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The isospin admixture of the ground state and the properties of the isobar analog resonances in medium and heavy mass nuclei

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Abstract. Within the framework of quasiparticle random phase approximation (QRPA), Pyatov–Salamov method [23] for the self-consistent determination of the isovector effective interaction strength parameter, restoring a broken isotopic symmetry for the nuclear part of the Hamiltonian, is used. The isospin admixtures in the ground state of the parent nucleus, and the isospin structure of the isobar analog resonance (IAR) state were investigated with the inclusion of the pairing correlations between nucleons for the medium and heavy mass regions: $80 < A < 90$, $102 < A < 124$, and $204 < A < 214$. It was determined that the influence of the pairing interaction between nucleons on the isospin admixtures in the ground state and the isospin structure of the IAR state is more pronounced for the light isotopes ($N \approx Z$) of the investigated nuclei.

Keywords. Shell model; Hartree–Fock and random-phase approximation; beta decay.

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1. Introduction

With the development of the heavy mass ion accelerator technology, many proton-rich isotopes within the mass region of $A = 80$–100 have been established. The investigations on these isotopes are very useful in understanding many nuclear aspects such as the isospin mixing effects of the nuclear states which are very important in estimating the effective vector coupling constants based on Fermi transitions, and in the description of the energies and the widths of the analog states and the isospin multiplets [1–4].

The isospin mixing is basically caused by the Coulomb potential. Since the very small symmetry energy tries to minimize the difference in the proton and the neutron systems, Coulomb forces are more dominant in nuclei for which the proton

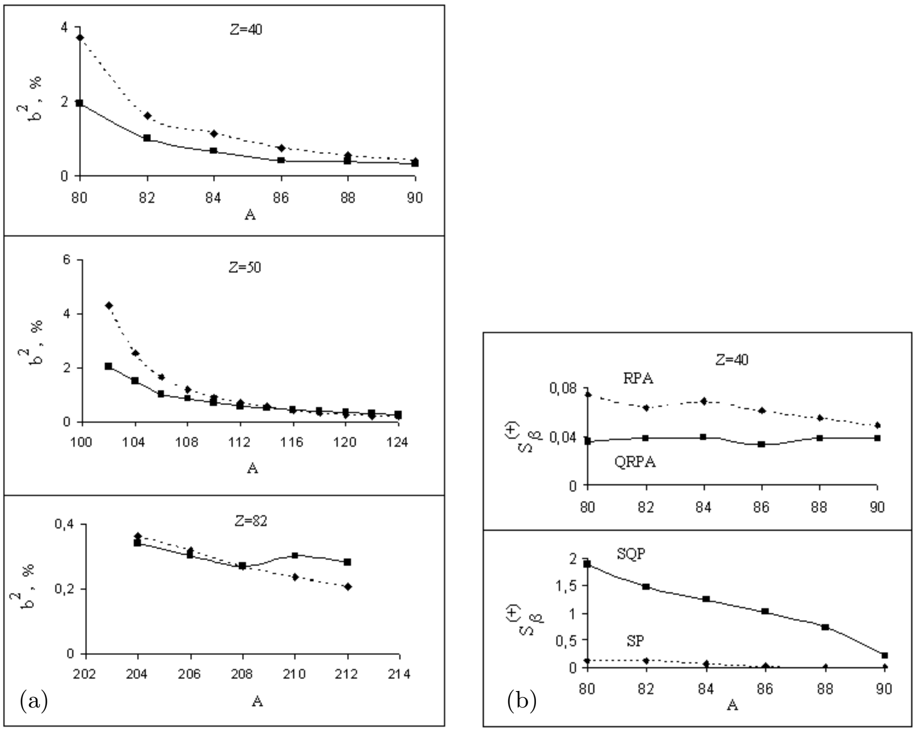

Figure 1. (**a**) The variation of the isospin admixture (b^2%) in the ground state of the parent nucleus with respect to the mass number A for the $Z = 40$, 50, 82 nuclei. (**b**) The total β^+ transition strength for $Z = 40$ nuclei within the SP, SQP, RPA and QRPA formalisms.

and the neutron numbers are close to each other. Therefore, the isospin mixing values will be large in the ground state of the proton-rich nuclei.

The isospin admixtures in the nuclear ground states have been investigated in different theoretical and experimental studies [5–23], but we do not want to go into the details of these studies in this article since they are given in our previous studies [24,25]. One of the most striking features in the theoretical studies is that the pairing correlations between nucleons are not considered. This shortcoming has given us the physical motivation to our previous study in which we analyzed the effect of the pairing interaction on the energies of the isobar analog resonances in $^{112-124}$Sb and the isospin admixtures in the ground states of $^{100-124}$Sn isotopes. In the present study, based on Pyatov–Salamov method, the calculations for the isospin structure of the IAR state and isospin admixtures in the ground state of the parent nucleus within the mass regions of $80 < A < 90$, $102 < A < 124$ and $204 < A < 214$ have been done by considering the pairing correlations between nucleons. The details of the corresponding formalism are given in ref. [24].

2. Results and discussion

The numerical calculations were done using eqs (26) and (29) of ref. [24]. In these calculations, the Woods–Saxon potential with Chepurnov parametrization [26] has been used. For open-shell nuclei, the BCS equations have been solved for a given value of $C_{\mathrm{n}} = C_{\mathrm{p}} = 12/\sqrt{A}$ ($C_{\mathrm{p}} = 0$ for $Z = 50$ and $Z = 82$ isotopes) [5]. The Ikeda sum rule is fulfilled with ≈1% accuracy. The calculation results for the $T_0 + 1$ isospin admixture (b^2%) in the ground state of the parent nucleus for different isotopes with $Z = 40$, 50, 82 have been depicted in figure 1a. In all figures, the solid and dashed lines correspond respectively to the cases with and without the pairing correlations between nucleons. Through these calculations, it has been observed that the pairing forces make their presence felt much more in the light isotopes ($N \approx Z$) of the investigated nuclei, and the isospin admixture values generally decrease as the mass number increases. This decline may stem from both the T_0 isospin and the total β^+ transition strength $S_\beta^{(+)}$ (see eq. (26)). It can be seen obviously from eq. (26) that the T_0 isospin in the denominator will certainly decrease the $T_0 + 1$ isospin admixture because the increase in mass number implies the increase in the T_0 isospin value. However, the analysis of the total β^+ transition strength values will be very beneficial in both clarifying the relationship between the $S_\beta^{(+)}$ values and the $T_0 + 1$ isospin admixture, and investigating the influence of the pairing correlations between nucleons on the total β^+ transition strength. For this purpose, they have been calculated for different isotopes with $Z = 40$, 50, 82 within four different models: single particle (SP), single quasiparticle (SQP), random phase approximation (RPA), and quasiparticle random phase approximation (QRPA). The results have been presented in figures 1b and 2. It is obvious that the calculated values for the total β^+ transition strength in the SQP formalism are quite larger than the corresponding SP ones. This is an expected result because the β^+ transitions in the isotopes with $N \geq Z$ allow only the proton–neutron transitions with $\Delta n \neq 0$ in case of no pairing forces. However, the inclusion of the pairing interaction will change the situation such that both the p–n transitions with $\Delta n = 0$ and $\Delta n \neq 0$ will be possible. Thus, the number of the allowed states which make β^+ transitions and in turn the total β^+ transition strength will considerably increase.

We now investigate the general tendency of the $S_\beta^{(+)}$ values in the RPA and QRPA formalisms. It can be easily seen from these figures that the $S_\beta^{(+)}$ values in the QRPA method remain constant with some fluctuations although the corresponding RPA values decrease as the mass number increases. This means that the decrease in the QRPA value for the $T_0 + 1$ isospin admixture (the curve shown in figure 1a with the solid line) can only result from the isospin term in the denominator of eq. (26) of ref. [24].

Let us note that the isospin admixtures in the RPA and QRPA formalisms have been completely refined from the spurious mixing although it is not possible to separate this mixing from the real one in the SP and SQP formalisms due to the violation of the isotopic invariance for the nuclear part of the H_{SP} and H_{SQP} Hamiltonians. This can be easily checked by keeping the Coulomb potential as constant in the calculations. In this case, the magnitude of the isospin admixture is certainly

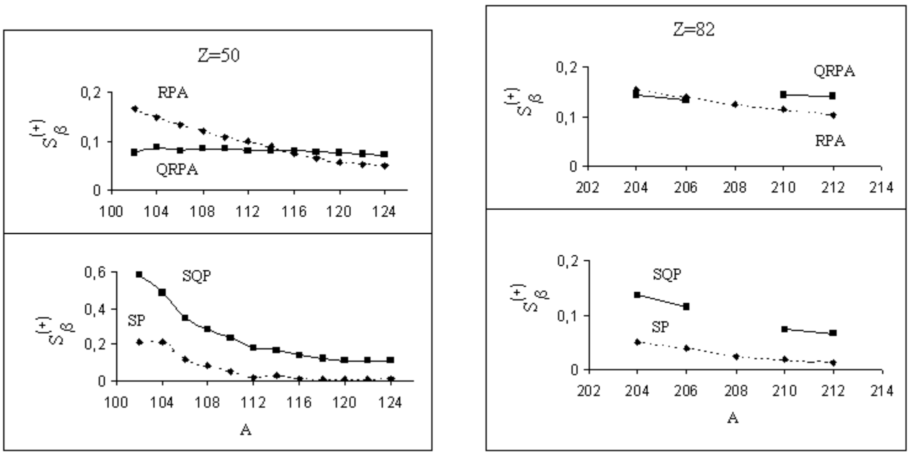

Figure 2. The total β^+ transition strength for $Z = 50$ nuclei (left) and $Z = 82$ (right) nuclei within the SP, SQP, RPA and QRPA formalisms.

Table 1. The comparison of our results for b^2 with the other calculations.

Nucleus	HF calculations [13]	From eq. (11) of ref. [13]	This study
^{80}Zr	3.90	2.21	1.95
^{100}Sn	4.25	4.07	4.40
^{208}Pb	≤0.7	0.30	0.51

zero. This is the indication of how Pyatov–Salamov method is a valid method for the restoration of the violated isotopic invariance of the nuclear part in the Hamiltonian. The comparison of our results for the $T_0 + 1$ isospin admixture with the other calculations has been presented in table 1. As seen from this table, the b^2 values for the ^{80}Zr, ^{100}Sn, and ^{208}Pb isotopes are close to the other model results.

The investigation of the isospin structure of the IAR state among the isobaric 0^+ states are very important. The important contribution to the IAR state comes from the states having the $T_0 - 1$, T_0 and $T_0 + 1$ isospins. The contribution of the T_0 isospin state to the IAR state ($\alpha^2, \%$) has been shown in figure 3 (left). This contribution changes to 90–98% in all other nuclei, except for the ^{82,84}Nb and ^{102,104}Sb isotopes. The α^2 values for these isotopes decrease to 60% (for ^{82}Nb and ^{102}Sb isotopes) and 85% (for ^{84}Nb and ^{104}Sb isotopes) when the pairing correlations between nucleons are considered. This can happen due to the division of the IAR state when the IVMR and IAR states approach each other by an amount of the proton–neutron chemical potential difference ($\lambda_n - \lambda_p$). It is obvious that the pairing correlations considerably influence the α^2 values in the light isotopes ($N \approx Z$) of the investigated nuclei. However, the α^2 values generally increase as the mass number increases. This result confirms the known fact that the isospin is better conserved in the heavy mass isotopes.

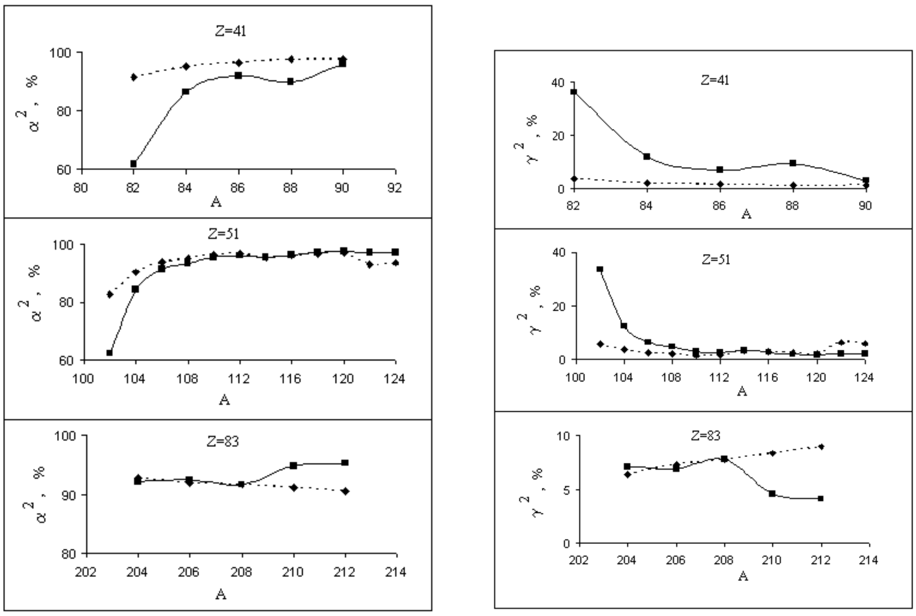

Figure 3. The variation of α^2 (the contribution of the T_0 isospin state to the IAR) (left) and γ^2 (the contribution of the $T_0 - 1$ isospin state to the IAR) (right) with respect to the mass number A.

The contribution of the $T_0 - 1$ isospin state to the IAR state ($\gamma^2, \%$) has been plotted in figure 3 (right). The QRPA calculations for the γ^2 values (solid line) show that the general tendency is such that the γ^2 values decrease as the mass number increases. However, the influence of the pairing correlations on this quantity is not the same in different mass regions. The pairing correlations generally increase the γ^2 values. This increase is more pronounced in the light mass region. For example, the γ^2 values for the ^{82}Nb and ^{102}Sb isotopes increase from 2 to 36%. The difference between the γ^2 values obtained for two different cases is getting smaller as the mass number increases. Even the γ^2 values in the case of the pairing correlations is lower in some isotopes such as ^{122,124}Sb and ^{210,212}Pb isotopes than the corresponding values in the case of no pairing correlations. In these mass regions, the IVMR state is getting far from the IAR state with the inclusion of the pairing correlations.

We expect that the β^2 values show a similar behavior with the $T_0 + 1$ isospin admixture values in the ground state of the parent nucleus since this quantity is proportional to the b^2 values as seen from eq. (29) [24]. Our results confirm this similarity. Therefore, the same physical arguments and interpretations for the b^2 values can also be given for the β^2 values.

As a result of our calculations, the following conclusions have been drawn:

- The influence of the pairing correlations between nucleons on the isospin admixture was dominantly seen in the light isotopes ($N \approx Z$) of the investigated nuclei. This implies that these correlations must be certainly taken into account in the isospin admixture calculations for the light isotopes although they play less important role in the heavy mass isotopes. The fact that the values of b^2 calculated by our model for ^{80}Zn, ^{100}Sb and ^{208}Pb nuclei in which the pairing interaction between nucleons does not occur are in good agreement with those calculated by other models proves that the b^2 values calculated for the nuclei for which the pairing interaction occurs are reliable.

- The dependence of the total β^+ transition strength on the mass number A in the QRPA formalism remains constant with some fluctuations. Therefore, the reason for the general decrease in the isospin admixture values can only be attributed to the $2(T_0 + 1)$ term in the denominator of eq. (26) [24].
- When the isospin structure of the IAR state is investigated, it is determined that the pairing interaction between nucleons are of primary importance for the light isotopes ($N \approx Z$) of the investigated nuclei. This is very important in order to calculate the isospin forbidden β-decay strength theoretically.

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