active galaxies (AGNs) (Shapiro & Silberberg 1983). Difficulties of this interpretation are (1) the huge overall energy requirement, unless one restricts the claim to energies above 10^{19} eV which are comparatively rare, (2) a monotonic increase and reorientation of the anisotropy (in arrival directions) with energy (Hillas 1983), even though one would sample over volumes which extend far beyond the Virgo cluster, and whose sizes change rapidly with energy, (3) a likely rareness of high-energy ions in AGNs: the total energy radiated by high-energy electrons is already so large that ions cannot store a much larger fraction thereof; the jets may predominantly consist of electrons and positrons (Kundt & Gopal-Krishna 1980; Kundt 1982a). They may share this property with pulsars whose high-energy wind is thought to be mainly leptonic (Sturrock 1971; Cheng & Ruderman 1980; Kundt & Krotscheck 1980; Kundt 1980).

So far, the above discussion has not thrown suspicion upon any astrophysical source as the booster to the highest observed energies. This absence of clues has led theorists to revive Fermi's idea of accelerating charges in many small steps, by bouncing them off approaching walls (Axford, Leer & Skadron 1977; Bell 1978; Blandford & Ostriker 1978; Krymsky 1977; Drury 1983). Such walls, or shocks, are believed to exist in the form of supernova shells, strong stellar winds and the like. The idea of tapping strong shocks—often called '*in-situ*-acceleration'—has become so widespread that researchers speak of little synchrotrons drifting down the jets of extragalactic radio sources', which are conceived capable of upgrading the bulk energy of beam motion into extremely relativistic electrons *(cf.* Rees 1982). In this way, the kinetic energy of marginally relativistic or even non-relativistic protons is assumed to be converted into a power-law distribution of electrons, with Lorentz factors exceeding the value of 10^6, with the electron power distributed almost evenly up to the high-energy cutoff, at efficiencies reaching 30 per cent, and possibly even in an anisotropic fashion such that we can only see the approaching jet. The existence of similar mechanisms has been claimed for laboratory plasmas.

If such were true, we should wonder why engineers build intricate machines to convert 1 MW of electric power into $\lesssim$ 1 kW of electrons with Lorentz factors $\gamma \lesssim 10^4$ where on thermodynamic grounds we expect a near-l00-per-cent efficiency. All they should do is blow a plasma beam into plasma in order to transform the kinetic energy of ordered proton motion into a power-law-distribution of relativistic electrons, with part of the electron power stored at several hundred times the proton streaming energy.

2. Energy estimates

Let us look in somewhat more detail at the constraints imposed upon a strong shock in the head of an extragalactic radio source. We can estimate the injected power and spectrum $\dot{N}_{\mathrm{e,E}}\mathrm{d}E$ of the relativistic electrons from their synchrotron radiation. This power is often $\gtrsim$ 1 per cent of the total radiation from the active nucleus, *i.e.* corresponds to an efficiency $\eta \gtrsim$ 1 per cent of the central engine which generates the beams (Kundt & Gopal-Krishna 1980). Clearly, there is not much room in the overall power budget for energetic protons or ions: their power $\dot{N}_{\mathrm{p}}E_{\mathrm{p}}$ should be at most comparable. *If* $c\beta_{\mp}$ is their bulk velocity $\left\{\begin{matrix}\text{before}\\ \text{after}\end{matrix}\right\}$ the shock, conservation of kinetic energy implies for an incoming 1-temperature hydrogen plasma assumed to segregate (completely) into thermal protons and relativistic electrons:

$$\dot{N}_{\mathrm{p}}[m_{\mathrm{p}}c^2(\gamma_- - 1) + 3kT_-] \simeq \dot{N}_{\mathrm{p}}[m_{\mathrm{p}}c^2(\gamma_+ - 1) + 3kT_+/2] + \int E\dot{N}_{\mathrm{e},E}\mathrm{d}E. \tag{6}$$

Entropy problems are least for an incoming cold beam. In this case, *i.e.* for $3kT_- \ll m_{\mathrm{p}}c^2(\gamma_- - 1)$, and in the absence of positrons we infer

$$\dot{N}m_{\mathrm{p}}c^2(\gamma_- - \gamma_+ - 3kT_+/2m_{\mathrm{p}}c^2) \simeq \int E\dot{N}_E\mathrm{d}E, \tag{7}$$

With

$$\dot{N} = \dot{N}_p = \dot{N}_e = \int_{E_{min}}^{E_{max}} \dot{N}_E \mathrm{d}E.$$

Observations give $\dot{N}_E \sim E^{-g}$ for $E_{min} \leqslant E \leqslant E_{max}$ with g = 2.2 ± 0.2, $E_{min} \simeq 10^2$ Me V, $E_{max} \gtrsim 10^2$ GeV, so that the eadic* $E^2\dot{N}_E$ peaks at the lower cutoff energy, and

$$\frac{\int E \dot{N}_E \mathrm{d}E}{\dot{N}} = E_{min} \frac{(g-1)}{(g-2)} \frac{[1-(E_{min}/E_{max})^{g-2}]}{[1-(E_{min}/E_{max})^{g-1}]} \simeq \frac{E_{min}}{g-2}, \tag{8}$$

whence from Equation (7)

$$E_{min} \simeq (g-2) m_p c^2 (\gamma_- - 1)\left[1 - \frac{\gamma_+ - 1 + 3kT_+/2m_p c^2}{\gamma_- - 1}\right]. \tag{9}$$

In particular, for a non-relativistic incoming bulk velocity $c\beta_-$ this expression implies the inequality

$$E_{min} \lesssim (g/2-1) m_p c^2 \beta_-^2 \simeq 10^2 \beta_-^2 \text{ MeV}. \tag{10}$$

Comparison with the 'observed' E_{min} shows that either $\beta_- \simeq 1$ must hold, in mild conflict with the assumption of anon-relativistic incoming flow, or else there must be a large number of low-energy electrons in addition to the high-energy power-law. In both cases, most of the electrons would have lower energies than the (shocked) protons, which can cause problems with fundamental constraints by nonequilibrium thermodynamics, in particular with the second law.

3. Entropy estimates

In order to see this, remember that the μ-space entropy S of a homogeneous system of N classical particles reads

$$S/Nk = 1 - \int f \ln f \,\mathrm{d}^3 p/nh^3, \tag{11}$$

where the phase space density $f = f(\mathbf{p})$ is normalized according to $\int f \mathrm{d}^3 p = n\mathrm{h}^3$, and n: = N/volume. Call s: = S/Nk the (dimensionless) 'entropy per particle'. A Maxwell distribution f (of temperature T) has

$$s_{th} = 5/2 - \ln(n\lambda^3) \quad \text{with} \quad \lambda := h/(2\pi mkT)^{1/2}, \tag{12}$$

where for relativistic temperatures, m has to be replaced roughly by $3\,kT/c^2$. A supersonic bi-Maxwellian beam has

$$s_{be} = 5/2 - \ln(n\lambda_\parallel \lambda_\perp^2) \quad \text{with} \quad \lambda_j := h/(2\pi mkT_j)^{1/2}, \tag{13}$$

and an isotropic truncated relativistic power-law distribution with $f \sim p^{-g-2}$ for $P_{min} \leqslant p \leqslant p_{max}$, $E = pc$, $g > 1$:

$$s_{po} \simeq (2g+1)/(g-1) - \ln[nl^3(g-1)/4\pi] \quad \text{with} \quad l := hc/E_{min}. \tag{14}$$

The particle density n in the beam of an extragalactic radio source is of order $n = L/A\beta cE \simeq 10^{-7}$ cm$^{-3}/\beta$ where L = power, A = beam cross section and E = typical particle energy; $\lambda = 10^{-13}$ cm$/T_{12.5}^{1/2}$ for protons,

* power per energy e-folding interval ($eade$).

$l = 10^{-12}$ cm (10^2 Me V/E). We thus get:

$$
\begin{aligned}
s^{\mathrm{p}}_{\mathrm{th}} &\simeq \ln\left(10^{46} T_{12.5}{}^{3/2}\right) + 5/2 = 108, \\
s^{\mathrm{p}}_{\mathrm{be}} &\simeq \ln\left(10^{43} T_{10.5}{}^{3/2}\right) + 5/2 = 102, \\
s_{\mathrm{po}} &\simeq \ln\left(10^{42} E_{-4.5}{}^{3}\right) + 9/2 = 101,
\end{aligned}
\tag{15}
$$

for the respective entropies per particle (an upper 'p' standing for 'proton', $T_{\|}T_{\perp}^{2} =: T^3$, $\beta \simeq 1$). These numbers depend somewhat on one's preferred values for the densities, temperatures and velocities before and after shocking, but satisfy the strict inequalities s_{be}, $s_{\mathrm{po}} < s_{\mathrm{th}}$. The comparatively low entropy of a beam is due to its low temperature, and that of a soft ($g > 2$) power-law distribution (of comparable total power, Equations 9, 10) is due to the preponderance of cold particles ($E \simeq E_{\min}$).

For the joint system of protons and electrons, the average entropy per particle $s_{\mp} \left\{\begin{matrix}\text{before}\\ \text{after}\end{matrix}\right\}$ shocking depends on the relative numbers ν of thermalized (ν_{th}), power-law (ν_{po}) and 'cold' (ν_{co}) particles. Somewhat more generally than above, I assume a mixture of incoming thermal protons and electrons ($T = T_{-}$) plus relativistic power-law electrons, and an outgoing shocked composition of thermal protons ($T = T_{+}$) plus relativistic power-law electrons and protons plus cold electrons (demanded by Equation 10), and obtain

$$
\begin{aligned}
2s_{-} &= s^{\mathrm{p}}_{\mathrm{be}} + \nu^{\mathrm{e}}_{\mathrm{th},-} s^{\mathrm{e}}_{\mathrm{th}} + (1 - \nu^{\mathrm{e}}_{\mathrm{th},-}) s^{\mathrm{e}}_{\mathrm{po}} \\
2s_{+} &= \nu_{\mathrm{th}} s^{\mathrm{p}}_{\mathrm{th}} + (1 - \nu_{\mathrm{th}}) s^{\mathrm{p}}_{\mathrm{po}} + \nu^{\mathrm{e}}_{\mathrm{po}} s^{\mathrm{e}}_{\mathrm{po}} + (1 - \nu^{\mathrm{e}}_{\mathrm{po}}) s^{\mathrm{e}}_{\mathrm{co}}
\end{aligned}
\tag{16}
$$

with $\nu^{\mathrm{e}}_{\mathrm{po}} := \nu_{\mathrm{po}}$ for electrons, $0 < \nu_j < 1$, $s_{\mathrm{co}} \lesssim s_{\mathrm{po}}$. Proponents of cosmic-ray production via shocks want $\nu^{\mathrm{p}}_{\mathrm{po}} \simeq \nu^{\mathrm{e}}_{\mathrm{po}}$. The power budget in extragalactic radio sources wants the energy in protons to be small. The second law of thermodynamics demands $s_{-} < s_{+}$ and is not automatically satisfied: it needs a sufficient number of thermalized protons (ν_{th} not too small) and/or a sufficiently low temperature T_{-} of the incoming beam:

$$
\nu_{\mathrm{th}} (s^{\mathrm{p}}_{\mathrm{th}} - s^{\mathrm{p}}_{\mathrm{be}}) > (1 - \nu_{\mathrm{th}})(s^{\mathrm{p}}_{\mathrm{be}} - s^{\mathrm{p}}_{\mathrm{po}}) + \nu^{\mathrm{e}}_{\mathrm{th},-} (s^{\mathrm{e}}_{\mathrm{th}} - s^{\mathrm{e}}_{\mathrm{po}}) + (1 - \nu^{\mathrm{e}}_{\mathrm{po}})(s^{\mathrm{e}}_{\mathrm{po}} - s^{\mathrm{e}}_{\mathrm{co}}). \tag{17}
$$

Note that all terms in this inequality are positive. Clearly, any proof of efficient shock acceleration (of electrons) that does not make use of a low enough pre-shock temperature ($s^{\mathrm{p}}_{\mathrm{be}}$ small) would violate the second law of thermodynamics, and hence must be inconclusive.

4. Further constraints

However, a growth of entropy is not the only condition imposed by non-equilibrium thermodynamics: The joint N-particle distribution function $f(\mathrm{x}_i, \mathbf{p}_i, t)$ of a closed system obeys a master equation which describes its evolution in time, as a consequence of which its spectral measures (phase-space integrals)

$$
\int_{f > \lambda} [f(t) - \lambda] \prod_i \mathrm{d}^3 x_i \, \mathrm{d}^3 p_i \tag{18}
$$

cannot grow with time for any positive λ (Schlögl 1980, Equation 4.3.13). This condition goes beyond the entropy theorem, and limits severely the (post-shock) number of 'cold' particles.

I therefore question results obtained on the shock-acceleration to extremely relativistic energies whenever significant efficiencies ($\eta \gtrsim 1$ per cent) are claimed, *i.e.* whenever the boosted charges leave the test particle regime, or, in other words, whenever the (joint) inertia of the boosted charges grows comparable to that of the scatterers.

5. Neutron stars and AGNs

It is my conviction that nature has found different solutions to the relativistic acceleration problem (*cf.* Fig. 1). We know of the existence of fast-spinning magnetized neutron stars. Their strong outgoing waves involve potential drops which are a factor of order 10^{10} above those of interstellar shocks (*cf* Equations 4 and 5) and which may well be responsible for most of the high-energy cosmic-ray electrons and positrons. Quite likely, the central engines in the active galactic nuclei are likewise magnetized rotators and act like giant pulsars (Morrison 1969; Ozernoy & Usov 1977; Kundt 1979, 1982a). If they can generate the observed extremely relativistic $e^{\pm}$ -flows on the innermost scale of $10^{15\pm1}$ cm, there would be no need for any downstream (hot-spot) post-acceleration.

We also know of the existence of magnetized neutron stars in binary systems where the weak but nevertheless heavy wind of a companion star can quench the pulsar mechanism, impinge upon the corotating magnetosphere and be flung out like sparks from a grindstone. When confining the magnetosphere deep inside the speed-of-light cylinder, the falling matter can (in principle) generate voltages of order

$$E_{\text{grind}} \leqslant 2eBr = 2eB_s R^3/r^2 = 10^{20}(B_{13}/r_7^2)\text{eV}, \tag{19}$$

which reach high enough to explain the observed cosmic-ray spectrum. This potential cosmic-ray booster has been recently discussed in (Kundt 1983a). Note that whereas shock acceleration models derive a power-law distribution in energy from a balance between an exponentially increasing particle energy and an exponentially decreasing survival probability with time, the magnetic grindstone can explain a power-law output in energy $\dot{N}_E \, dE \sim E^{-27\pm0.3}\, dE$ as a number ratio of particle orbits traversing different electric potentials: For a distance-dependence $B \sim r^{-n}$, and $\Delta x \sim r$, β = const, Equation (3) implies $E \sim B\,\Delta x \sim r^{1-n}$; hence

$$E\dot{N}_E \sim r^3 \sim E^{-3/(n-1)}, \tag{20}$$

which agrees with the primary cosmic-ray spectrum for $n = 2.8 \pm 0.3$, *i.e.,* for an almost-dipole behaviour of the magnetic field.

A final word concerns SS 433. Its ultraviolet output is difficult to estimate, but both the observed fluxes and the Eddington limit suggest that the central engine radiates less than 10^{39} erg s^{-1}. At an efficiency η of order 1 per cent, this central engine should not be able to produce beams more powerful than some 10^{37} erg s^{-1}. This consideration stands against the hydrogen beam model, in favour of extremely relativistic $e^{\pm}$ –beams (Kundt 1981b).I even maintain that details of the radio spiral of Hjellming &Johnston (1981) and X-ray map of Seward *et al* (1980) can be understood better in terms of the ‘soft beam’ model (hair-drier, Kundt 1982b) than in terms of the ‘hard beam’ model (lawn sprinkler). If so, SS 433 would follow a pattern similar to that of the AGNs.

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