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Interference Between Collective and Direct Nucleon Radiative Capture.

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Summary. — The interference term between direct and collective dipole radiative capture of nucleons by nuclei is obtained. The cross-section is written in a Breit-Wigner form as a sum of three terms: direct capture, collective or « resonant » capture and interference term. The expression of the latter term is investigated and the conditions for constructive, destructive and zero interference are given. The nucleon capture cross-sections for ^{130}Te, ^{142}Ce and ^{208}Pb are calculated. Taking into account the interference between direct and collective processes, a better agreement between theory and experiment is achieved.

1. – Introduction.

In recent papers (1-3) the radiative capture of nucleons by heavy nuclei in the $(10 \div 50)$ MeV energy range has been discussed in terms of direct and collective mechanisms. According to the « direct » process (1), an incident nucleon in the mean nuclear potential field may emit a photon undergoing a transition to a single-particle bound state. In the « collective » picture (3) the target nucleus may have shape oscillations and an incident nucleon experiences a slightly deformed potential, which can excite collective modes of the target. In the collective capture process the nucleon is scattered into a bound state and the nucleus is excited to its giant dipole state. The latter then decays emitting a γ-ray.

(1) A. M. LANE: *Nucl. Phys.*, **11**, 625 (1959); A. M. LANE and J. E. LYNN: *Nucl. Phys.*, **11**, 646 (1959).

(2) G. E. BROWN: *Nucl. Phys.*, **57**, 339 (1964).

(3) C. F. CLEMENT, A. M. LANE and J. R. ROOK: *Nucl. Phys.*, **66**, 273, (1965); **66**, 293 (1965).

The direct model does not explain the cross-section peak in the $(10 \div 20)$ MeV nucleon-energy range ([4]), while calculations, which take into account the collective process too, give peak values lower than the experimental ones ([3]). As pointed out in ref. ([3]), the interference contribution between the two processes could improve the agreement between theory and experiment.

In this paper an expression of the « interference term » between direct and collective radiative capture of nucleons is obtained. The cross-section is written in a Breit-Wigner form as a sum of three terms: direct capture, collective or « resonant » capture and interference term.

The expression of the interference term is investigated and the conditions for constructive, destructive and zero interference are given. These conditions are illustrated by a proton-capture transition for ^{142}Ce.

The dipole-capture cross-sections for the ^{130}Te(p, γ), ^{142}Ce(p, γ), ^{208}Pb(p, γ) and ^{208}Pb(n, γ) reactions are calculated. As will be shown, the interference between direct and collective capture is destructive below the collective resonance energy and constructive above it. So, the interference raises the total cross-section peak, slightly displacing its position in the direction of higher energies, as required for a better agreement between theory and experiment.

2. – Basic formulation.

Let us consider the interaction of an incident nucleon with a target nucleus of A particles. The total Hamiltonian of the system is

$$H = H_\xi + T(r) + V(r, \xi) \tag{2.1}$$

with H_ξ the Hamiltonian of the nucleus, $T(r)$ the kinetic energy of the incoming particle, and $V(r, \xi)$ the potential of the interaction between the A nuclear particles and the incident nucleon, *i.e.*

$$V(r, \xi) = \sum_{i=1}^{A} V(r, \xi_i)\,. \tag{2.2}$$

The variable r labels the incoming nucleon co-ordinates and ξ the totality of the co-ordinates of the A particles inside the nucleus. The incident and target nucleons are treated as distinguishable.

By $\Phi(r, \xi)$ we indicate the solution of the Schrödinger equation

$$H\Phi^{(\pm)} = E\Phi^{(\pm)} \tag{2.3}$$

([4]) P. J. DALY, J. R. ROOK and P. E. HODGSON: *Nucl. Phys.*, **56**, 331 (1964).

and by Ψ the initial wave function satisfying

$$H_0 \Psi = E\Psi , \tag{2.4}$$

where $H_0 = H_\xi + T(r)$. In future we shall drop the upper $+$ or $-$ suffix for the outgoing or incoming waves.

Then, in order to obtain the connection with the optical model, we introduce the optical complex potential $\tilde{V}$, so that $V = \tilde{V} + H'$, where the « residual potential » H' can be treated as a perturbation. The distorted wave function $\tilde{\Psi}$ obeys the equation

$$\tilde{H}_0 \tilde{\Psi} = E\tilde{\Psi} , \tag{2.5}$$

with $\tilde{H}_0 = H_0 + \tilde{V}$.

The solution of eq. (2.3) satisfies the integral equation

$$\Phi = [1 + GH'] \tilde{\Psi} , \tag{2.6}$$

where G is the Green's operator. Replacing G by $(E - H \pm i\varepsilon)^{-1}$, where the magnitude of ε is arbitrary and, in the final expressions, the limit $\varepsilon \to 0$ is taken, we have

$$\Phi = \left[1 + \frac{1}{E - H \pm i\varepsilon} H'\right] \tilde{\Psi} . \tag{2.7}$$

Consequently the matrix element of an electric transition for the radiative capture process may be written as

$$M_{i\to f} = \langle \Psi_f | \mathscr{H} | \tilde{\Psi}_i \rangle + \sum_\lambda \frac{\langle \Psi_f | \mathscr{H} | \Phi_\lambda \rangle \langle \Phi_\lambda | H' | \tilde{\Psi}_i \rangle}{E - H \pm i\varepsilon} , \tag{2.8}$$

where Φ_λ is a member of a complete set of eigenfunctions of the nuclear Hamiltonian H.

Let us now consider the case of the dipole emission. We split the total dipole operator into the nucleon and the target part

$$\mathscr{H} = \mathscr{H}^{(\mathscr{N})} + \mathscr{H}^{(t)} , \tag{2.9}$$

with $\mathscr{H}^{(\mathscr{N})} \equiv \mathscr{H}(r)$ and $\mathscr{H}^{(t)} = \sum_{i=1}^{A} \mathscr{H}(\xi_i)$. Taking into account the fact that the intermediate giant-dipole state Φ_{int}, of a fairly well defined energy E_r, has a finite decay width Γ, the matrix element can eventually be written as

$$M_{i\to f} = \langle \Psi_f | \mathscr{H}^{(\mathscr{N})} | \tilde{\Psi}_i \rangle + \frac{\langle \Psi_f | \mathscr{H}^{(t)} | \Phi_{\text{int}} \rangle \langle \Phi_{\text{int}} | H' | \tilde{\Psi}_i \rangle}{E - E_r + \frac{1}{2} i\Gamma} , \tag{2.10}$$

where the first and second terms may be interpreted by means of the « direct » [1] and the « collective » [3] mechanisms respectively.

The cross-section for the total dipole capture process has the usual form

$$\sigma_{i\to f} = \frac{16\pi}{9}\,\frac{Mk_\gamma^3}{\hbar^2 k'}\,|M_{i\to f}|^2\,, \tag{2.11}$$

where M is the reduced mass, k' and k_γ are the incident and photon wave numbers respectively.

3. – Cross-section: direct, collective and interference term.

By using the notation of ref. [5], the dipole-capture cross-section for an individual final single particle bound state (l, j) can be written as

$$\sigma_{lj}^{(\text{tot})} = \frac{16\pi}{9}\,\frac{Mk_\gamma^3}{\hbar^2 k'}\sum_{\mu m}\left|\langle\Psi_{\text{fin}}|\mathscr{H}_{1\mu}^{(\mathcal{N})}|\Psi_{\text{in}}\rangle + \frac{\langle\Psi_{\text{fin}}|\mathscr{H}_{1\mu}^{(t)}|\Psi_{\text{int}}\rangle\langle\Psi_{\text{int}}|H'|\Psi_{\text{in}}\rangle}{\varepsilon_{l'j'} - \hbar\omega_1 - \varepsilon_{lj} + i\Gamma/2}\right|^2, \tag{3.1}$$

where $\varepsilon_{l'j'}$ and ε_{lj} are the initial and final nucleon energies and $\hbar\omega_1$ the excitation energy of the dipole state in the target nucleus (A, N, Z).

The initial, intermediate and final states are given respectively by

$$\left\{\begin{aligned} \Psi_{\text{in}} &= \sum_{l'j'} i^{l'}[4\pi(2l'+1)]^{\frac{1}{2}}(l'\,0\,\tfrac{1}{2}\,\tfrac{1}{2}|j'\,\tfrac{1}{2})\,\Phi_{j'm'}(\Omega, \sigma)\,\psi_{l'j'}(r)\,\varphi_{00}\,,\\ \Psi_{\text{int}} &= \Phi_{jm}(\Omega, \sigma)\,u_{lj}(r)\,\varphi_{1-\mu}\,,\\ \Psi_{\text{fin}} &= \Phi_{jm}(\Omega, \sigma)\,u_{lj}(r)\,\varphi_{00}\,, \end{aligned}\right. \tag{3.2}$$

where l', j', m' and l, j, m are the quantum numbers of the initial and final states of the incident nucleon, φ the target functions and Φ the spin-angular wave functions

$$\Phi_{JM} = \sum_{\lambda}(l, M-\lambda, \tfrac{1}{2}, \lambda|J\,M)\,Y_{l,M-\lambda}\,\chi_{\frac{1}{2},\lambda}\,. \tag{3.3}$$

The direct dipole transition operator $\mathscr{H}_{1\mu}^{(\mathcal{N})}$ is given by

$$\mathscr{H}_{1\mu}^{(\mathcal{N})} = e\sum_{i=1}^{Z} r_i Y_{1\mu}(\theta_i\varphi_i)\,, \tag{3.4}$$

the variables θ_i, φ_i, r_i being the spherical co-ordinates of the i-th proton with respect to the centre-of-mass of the target nucleus. The collective dipole radia-

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[5] G. LONGO and F. SAPORETTI: *Nuovo Cimento*, **52** B, 539 (1967).

tion operator and the component of the particle-vibration coupling for the dipole mode which leaves the charge of the particle unchanged are respectively

$$\mathscr{H}^{(t)}_{10} = \text{const} + e\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\frac{NZ}{A}\eta_z\,, \tag{3.5}$$

$$H' = \left(\frac{v_1}{4}\right)2\frac{NZ}{A^2}\delta_a(r-R_0)\frac{\boldsymbol{r}\cdot\boldsymbol{\eta}}{|\boldsymbol{r}|}\tau_3\,, \tag{3.6}$$

where $\boldsymbol{\eta} = \boldsymbol{R}_n - \boldsymbol{R}_p$ is the separation of the centroids of the neutron and proton systems, $\boldsymbol{r}$ the position of the incident particle, v_1 the strength of the isotopic spin term of the optical potential and $\delta_a(r-R_0)$ the « finite-width δ-function » with a equal to the surface thickness and R_0 the nuclear mean radius.

For the matrix element of the direct transition one then obtains

$$\langle\Psi_{\text{fin}}|\mathscr{H}^{(N)}_{1\mu}|\Psi_{\text{in}}\rangle = \frac{e}{A}\sum_{l'j'} i^{l'}(2l'+1)^{\frac{1}{2}}\cdot \\ \cdot(l'\,0\,\tfrac{1}{2}\,\tfrac{1}{2}|j'\,\tfrac{1}{2})(4\pi)^{\frac{1}{2}}\langle\Phi_{jm},\,Y_{1\mu}\Phi_{j'm'}\rangle\,\tilde{e}D_{l'j'}\,, \tag{3.7}$$

where $\tilde{e}$ is equal to N and $-Z$ for an incident proton and neutron respectively, and $D_{l'j'}$ is the direct radial integral expressed as

$$D_{l'j'} = \int u_{lj}(r)\,\psi_{l'j'}(r)\,r^3\,\mathrm{d}r\,. \tag{3.8}$$

Using the expressions (3.5) and (3.6), the collective transition matrix elements are

$$\langle\Psi_{\text{int}}|H'|\Psi_{\text{in}}\rangle = \pm(-)^{\mu}\frac{v_1}{2}\frac{NZ}{A^2}\left(\frac{4\pi}{3}\right)^{\frac{1}{2}}\cdot \\ \cdot\sum_{l'j'} i^{l'}(2l'+1)^{\frac{1}{2}}(l'\,0\,\tfrac{1}{2}\,\tfrac{1}{2}|j'\,\tfrac{1}{2})(4\pi)^{\frac{1}{2}}\langle\Phi_{jm},\,Y_{1\mu}\Phi_{j'm'}\rangle\langle 1|\eta_z|0\rangle C_{l'j'} \tag{3.9}$$

and

$$\langle\Psi_{\text{fin}}|\mathscr{H}^{(t)}_{1\mu}|\Psi_{\text{int}}\rangle = (-)^{\mu}e\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\frac{NZ}{A}\langle 0|\eta_z|1\rangle\,, \tag{3.10}$$

where the upper and lower signs in (3.9) refer to the proton and neutron case, respectively; the « collective » radial integral $C_{l'j'}$ is given by

$$C_{l'j'} = -\int u_{lj}(r)\,\frac{1}{V}\,\frac{\mathrm{d}V(r)}{\mathrm{d}r}\,\psi_{l'j'}(r)\,r^2\,\mathrm{d}r\,, \tag{3.11}$$

with $V(r)$ the real part of the central nuclear potential, while the matrix element $\langle 1|\eta_z|0\rangle$, according to the sum rule, is

$$\langle\eta_z\rangle^2 = \langle 1|\eta_z|0\rangle^2 = \frac{A}{NZ}\frac{\hbar^2}{2M}\frac{1+0.8x}{\hbar\omega_1}, \tag{3.12}$$

with x the exchange force factor.

Inserting in eq. (3.1) the matrix elements (3.7), (3.9), (3.10) and (3.12) and summing over all the bound states, the total cross-section becomes

$$\sigma^{(\text{tot})} = \frac{8\pi}{9A^2}\frac{e^2M}{\hbar^2k'}\sum_{lj j'}(2j'+1)k_\gamma^3 S_{jj'}\cdot \left|\tilde{e}D_{ljj'} \pm \frac{C_{ljj'}}{\varepsilon_{j'}-\hbar\omega_1-\varepsilon_{lj}+i(\Gamma/2)}\frac{v_1}{2}\frac{NZ}{A}\frac{\hbar^2}{2M}\frac{1+0.8x}{\hbar\omega_1}\right|^2, \tag{3.13}$$

where $S_{jj'}$ is the statistical factor

$$S_{jj'} = 4\pi\sum_{\mu m}|\langle\Phi_{jm},\, Y_{1\mu}\Phi_{j'm'}\rangle|^2,$$

for which tables ([6]) are available.

Squaring the first and the second terms separately, we obtain respectively the expressions for the direct and the collective capture cross-sections given in ref. ([5]).

The mixed term gives the interference between direct and collective capture

$$\sigma^{(\text{int})} = \frac{8\pi}{9A^2}\frac{e^2M}{\hbar^2k'}\sum_{ljj'}(2j'+1)k_\gamma^3 S_{jj'}\cdot 2|\tilde{e}|\left(\frac{v_1}{\Gamma}\frac{NZ}{A}\frac{\hbar^2}{2M}\frac{1+0.8x}{\hbar\omega_1}\right)\mathrm{Re}\left(\frac{D^*_{ljj'}C_{ljj'}}{X+i}\right), \tag{3.14}$$

where the « energy » X (in units of $\frac{1}{2}\Gamma$) is given by

$$X = \frac{\varepsilon_{j'}-\hbar\omega_1-\varepsilon_{lj}}{\frac{1}{2}\Gamma}. \tag{3.15}$$

The total cross-section can now be written in a more compact Breit-Wigner form as

$$\sigma_{(\text{tot})} = \frac{\pi}{k'^2}\sum_{ljj'}(2j'+1)\cdot\left\{a^2 + 2a\frac{(\Gamma_\gamma\Gamma_{\text{in}})^{\frac{1}{2}}}{\Gamma}\frac{X\cos(\alpha-\beta)-\sin(\alpha-\beta)}{(1+X^2)} + \frac{\Gamma_\gamma\Gamma_{\text{in}}}{\Gamma^2(1+X^2)}\right\}, \tag{3.16}$$

([6]) See for example: S. A. MOSZKOWSKI: *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. SIEGBAHN (Amsterdam, 1966), p. 879.

by introducing the « resonance widths »

$$(3.17)\quad \begin{cases} \Gamma_\gamma = \dfrac{16}{9}\pi k_\gamma^3 \langle \Phi_{jm} u_{lj}\varphi_{00} | \mathscr{H}^{(t)}_{1\mu} | \Phi_{jm} u_{lj} \varphi_{1-\mu}\rangle^2 = \dfrac{4}{3}\dfrac{NZ}{A} e^2 k_\gamma^3 \dfrac{\hbar^2}{2M}\dfrac{1+0.8x}{\hbar\omega_1}\,, \\[2ex] \Gamma_{\text{in}} = 4\dfrac{Mk'}{\hbar^2}\sum_{\mu m} |\langle \Phi_{jm} u_{lj}\varphi_{1-\mu}|H'|\Phi_{j'm'}\psi_{l'j'}\varphi_{00}\rangle|^2 = \\ \qquad\qquad \dfrac{2}{3}\dfrac{Mk'}{\hbar^2}\dfrac{NZ}{A^3} v_1^2 \dfrac{\hbar^2}{2M}\dfrac{1+0.8x}{\hbar\omega_1} S_{jj'}|C_{ljj'}|^2\,, \end{cases}$$

and by introducing the modulus a of the direct capture amplitude, defined by

$$(3.18)\quad a^2 = \frac{8}{9}\frac{Mk'}{\hbar^2}\left(\frac{\tilde{e}}{A}\right)^2 e^2 k_\gamma^3 S_{jj'}|D_{ljj'}|^2\,,$$

and the phases α and β

$$(3.19)\quad \begin{cases} \alpha = \operatorname{arctg}\dfrac{\operatorname{Im}(D_{ljj'})}{\operatorname{Re}(D_{ljj'})}\,, \\[2ex] \beta = \operatorname{arctg}\dfrac{\operatorname{Im}(C_{ljj'})}{\operatorname{Re}(C_{ljj'})}\,. \end{cases}$$

The first, second and third terms of the eq. (3.16) represent the direct capture, the interference term and the collective or « resonant » capture respectively.

4. – The interference term.

4·1. *Conditions for constructive, destructive and zero interference* – In order to study the influence of the interference term on the cross-section, let us consider a single transition from a given initial state $(l'j')$ to an individual-particle bound state (lj). For such a transition the cross-section (3.16) may be expressed as

$$(4.1)\quad \sigma_{l'j'\to lj} = \frac{\pi}{k'^2}(2j'+1)|\mathscr{A}_d + \mathscr{A}_c|^2\,,$$

where $\mathscr{A}_d$ and $\mathscr{A}_c$ are the direct and collective capture amplitudes given by

$$(4.2)\quad \begin{cases} \mathscr{A}_d = a\exp[i\alpha]\,, \\[1ex] \mathscr{A}_c = \dfrac{(\Gamma_\gamma\Gamma_{\text{in}})^{\frac{1}{2}}}{\Gamma}\cos\theta\exp\left[i\left(\beta+\theta-\dfrac{\pi}{2}\right)\right]\,, \end{cases}$$

with

$$(4.3)\quad \theta = \operatorname{arctg} X\,.$$

The sign of the interference term is determined by the phase-shift δ

$$\delta = \alpha - \beta - \theta + \frac{\pi}{2}\,. \tag{4.4}$$

The interference vanishes for $\delta = (n \pm \frac{1}{2})\,\pi$ $(n = 0, \pm 1, \pm 2 \ldots)$ or

$$X_0 = \operatorname{tg}(\alpha - \beta) \tag{4.5}$$

and is entirely constructive or destructive for $\delta = n\pi$ or

$$\tilde{X} = \operatorname{ctg}(\beta - \alpha)\,. \tag{4.6}$$

The values of X_0 and $\tilde{X}$ can be found by calculating the expression

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{Im}(D_{ljj'})\cdot\operatorname{Re}(C_{ljj'}) - \operatorname{Re}(D_{ljj'})\operatorname{Im}(C_{ljj'})}{\operatorname{Re}(D_{ljj'})\cdot\operatorname{Re}(C_{ljj'}) + \operatorname{Im}(D_{ljj'})\operatorname{Im}(C_{ljj'})}\,. \tag{4.7}$$

The eq. (4.5) gives the exact position of the zero-values of the interference term, while (4.6) obviously gives only the approximate position of the interference peaks because the energy-dependence of the amplitude moduli influences such a position.

The interference is constructive for $-\pi/2 < \delta < \pi/2$ and is destructive for $\pi/2 < \delta < 3\pi/2$. For incident nucleon energies higher than the resonance energy $(X > 0)$, it is $0 < \theta < \pi/2$. Therefore in this energy region the interference is always destructive for $\alpha - \beta$ in the second quadrant and constructive for $\alpha - \beta$ in the fourth quadrant.

Below the resonance energy $(X < 0)$, the θ-value can vary in the range $-\pi/2 + \Delta\theta < \theta < 0$, where $\Delta\theta = (\pi/2) - \operatorname{arctg}[2(\hbar\omega_1 + \varepsilon_{l'j'})/\Gamma]$. Therefore the interference is always destructive or constructive for $0 < \alpha - \beta < (\pi/2) + \Delta\theta$ and $\pi < \alpha - \beta < (3\pi/2) + \Delta\theta$ respectively. For typical dipole state and bound state parameters, it is $\Delta\theta \simeq (5 \div 10)°$. The above conclusions are sumarized in Table I.

In the cases not considered in Table I (for example, $X > 0$ and $0 < \alpha - \beta < \pi/2$) the interference may be either constructive or destructive.

TABLE I.

	Interference	
	Always destructive	Always constructive
For $X < 0$	$0 < \alpha - \beta < (\pi/2) + \Delta\theta$	$\pi < \alpha - \beta < (3\pi/2) + \Delta\theta$
For $X > 0$	$\pi/2 < \alpha - \beta < \pi$	$3\pi/2 < \alpha - \beta < 2\pi$

Thus the sign of the single-transition interference term may be determined from the conditions listed in Table I and those given by eqs. (4.5) and (4.6), by calculating the expression (4.7).

4'2. *Typical behaviour of the interference term.* – We have studied the interference term for the transitions of the ^{130}Te(p, γ), ^{142}Ce(p, γ), ^{208}Pb(p, γ) and ^{208}Pb(n, γ) reactions. The set of parameters used in the calculations is listed in Sect. **5**.

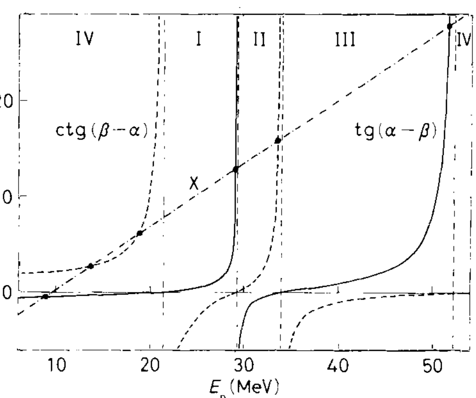

Fig. 1. – The calculated curves $\mathrm{tg}(\alpha-\beta)$ and $\mathrm{ctg}(\beta-\alpha)$ for the $f_{\frac{7}{2}}\to d_{\frac{5}{2}}$ transition of the ^{142}Ce(p, γ) reaction *vs.* the proton energy. The intersections of these curves with the straight line $X=\mathrm{tg}\,\theta$ give three points of zero-interference and three of entirely constructive or destructive interference. The quadrants for $\alpha-\beta$ are indicated by Roman numbers.

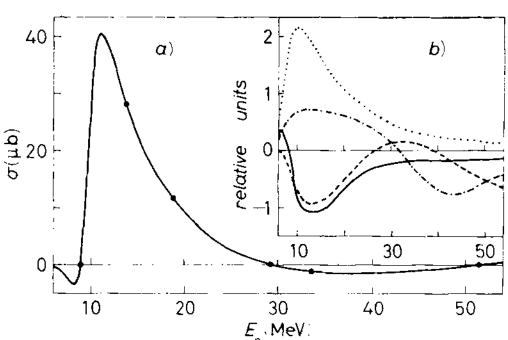

Fig. 2. – *a*) The calculated interference term for the $f_{\frac{7}{2}}\to d_{\frac{5}{2}}$ transition of the ^{142}Ce (p, γ) reaction *vs.* the proton energy. *b*) The imaginary and real part of the direct and collective integrals $D_{ljj'}$ and $C_{ljj'}$ for the same transition.

The typical behaviour of a single transition interference term is illustrated by the example of the $f_{\frac{7}{2}}\to d_{\frac{5}{2}}$ transition for the proton capture by ^{142}Ce. The values of $\mathrm{tg}(\alpha-\beta)$ and $\mathrm{ctg}(\beta-\alpha)$, calculated by using the expression (4.7) together with those of X *vs.* proton energy are shown in Fig. 1. The intersection of the curve $\mathrm{tg}(\alpha-\beta)$ and X gives the zero values of the interference term. The latter have been found at 8.8, 29.1 and 51.5 MeV proton energy. The condition (4.6) for entirely constructive or destructive interference occurs at 13.7, 18.7 and 33.5 MeV energy, where X and $\mathrm{ctg}(\beta-\alpha)$ cross.

The conditions listed in Table I imply that the interference is constructive in the energy interval $(9.6\div21.4)$ MeV. The first zero-interference points external to this interval occur at 8.8 and 29.1 MeV, so that the interference remains constructive between these energies. In this region the condition (4.6) gives entirely constructive interference at 13.7 and 18.7 MeV. Before the zero-point at 8.8 MeV and after the one at 29.1 MeV, the interference becomes destructive, as it must for energies between 29.3 and 33.9 MeV (see Table I). In this region a point of entirely destructive interference occurs at 33.5 MeV. Finally, for energies higher than 51.5 MeV there is a new positive peak.

The contribution to the capture cross-section due to the interference term for the $f_{\frac{7}{2}} \to d_{\frac{5}{2}}$ transition of the ^{142}Ce(p, γ) reaction has been calculated from eq. (3.16) and is shown in Fig. 2*a*). It confirms the predictions derived from Fig. 1 regarding the position of the zero points and the regions of constructive or destructive interference. It can be seen that, though the interference is entirely constructive or destructive at 13.7, 18.7 and 33.5 MeV, the maximum and minimum values are displaced with respect to these energies. This is due to the variation with energy of the absolute value of the direct and collective capture amplitudes, as shown in Fig. 2*b*), where the real and imaginary parts of the direct and collective integrals $D_{ljj'}$ and $C_{ljj'}$ for the transition studied are drawn.

The point $X = 0$ corresponds to the collective capture-resonance energy. The examination of Fig. 1, confirmed by Fig. 2*a*), indicates the presence of a negative interference below $X \approx 0$ and a positive one above this X-value. Therefore the position of the peak for the total cross-section of the $f_{\frac{7}{2}} \to d_{\frac{5}{2}}$ proton transition is slightly displaced with respect to the collective peak in the direction of higher energies.

The behaviour of the other single transition interference terms for proton capture by ^{130}Te, ^{142}Ce and ^{208}Pb is similar to the one described above. For a given final state (*e.g.*, the proton $2f_{\frac{7}{2}}$ bound state of ^{208}Pb), the function $(\alpha - \beta)$ increases with energy more rapidly for a transition from an initial state with higher momenta $(l'j')$. This gives intersection points on the curve $\operatorname{tg}(\alpha - \beta)$ which are nearer one another. So, *e.g.*, for the $g_{\frac{9}{2}} \to f_{\frac{7}{2}}$, $g_{\frac{7}{2}} \to f_{\frac{7}{2}}$, and $d_{\frac{5}{2}} \to f_{\frac{7}{2}}$ proton capture transitions in Pb the constructive interference region extends from about $X \approx 0$ up to the energy of 31.4, 33.5 and 41 MeV respectively. Similar considerations are feasible for the transitions from an initial state $(l'j')$ fixed with respect to the corresponding final state (lj) (*e.g.* the transitions to each bound state from the initial state $l' = l + 1$ and $j' = j + 1$). In this case, the function $(\alpha - \beta)$ increases with energy more rapidly for the transitions to bound states lower in the succession given by shell-model calculations (⁷) (*e.g.*, for the tellurium proton transitions of the sequence $i_{\frac{13}{2}} \to h_{\frac{11}{2}}$, $p_{\frac{3}{2}} \to s_{\frac{1}{2}}$, $f_{\frac{5}{2}} \to d_{\frac{3}{2}}$, $f_{\frac{7}{2}} \to d_{\frac{5}{2}}$, $h_{\frac{9}{2}} \to g_{\frac{7}{2}}$).

The above-described behaviour of the proton transition interference term is such that it will cause an enhancement and a slight displacement of the total cross-section peak in the direction of higher energies.

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(7) M. GOEPPERT MAYER, J. HANS and D. JENSEN: *Elementary Theory of Nuclear Shell Structure* (New York, London, 1957), p. 58.

5. – Calculations.

The dipole capture cross-sections of ^{130}Te(p, γ), ^{142}Ce(p, γ), ^{208}Pb(p, γ) and ^{208}Pb(n, γ) reactions are calculated by using the formula (3.16), which includes the interference term. For capture by ^{208}Pb, more recent experimental bound states and an adjusted programme are used with respect to the ones of ref. (5). In the present work the calculations for proton capture by ^{142}Ce differ from those of ref. (3,4) in that the spin-orbit interaction is also included.

5·1. *Radial part of the bound state wave functions.* – According to the model, $u_{lj}(r)$ is the radial part of the final particle wave function, normalized by

$$\int_0^\infty [u_{lj}(r)]^2 r^2 \,\mathrm{d}r = 1\,,$$

and satisfying the Schrödinger equation. The potential for the bound-state functions is assumed to be of the Woods-Saxon form containing a spin-orbit term of the Thomas type as well as the Coulomb potential of a uniformly charged sphere. The potential chosen is

$$U(r) = -V^{(\substack{\mathrm{p}\\ \mathrm{n}})} f(r) - V_s h(r)\boldsymbol{\sigma}\cdot\boldsymbol{l} + \begin{cases} U_c(r) & \text{(for protons)} \\ 0 & \text{(for neutrons)}\,, \end{cases}$$

where the spin-orbit strength V_s and the Coulomb potential $U_c(r)$ are given by

$$V_s = \lambda \left(\frac{m_\pi}{2M}\right)^2 V^{(\substack{\mathrm{n}\\ \mathrm{p}})}\,,$$

$$U_c(r) = \begin{cases} \dfrac{Ze^2}{r} & r > R \\[2mm] \dfrac{Ze^2}{2R}\left(3 - \dfrac{r^2}{R^2}\right) & r < R \end{cases}\,,$$

and the form factors are $f(r) = \left[1 + \exp[r - r_0 A^{\frac{1}{3}}]/a\right]^{-1}$ and $h(r) = -\lambda_\pi^2 \,\mathrm{d}f(r)/r\,\mathrm{d}r$. The Coulomb radius R is taken as being equal to the nuclear one.

This potential contains four parameters: the nuclear potential depth $V^{(\substack{\mathrm{p}\\ \mathrm{n}})}$ for protons and neutrons, the radius parameter r_0, the diffuseness a and the positive dimensionless parameter λ.

For the nucleon capture by ^{208}Pb, the set of bound-state parameters is $\lambda = 33$, $a = 0.67$ fm and $r_0 = 1.30$ fm. The potential depth $V^{(\substack{\mathrm{p}\\ \mathrm{n}})}$ for each bound state is adjusted to give the experimentally known binding energy [8,9]. The final bound states taken into account in the calculations are $3d_{\frac{3}{2}}(1.40)$, $2g_{\frac{7}{2}}(1.47)$, $4s_{\frac{1}{2}}(1.91)$, $3d_{\frac{5}{2}}(2.38)$, $1j_{\frac{15}{2}}(2.53)$, $1i_{\frac{11}{2}}(3.15)$, $2g_{\frac{9}{2}}(3.94)$ for neutron capture and $3p_{\frac{3}{2}}(0.66)$, $2f_{\frac{5}{2}}(0.96)$, $1i_{\frac{13}{2}}(2.19)$, $2f_{\frac{7}{2}}(2.9)$, $1h_{\frac{9}{2}}(3.8)$ for proton capture; the binding energy in MeV is given in brackets.

For the proton capture by ^{142}Ce the $2d_{\frac{5}{2}}$, $3s_{\frac{1}{2}}$, $2d_{\frac{3}{2}}$, $1h_{\frac{11}{2}}$ final bound states are considered as in ref. [3,4]. In this case the experimental binding energies are not well known. Therefore the eigenfunctions are calculated by using the bound states parameters $V^{(p)} = 56$ MeV, $\lambda = 33$, $a = 0.67$ fm, $r_0 = 1.27$ fm. The same set of parameters is used in the case of the ^{130}Te(p, γ) reaction for the $1g_{\frac{7}{2}}$, $2d_{\frac{5}{2}}$, $3s_{\frac{1}{2}}$, $2d_{\frac{3}{2}}$ and $1h_{\frac{11}{2}}$ final states.

5'2. *Radial part of the continuum-state wave functions.* – The radial part $\psi_{l'j'}(r)$ of the incident wave function obeys the Schrödinger equation, however the potential experienced by the incident particle has now an absorption term too, expressed as $-iW^{(\mathrm{p}/\mathrm{n})}g(r)$. We assume this imaginary part of the potential to be peaked at the surface in the form

$$g(r) = -4b\,\mathrm{d}\big[1 + \exp[r - r_0 A^{\frac{1}{3}}]/b\big]^{-1}/\mathrm{d}r\,,$$

with b the width parameter. The optical potential is now defined by the six optical parameters $V^{(\mathrm{p}/\mathrm{n})}$, $W^{(\mathrm{p}/\mathrm{n})}$, V_s, a, b, r_0.

The Rosen *et al.* optical parameters [10], which describe many experimental data, are chosen to calculate the free functions of the incident particle, *i.e.*

$$V^{(\mathrm{p}/\mathrm{n})}(\mathrm{MeV}) = \left\{\begin{matrix} 53.8 \\ 49.3 \end{matrix}\right. - 0.33E\,, \qquad W^{(\mathrm{p}/\mathrm{n})}(\mathrm{MeV}) = \left\{\begin{matrix} 7.5 \\ 5.75 \end{matrix}\right.\,, \qquad V_s(\mathrm{MeV}) = 5.5\,,$$

$$r_0(\mathrm{fm}) = 1.25\,, \qquad a(\mathrm{fm}) = 0.65\,, \qquad b(\mathrm{fm}) = 0.70\,.$$

The strength v_1 of the symmetry term of the nuclear optical potential is put equal to 160 MeV as in ref. [3]. The excitation energy $\hbar\omega_1$ and the width Γ of the dipole state are taken as $\hbar\omega_1 = 15$ MeV, $\Gamma = 3$ MeV for tellurium and cerium (see ref. [3]), and $\hbar\omega_1 = 14.5$ MeV, $\Gamma = 2.5$ MeV for lead. The papers of ref. [11] have been taken as a guide in choosing these values. Since the cal-

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(8) *Nuclear Data Sheets*, Part. 11 (New York, 1966).

(9) J. S. Lilley and N. Stein: *Phys. Rev. Lett.*, **19**, 709 (1967).

(10) L. Rosen, J. G. Beery, A. S. Goldhaber and E. M. Auerbach: *Ann. of Phys.* **34**, 96 (1965); F. P. Agee and L. Rosen: LA-3538-MS (1966).

(11) B. I. Goryachev: *Atomic Energy Rev.*, **2**, No. 3, 71 (1964); P. Oliva and D. Prosperi: *Nuovo Cimento*, **49** B, 161 (1967).

culated collective cross-section is a sum of several single transition terms, the resulting width is larger than the Γ-value used in the calculations. Taking into account this effect, the $\hbar\omega_1$- and Γ-values can be considered as being in agreement with the giant-resonance data of the reference mentioned. As usual the exchange form factor x has been put equal to 0.5.

6. – Results.

In Figs. 3 and 4 a comparison between calculated and experimental cross-sections [12,13] for proton radiative capture by ^{142}Ce and ^{130}Te is given. To show the weight of the different terms, the total, direct, collective and inter-

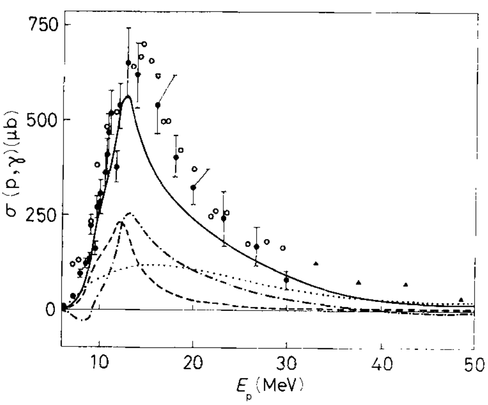

Fig. 3. – The ^{142}Ce(p, γ) experimental and calculated cross-sections *vs.* the proton energy. The experimental points are: closed circles from ref. [13], open circles and triangles from ref. [12]. ——— total capture, — — — collective capture, · · · · · direct capture, —·—·— interference term.

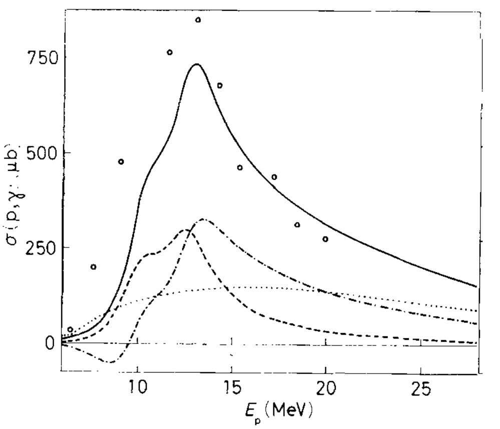

Fig. 4. – The ^{130}Te(p, γ) experimental and calculated cross-sections *vs.* the proton energy. The experimental data [12] are indicated by open circles. ——— total capture, — — — collective capture, · · · · · direct capture, —·—·— interference term.

ference curves are drawn separately. The interference term affects the calculated cross-section increasing the peak value and slightly displacing the position of the peak towards the experimental one.

The calculated cross-sections for proton capture by ^{208}Pb are plotted in Fig. 5, together with the experimental points of the proton capture by ^{209}Bi.

([12]) P. J. Daly and P. F. D. Shaw: *Nucl. Phys.*, **56**, 322 (1964).
([13]) E. V. Verdieck and J. M. Miller: *Phys. Rev.*, **153**, 1253 (1967).

As can be seen, the experimental cross-section values and their trend can be obtained.

The calculated curves and experimental cross-sections of the ^{208}Pb(n, γ) reaction are shown in Fig. 6. In the energy range considered, only the meas-

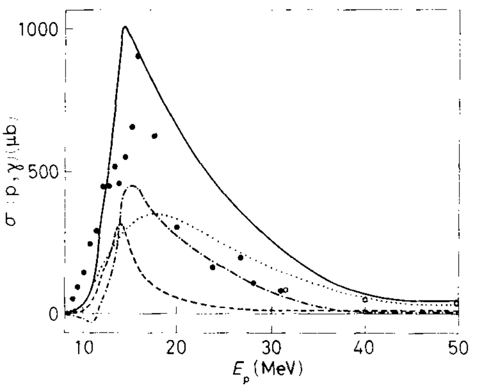

Fig. 5. – The ^{208}Pb(p, γ) calculated cross-sections *vs.* the proton energy. Closed and open circles are the experimental data for proton capture by ^{209}Bi quoted in ref. ([14]) and ([12]) respectively. ——— total capture, – – – – collective capture, · · · · · direct capture, —·—·— interference term.

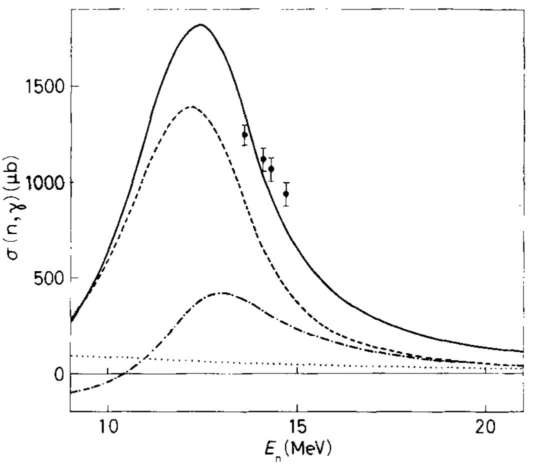

Fig. 6. – The ^{208}Pb(n, γ) experimental and calculated cross-sections *vs.* the proton energy. The points are the experimental data quoted in ref. ([15]). ——— total capture, – – – collective capture, · · · · · direct capture, —·—·— interference term.

ured values in the (13.4 ÷ 15.0) MeV interval are available ([15]). Here, the theoretical predictions give satisfactory results.

So, the calculations show that the contribution of the interference between direct and collective capture improves the agreement between theory and experiment.

* * *

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([14]) E. L. Kelly: Report UCRL-1044 (1950), Table IV.

([15]) J. Csikai, G. Petö, M. Buczkó, Z. Miligy and N. A. Eissa: *Nucl. Phys.*, **A 95**, 229 (1967).

RIASSUNTO

Si ricava l'espressione del termine di interferenza fra la cattura radiativa dipolare diretta e collettiva da parte dei nuclei. La sezione d'urto è scritta nella forma di Breit-Wigner come somma di tre termini: cattura diretta, cattura collettiva o di « risonanza » e termine di interferenza. Si studia l'espressione di questo ultimo termine e si danno le condizioni di interferenza costruttiva, distruttiva e nulla. Si calcolano le sezioni d'urto di cattura di nucleoni per il ^{130}Te, il ^{142}Ce ed il ^{208}Pb. Tenendo conto dell'interferenza fra i processi diretto e collettivo si ottiene un migliore accordo fra teoria ed esperimento.

Интерференция между коллективным и прямым радиационным захватом нуклонов.

Резюме (*). — Выводится интерференционный член между прямым и коллективным дипольным радиационным захватом нуклонов ядрами. Поперечное сечение записывается в форме Брайта-Вигнера, как сумма трех членов: прямого захвата, коллективного или « резонансного » захвата и интерференционного члена. Исследуется выражение для последнего члена, и приводятся условия для положительной, отрицательной и нулевой интерференции. Вычисляются поперечные сечения захвата нуклонов для ^{130}Te, ^{142}Ce и ^{208}Pb. Учитывая интерференцию между прямым и коллективными процессами, получается лучшее согласие между теорией и экспериментом.

(*) *Переведено редакцией.*