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Collective $E1$ Transitions in Even-Mass Rare-Earth Nuclei (*).

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During the last five years a considerable amount of work has been devoted to the task of describing microscopically low-energy properties in even-mass, deformed (heavy) nuclei, using the pairing plus quadrupole (or octupole) force model (PQF or POF) (1,2). The results obtained in that way can undoubtedly be considered as very encouraging. More recently a more realistic force, the surface delta interaction (SDI) (3), has been succesfully applied to those nuclei (4,5). However, none of the referred papers, with the exception of a rough estimation of the order of magnitude given in ref. (2), has been concerned with the problem of calculating reduced electric-dipole transition probabilities from the octupole vibrational band to the ground state in such nuclei. This might be attributed to the fact that very few experimental results are available. Here we calculate such reduced $E1$ transitions, within the framework of the quasi-particle random-phase approximation (QRPA), for the rare-earth nuclei. Both the SDI and the POF have been utilized.

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It should be pointed out that, in the phenomenological model, the $E1$ strength for the $E1$ transition from the octupole band to the ground-state rotational band is zero in the usual approximation, due to momentum conservation (⁶) (the ratio of the charge and mass density is constant in this case).

The reduced electric multipole from a state $|I_i g\rangle$ in the ground-state rotational band into a vibrational band is given by the expression

$$B(E\lambda;\, I_i g \to I_f K_f) = C^2(I_i \lambda I_f|0KK)\, M^2 \tag{1}$$

with

$$M = \sum_{\mu \geq \nu > 0} (u_\mu v_\nu + u_\nu v_\mu)(e_p + e_{\text{eff}}) \Bigg[Q^{\lambda K}_{\mu\nu}(1-\delta_{\mu,\nu})(\xi_{\mu\nu}+\eta_{\mu\nu}) + \\ + Q^{\lambda K}_{\mu\bar{\nu}} \frac{(1+\delta_{K,0})^{\frac{1}{2}}}{(1+\delta_{\mu,\nu})^{\frac{1}{2}}} (\xi_{\mu\bar{\nu}}+\eta_{\mu\bar{\nu}}) \Bigg] . \tag{2}$$

The u's and v's are the coefficients of the Bogoliubov-Valatin transformation. The charge e_p has the value 1 for protons and 0 for neutrons, and e_{eff} is the effective charge. The bar ($\bar{\nu}$) indicates the time-reversed state. The multipole matrix elements

$$\begin{cases} Q^{\lambda K}_{\mu\nu} = -\langle \mu | r^\lambda\, Y_{\lambda K} | \bar{\nu} \rangle\,, \\ Q^{\lambda K}_{\mu\bar{\nu}} = \langle \mu | r^\lambda Y_{\lambda K} | \nu \rangle \end{cases} \tag{3}$$

and the mixing coefficients $\xi_{\mu\nu}$ and $\eta_{\mu\nu}$ for the two quasi-particles states are given in ref. (³), formulae (6), (7) and (9). (The Q's are called D's in that paper.) The minus sign in formula (3) has been introduced in order to obtain a more symmetric expression (⁴). Due to momentum conservation the effective charge has the value $-Z/A$.

Theoretical values for the reduced electric-dipole transition probabilities from the ground state to the $K=0$ octupole vibrational band in even rare-earth nuclei are given (in single-particle units) in Table I for both the SDI and the POF.

TABLE I. – $B(E1;\, 0^+g \to 1^-)[(\text{S.P.U.})\cdot 10^{-3}]$ for $e_{\text{eff}} = -Z/A$.

Nucleus	Z	SDI	POF	Nucleus	Z	SDI	POF
^{150}Nd	60	9.41	2.32	^{170}Er	68	0.72	2.02
^{152}Sm	62	3.77	0.20	^{168}Yb	70	5.89	0.13
^{154}Sm	62	4.63	0.02	^{172}Yb	70	2.50	0.88
^{154}Gd	64	1.85	0.01	^{174}Yb	70	0.42	1.92
^{156}Gd	64	2.34	0.23	^{176}Yb	70	0.11	3.69
^{158}Gd	64	2.90	0.40	^{174}Hf	72	1.65	1.70
^{160}Gd	64	3.21	0.38	^{176}Hf	72	1.05	1.14
^{158}Dy	66	1.23	0.64	^{178}Hf	72	0.01	3.57
^{160}Dy	66	1.63	0.89	^{180}W	74	0.10	2.78
^{162}Dy	66	1.97	0.76	^{184}W	74	0.03	1.30
^{164}Dy	66	1.56	0.13	^{186}W	74	0.16	0.44
^{164}Er	68	3.40	0.33	^{186}Os	76	0.01	2.03
^{166}Er	68	3.08	0.63	^{188}Os	76	0.01	0.98
^{168}Er	68	1.92	1.48	^{190}Os	76	0.13	0.29

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The only experimental value found in the literature has been measured by METZGER (7) and corresponds to ^{152}Sm, where we have

$$B(E1,\, 1^- \rightarrow 0^+g)\,[\text{S.P.U.}] = [4.0 \pm 0.4]\cdot 10^{-3}\,.$$

It is seen that the value obtained using the SDI is in better agreement with experiment than that yielded by the POF. The estimated value given by VOGEL (2) lies between $4\cdot 10^{-2}$ and $8\cdot 10^{-2}$. However, it should be pointed out that, due to cancellation effects between the proton and neutron contributions, the theoretical $B(E1)$ values are very sensitive to the value of the effective charge. In Table II we give the theoretical results obtained using three different values for e_{eff}, all of them very close to $-Z/A$, for a few rare-earth nuclei. These results illustrate the above-mentioned fact about the sensitivity of the $B(E1)$ values with respect to e_{eff}.

TABLE II. – $B(E1;\, 0^+g \rightarrow 1^-)[(\text{S.P.U.})\cdot 10^{-3}]$.

e_{eff}	Model	Nucleus				
		^{150}Nd	^{152}Sm	^{154}Sm	^{154}Gd	^{156}Gd
-0.35	SDI	31.31	26.61	28.06	26.03	27.43
	POF	18.27	14.34	12.17	13.22	10.98
-0.40	SDI	9.41	5.67	5.32	5.06	4.67
	POF	2.32	0.82	0.09	0.61	0.03
-0.45	SDI	2.90	0.16	0.47	0.36	0.84
	POF	1.51	3.91	8.27	4.31	8.87

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