Mixing Phenomena through the Heat and Salt Transports Across a Density Interface in a Stratified Two-layer System

Eunsook Cho and Hyo Kim*

Department of Chemical Engineering, University of Seoul, 90 Jeonnong-Dong, Dongdaemun-Gu, Seoul 130-743, Korea (Received 9 April 2001 • accepted 24 September 2001)

Abstract–We have investigated the mixing phenomena in a stratified two-layer fluid system where a layer of fresh water is initially put on top of salty water. When this stabilized system by a salt gradient is heated from below, it becomes unstable by an onset of convection in the lower layer due to the thermal buoyancy effect. Thereafter the heat and salt is ready to diffuse into the upper layer through the adjacent diffusive interface. To examine quantitatively the merging process of the two layers as the destabilizing temperature gradient gets more dominant, we have measured many profiles of temperature and salinity in both layers by using accurate micro-scale measurement probes. Each run of experiment with several different initial concentrations of salt is followed until there appears a sudden overturning into a perfect mixing state. The order of thermal Rayleigh number has been kept as order of 10⁷ around which we have observed uniform temperature and salt profiles in the upper layer without any external mixing force. Since the employed measuring probes shows good reproducibility and very fast response time to the variations of the temperature and salt concentration, the mixing phenomena with the double-diffusive convection has been pursued easily.

Key words: Mixing Phenomena, Heat and Salt Transports, Diffusive Interface, Micro-scale, Measurement Probes, Interface Stability

INTRODUCTION

Double-diffusive convective motions can take place in a stably stratified fluid under gravity with a thin density interface when one of the two components, usually heat and solute contributing to the density distribution, is disturbed. There are two kinds of interface, that is, diffusive or finger, depending on whether the destabilizing component has the greater or lesser diffusivity. In the diffusive regime, the greater diffusive component, the heat if comparing with the solute, makes an unstable buoyancy flux across the interface and drives convection in the stratified layers. If the destabilizing component has the lesser diffusivity, there will appear long narrow convection cells like fingers and the interface is called a finger interface [Turner, 1973].

Recently double-diffusive convection has attracted much attention in several areas of science and engineering because it exhibits many systematic explanations for various natural aspects involving heat and mass transfers. We find easily the large-scale applications of this convection in the design and operation of solar pond and liquefied natural gas (LNG) storage tank. In the solar pond an insulating liquid layer is put on a salt-gradient water which is prone to convection by absorbing solar energy. The operation problem comes from the instability occuring from the nonuniform salt gradient caused by the double-diffusive convection between the adjacent layers [Chen et al., 1984]. We can also see the problems arising from double-diffusive convection in huge LNG storage tanks. These problems have been investigated by Sarsten [1972], Germeles [1976], and Kim et al. [1994] etc. since 1971 when the LNG storage tank at the La Spezia LNG receiving terminal in Italy gave rise to a disaster of venting a lot of combustible gas into air. Because the LNG storage tanks are usually receiving LNG with different composition according to the sources, there may exist temporary stratified LNG layers which will reach an equilibrium in density by the diffusive convection mode. The equilibrium state means a sudden mixing of LNG, that is, an abrupt overturning of LNG which generates a great deal of boil-off natural gas from the free surface at the top layer. Besides these applications we have many other practical research fields using the double-diffusive behavior such as in determining transport coefficients in multicomponent fluids [Caldwell, 1973], observing the mixing processes through the rapid transport of salt and heat in oceanography [Veronis, 1965], and deciding the double-diffusive effects on the volcanism and magnetic crystallization during the formation of the Earth's crust [Turner et al., 1981].

Here in our laboratory experiment we have prepared an artificially stratified fluid which is composed of a fresh water layer, a thin stable density interface and a salty layer from the top. Under an isothermal condition it remains linearly stable. However, to understand and trace the behavior of heat and solute transports through the double-diffusive convective motion we have heated the bottom wall with a constant hot temperature. As the salty layer gets warmer it succumbs to convection due to the buoyancy and the convection is confined only in the layer by the stable interface. The diffusivity of heat is well known to be much greater than that of salt. Thus the unstable heat energy will move continuously into the fresh water zone across the diffusive interface and that causes another convective motion in the heat acceptor layer. The convections in both layers separated by a density interface are keeping until they reach an equilibrium, that is, the interface is broken and two layers are mixed. The changes of temperature and concentration in each layer have been measured by accuracy-proved probes MSCTI (MicroScale Conductivity and Temperatures Instrument) which was devised by

[†]To whom correspondence should be addressed. E-mail: hkim@uos.ac.kr

Head [1983]. Turner [1965] performed an experiment similar to ours, but he applied additional mechanical mixing in the upper layer to obtain instantly uniform distributions of temperature and concentration while the diffusive transports advanced, which might be useful to calculate the changes of potential energy and stability. However, if there is any other external mixing force added to heat and mass transfer system, its effect has to be taken into account [Crapper, 1975]. To avoid the discrepancy from this incorrect situation and ensure the calculations for non-dimensional flux ratio of heat and salt as long as the double-diffusive convection proceeds, we have used the thermal Rayleigh number at the large value of about 10^7 to 10° throughout the experiment, where the temperature and concentration are nearly constantly distributed in the upper layer [Cho et al., 1999].

In the stratified fluid the Nusselt numbers for heat Nu_{τ} and salinity Nu_{ε} have the relationships with the density ratio R_{ρ} and the Rayleigh numbers for heat Ra_{τ} and salinity Ra_{ρ} , respectively, as suggested by Turner [1965]:

$$Nu_{T} = f_{1}(R_{p})Ra_{T}^{1/3} = \frac{F_{T}}{k_{T}\Delta T/h},$$
(1)

and

$$Nu_{s} = f_{a}(R_{\rho})Ra_{s}^{1/3} = \frac{F_{s}}{k_{s}\Delta S'h}.$$
(2)

Here $Nu_{\tau}=F_{\tau}/(k_{\tau}\Delta T/h)$ and $Nu_{s}=F_{s}/(k_{s}\Delta S/h)$ where F_{τ} and F_{s} are the heat and salt fluxes, k_{τ} and k_{s} are the molecular diffusivities for heat and salt, ΔT and ΔS stand for temperature and salinity differences between the lower and upper layers, respectively, and h is the layer thickness. $R_{p}=\beta\Delta S/\alpha\Delta T$, where α and β are thermal and solutal expansion coefficients. The definitions of Ra_{τ} and Ra_{s} are ($g\alpha\Delta Th^{3}$)/ ($\nu\kappa_{\tau}$) and ($g\beta\Delta Sh^{3}$)/($\nu\kappa_{s}$), respectively. Here g is the gravity, κ_{τ} and κ_{s} are the thermal and salt diffusivities. The functions f_{1} and f_{2} will be constant and zero, respectively, if the interface is replaced by the sheet of a perfect thermal conductor. At last, from (1) and (2) we can get the following equation for the ratio of the potential energy changes due to the transfers of heat and salt across the interface according to the stability R_{ρ} .

$$\frac{\beta F_s}{\alpha F_\tau} = f_3(R_\rho). \tag{3}$$

In addition, the ratio of the turbulent transport coefficients for salt K_z and for heat K_T can be given by

$$\frac{K_{\rho}}{K_{\tau}} = f_{t}(R_{\rho}), \tag{4}$$

where $K_s = F_s/\Delta S$ and $K_r = F_r/\Delta T$. From the measured data of the temperature and salinity variations in the stratified fluid as the lower layer is heated below, we will get the functions of f_3 and f_4 to describe the quantitative relation for the salt and heat transports by the double-diffusive convection and discuss the differences from the results obtained in the previous experiment done by Turner [1965].

The following sections are composed of experimental procedure, results and conclusions. In the section of experimental procedure we have explained the methods for the preparation of the stratified fluid with a density interface and the measurement and data collection for the temperature and salinity. The section of results shows



the temperature, concentration and density variations with time or layer depth, which are used for finding the relationship between the dimensionless flux ratio and the stability. And finally we have got several conclusions on the traces of the proposed double-diffusive convective behavior.

3. MSCTI

EXPERIMENTAL PROCEDURE

The schematic arrangement for our experimental apparatus is shown in Fig. 1. The experiments were performed in a cylindrical Pyrex tank approximately 30 cm in diameter and 30 cm high, whose base was designed to be heated by an electrical heating element with a controller. The side and top walls of the tank were kept in insulation, but in our preliminary experiments to check the unifrom temperature and salinity distributions to the layer depth as the heating of the bottom proceeded we used a tank uninsulated in the side wall for convenience sake. The stratified fluid with an about 12-cm depth was set up by putting a 6- to 8-cm thick layer of fresh (distilled) water in the tank and then carefully pouring saline solution, which had been prepared with a known amount of salt, into the tank bottom. The heating was then switched on and the temperature of the heating pad was kept as a constant temperature until the two layers were merged. As the lower layer starts to convect, it gradually entrains the upper layer. Hence the interface becomes more sharp and its location moves a little upward due to the volume expansion as the bottom-wall temperature goes up. We easily see the extent of turbulence in the lower layer is slightly greater than that of the upper layer. The order of thermal Rayleigh number has been kept throughout this experiment as order of 107 around which we have observed homogeneous temperature and salinity in the upper layer. The initial salinity difference in concentration units which are grams of salt per 100 g of solution was varied 0.27 to 0.7%.

In order to calculate the fluxes of heat and salt across the diffusive interface, we need the measured temperatures and salinities of each layer as functions of time. For these measurements we employed MSCTI sensors which had been proved in very good accuracy and reproducibility by Head [1983]. This micro-scale instrument is composed of two parts, that is, one is a fast temperature sensor and the other is a fast electrode conductivity sensor. The fast conductivity sensor which is consisted of four sphered-platinum electrodes provides two analog voltage outputs, that is, one represents the linearly proportional value to the solution conductivity and the other is for the compensation for the nonlinearity occuring from the temperature variation. These sensors were positioned at the midpoint of each layer when the measurements gave representative values or they were traversed through the layers if it was necessary to measure the values of the temperature and concentration with depth. And the sensors were connected to a Fluke data-acquisition unit (2625A model) to transform the continuous electric voltage changes into the instant values of temperature and salinity at the probe tips. These signals finally transmitted to a computer to collect the data over a 1-second interval. The measurement values were averaged over 2 minutes to calculate the relations in (3) and (4). Since the density was dependent on the temperature and salinity, we prepared calibration curves from the given values of density at some specific temperatures and salt concentrations. The other physical properties were taken at the average temperature and concentration within the experimental ranges. Preliminary experiments were also conducted to know whether the midpoint values were representative in our experiments. In the following section we show the calculated results as well as experimentally measured values.

RESULTS

To examine the mixing phenomena through the double-diffusive convection across the diffusive interface, we have to measure the correct values of temperature and salinity in both layers with time. In a weak stage of convection the temperature and salt concentration profiles are dependent on the layer thickness as well as time, where the calculation of heat and salt fluxes may be difficult and incorrect. Thus to acquire the homogeneous temperature and salinity in each layer, first of all we need to control the rate of thermal penetration into the fluid enough to make a vigorous convection in the upper layer. In our experiments the large value of thermal Rayleigh number, i.e., around 10^7-10^8 , sufficed this condition. As a preliminary experiment confirming the uniformity in each layer



Fig. 2. Temperature profiles with the layer depth. 0.40_0 saline solution was initially added to a fresh water. The side wall of tank was uninsulated. The first line represents a profile at t=10 min. and the time lapse between two successive lines is 10 min.



Fig. 3. Concentration profiles with the layer depth. 0.40 b saline solution was initially added to a fresh water. The side wall of tank was uninsulated. The first line represents a profile at t=10 min. and the time lapse between two successive lines is 10 min.

we measured the temperature and concentration according to the layer depth with time as in Fig. 2 and Fig. 3. Here we prepared a stratified fluid by putting a fresh water layer on top of 0.4% as salty layer in a tank whose side wall was not insulated in order to trace the position level of the measuring elements by eyes. Thus there was some heat loss through the side wall of the tank. Here we kept the bottom temperature of the electric heating pad at a constant value of 180 °C and the initial fluid temperature at 19 °C. However, the object of this experiment is a pretest to check the homogeneity at the specified range of Rayleigh number and the obtained results matter little to our main analysis, for example, as in Fig. 5 and Fig. 6, because the order of magnitudes in the temperature and salinity differences was almost commensurate to gurantee the required high



Fig. 4. Density profiles with the layer depth. 0.4^{0} basine solution was initially added to a fresh water. The side wall of tank was uninsulated. The first line represents a profile at t=10 min and the time lapse between two successive lines is 10 min.



Fig. 5. Temperature variations with time. 0.4% saline solution was initially added to a fresh water. The side wall of tank was insulated.



Fig. 6. Concentration variations with time. 0.4% saline solution was initially added to a fresh water. The side wall of tank was insulated.

Rayleigh number. The measuring elements traveled at approximately 0.5 cm/sec and it took about 25 sec to produce each measured profile across the whole layer depth. In Fig. 2 and Fig. 3 we plotted the data every 10 minutes with the layer depth. The temperature of the atmosphere was 10 °C. Hence the temperature of the fluid surface was always a little lower than that of the bulk fluid as shown in Fig. 2. At 10 minutes after the bottom wall was starting to be heated, the temperature of the lower layer was up to 23 °C while the one of the upper layer was almost kept at the initial value. We could see the linear temperature gradient which was steep at the early stage of heating time in the intermediate layer and this became gentle just before two layers merged. In Fig. 3 the initially measured value of the concentration was shown to be lower than in the salt solution which was prepared in another vessel as 0.4% in concentration to add to the fresh water for the stratification. The reason was that in injecting the salty solution into the bottom of the fresh water there between the two layers was born an interface which had a certain magnitude in depth and intermediate value of concentration, that is, the lower layer was diluted in the course of stratified-layer generation. After about 50 minutes the two layers were

January, 2002

getting into a perfect mixing stage with the differences of temperature and concentration fading away. As time advances from the beginning the convection becomes more active and it can also be seen the thickness of the diffusive interface gets smaller. This is for the entrainment of the fluid particles in each layer as the natural convection gets vigorous. From the temperature distributions throughout the system we see the temperature differences between two layers are about 4 °C at 10 min, 5 °C at 20 min, 4 °C at 30 and 40 min, and 0°C at the final time. The big temperature differences around the mid-time are attributed to the density interface which acts as a temporary barrier to the heat transfer into the upper layer by the thermal convection. That is, within the lower laver the heat will continuously accumulate so far as the heat influx into it is larger than the output. As soon as the temperature difference arrives at the maximum value, even though there is no convecting transfers from lower to upper layers, the major role of heat exchange between the layers will be played by the heat conduction until the system reaches an equilibrium state. Hence the temperature gaps gets smaller after the ultimate value and then disappears. From both graphs we can see the temperature and concentration were uniformly distributed at each time and thus the measurement only at the midpoint of each layer would be enough to give the representative values. In this case the order of Ra_7 was about 10⁷, which would be seen in the later calculation. The calculated density profile by using Fig. 2 and Fig. 3 is shown in Fig. 4.

From now on we have traced the mixing phenomena of the stratified fluid as the heat and salt diffuse across the density interface by measuring the temperature and concentration in each layer which has equal mass by using MSCTI. The measuring probes were mounted in the middle of each layer. Except the very early stage of the heating time (at about few minutes) the values of the temperature were good representative as presented by Cho et al. [1999]. In Fig. 5 and Fig. 6, the temperature and salt concentration are plotted every 30 sec, respectively. The experimental conditions were a little different from the case of the preliminary experiment. Here we employed an insulated tank and so as to get the same order of temperature differences to meet the specified range of the Rayleigh number. The heating temperature of the electric pad was set 160 °C. The initial fluid temperature was 20 °C. We can see the temperature gradient in the lower layer is more steep than that of the upper layer until 1,200 sec after which two slopes are increased almost parallel to each other until the two layers are mixed. In Fig. 6 we see the upper layer gets heavier in salt while the lower layer becomes dilute as time moves forward. At about 3,250 sec the two layers are at last on the verge of turnover when the fluctuations of the temperature and salinity in the upper layer are more severe than in the lower layer. Even though we turned off the heating system as soon as the layers were mixed, the temperature of the mixed bulk rose due to the remnant heat but the concentration was seen the exactly averaged value of 0.2%. Here we have to note the concentration of the lower layer at the initial stage is measured smaller than that in the few minutes later as seen in Fig. 6. From this fact we believe that the lower layer was stabilized by the salt, i.e., when we added the 0.4% solution into the bottom of fresh-water layer, however carefully and quickly it might be done, the layer would be slightly diluted with the fresh water and thus there existed a salt gradient made linearly with the lighter next to the interface and the heavier around the



Fig. 7. Density variations with time. 0.4^{0} to saline solution was initially added to a fresh water. The side wall of tank was insulated.



Fig. 8. Ra_T vs. R_r

tank bottom. Therefore the probe located at the midpoint of the layer is not well representative at this stage, but those few values do not affect the calculation of salt flux because at this stage the diffusion of salt across the density interface will be very small. Fig. 7 shows the density profiles in both layers with time. Even after they merged, the density goes down due to the temperature effect.

In Fig. 8 the Rayleigh number for heat Ra_r is plotted against the density ratio R_ρ in several concentrations such as 0.27, 0.4, 0.55 and 0.7% to check the magnitude of Ra_r , which lies between 10⁷-10⁸ where we found the evenly distributed profiles of temperature and salinity in each layer such as in Fig. 2 and Fig. 3. As the stratified fluid becomes more stable, i.e., R_ρ gets bigger, the Ra_r decreases since the temperature difference between the layers gets smaller at a fixed difference of salinity. At a given stability the Ra_r shows higher value as the concentration gets thicker because the temperature difference has to be higher to keep the same level of stability. As the stratified fluid gets unstable, i.e., as R_ρ gets smaller, all the Rayleigh numbers for different salinity linearly increase until they meet a critical value and then they go down to some extent. These phenomena can be explained by the transient data of the temperature shown in Fig. 2. In the log-log coordinates we have got the linear



Fig. 9. The potential energy ratio vs. the interface stability.

relations between R_{ρ} and Ra_{τ} according to the saline concentration in the regime of high stability.

We have calculated the nondimensional flux ratio to characterize the mixing process through the double-diffusive convection. This result can be related to the stability according to (3). Fig. 9 shows the flux ratio against the density ratio, where we know the buoyancy flux ratio has some constant value, around 0.2, in the stabilized zone where the heat flux is many times greater than the salt flux, and it appears to approach 1 as R_o goes to 1. For the low level of stability, i.e., around R_o=2, the variation of the ratio is very rapidly increasing and the salt flux begins to be commensurate to the heat flux. This means the fluid is to turnover in a few minutes with the interface slightly fluctuated [Cho et al., 1999] and thus the transports of heat and salt from the lower layer to the upper one is not simply attributed to the pure double-diffusive convection. Finally we calculated the ratio of the turbulent transport coefficients with the density ratio and the results are plotted in Fig. 10. During the double diffusion through the interface the bulk fluid of each layer undergoes a turbulent convection by which there exists the difference of potential energy between the two layers. Therefore we need the ratio of the transport coefficients versus the stability as in (4) to



Fig. 10. The ratio of turbulent transport coefficients vs. the interface stability.

characterize the mixing phenomena through the density interface. In the stability regime for $R_{\rho}>3$ we found the turbulent coefficient of heat is about 30 times greater than that of salt. When it is about to merge, the ratio becomes 1, that is, the heat and salt have the same transferring velocities by the mixing of the lower and upper layers.

CONCLUSIONS

We have examined the mixing phenomena in a stratified twolayer fluid system where a layer of saline water is set underneath a layer of salty water. As this stabilized system by a salt gradient is heated from below, it becomes unstable by an onset of convection in the lower layer due to the thermal buoyancy effect. Thereafter the heat and salt is ready to diffuse into the upper layer through the adjacent diffusive interface. To characterize the double-diffusion process in the two layers as the destabilizing temperature gradient gets more dominant, we have measured many profiles of temperature and salinity in both layers by using the micro-scale measuring probes, i.e., MSCTI. Each run of experiment with several different initial concentrations of salt is followed until there appears a sudden overturning into a perfect mixing state. The order of thermal Rayleigh number has been kept as order of 10^7 around which we have observed uniform temperature and salt profiles in the upper layer without any external mixing force such as stirring and agitating. We can trace the temperature and salinity very fast with space and time, and the measured data show very good reliability in comparison with the known properties in the process of calibration. When the interface is stable ($R_{u}>3$), the potential energy change is almost constant as 0.2. But when it is unstable ($R_a < 3$), that goes to 1 rapidly. The stable zone is called as 'constant regime' and considered to be dominated by the pure double-diffusive convection mechanism. At this regime, salt and heat are transferred by the diffusion and conduction, respectively. The other zone called as 'variable regime' has very vigorous convection that the density interface undulates and then ruptures. Thus heat and salt are transported simultaneously by the moving fluid elements. At the constant regime, Ks/K7, the ratio of turbulent transport coefficients is almost constant as 0.03. This means that heat is transported about 30 times faster than the salt when the interface is stable. At the variable regime, the coefficient ratio goes to 1. This means that heat and salt are transported at the same time by the fluid elements due to strong convection. Throughout this study, the results from MSCTI are very reliable because it produces very accurate data. And we have performed our experiments without any artificial mixing apparatus used in other previous experiments to guarantee the homogeneous temperature and salinity profiles. Therefore our analysis would be better to explain the naturally occuring double-diffusive convection systems than any other existings.

ACKNOWLEDGMENT

This study was supported by the grant from the University of Seoul in 2000 and the authors gratefully acknowledge it.

NOMENCLATURE

- f_i : nondimensional functions (i=1, 2, 3, 4)
- January, 2002

- F_s : salt flux across a diffusive interface [gr cm⁻² sec⁻¹]
- F_{τ} : real heat flux across a diffusive interface [cal $cm^{-2}\,sec^{-1}]$
- g \qquad : acceleration due to gravity [cm sec^2]
- h : layer depth [cm]
- K_s : turbulent transfer coefficient for salt [cm sec⁻¹]
- k_s : molecular diffusivity for salt [cm² sec⁻¹]
- K_{7} : turbulent transfer coefficient for heat [cal cm^{-2} sec^{-1} \, ^{o}\mathrm{C}^{-1}]
- $k_r = :$ molecular diffusivity for heat [gr cm sec⁻³ °C⁻¹]
- Nu_s : Nusselt number for salt $[=F_shk_s^{-1}\Delta S^{-1}]$
- Nu_{T} : Nusselt number for heat $[=F_{T}h k_{T}^{-1}\Delta T^{-1}]$
- R_{ρ} : interface stability [= $\beta \Delta S / \alpha \Delta T$]
- Ra_s : Rayleigh number for salt $[=g\beta\Delta Sh^3/v\kappa_s]$
- Ra_T : Rayleigh number for heat $[=g\alpha\Delta Th^3/v\kappa^T]$
- ΔT : temperature difference between two layers [°C]
- ΔS : salinity difference between two layers [gr cm⁻³]

Greek Letters

- α : thermal expansion coefficient [°C⁻¹]
- β : solute expansion coefficient [cm³ gr⁻¹]
- κ_s : solute diffusivity [cm² sec⁻¹]
- κ_{T} : thermal diffusivity [cm² sec⁻¹]
- \mathbf{v} : kinematic viscosity [cm² sec⁻¹]

REFERENCES

- Caldwell, D. R., "Thermal and Fickian Diffusion of Sodium Chloride in a Solution of Oceanic Concentration," *Deep-sea Res.*, 20, 1029 (1973).
- Chen, C. F. and Johnson, D. H., "Double-diffusive Convection," a report on an Engineering Foundation Conference. J. Fluid Mech., 138, 405 (1984).
- Cho, E. S. and Kim, H., "Convective Mixing Phenomena in a Two-layer Fluid with or without a Turbulent Patch," AIChE's 1999 Annual Meeting, 127e, Dallas, Texas, U.S.A. (1999).
- Crapper, P. F., "Measurements Across a Diffusive Interface," *Deep-Sea Research*, 22, 537 (1975).
- Germeles, A. E., "A Model for LNG Tank Rollover," Adv. Cryo. Eng., 21, 326 (1976).
- Head, M. J., "The Use of Miniature Four-Electrode Conductivity Probes for High Resolution Measurement of Turbulent Density of Temperature Variations in Salt-Stratified Water Flows," Ph. D. Thesis, University of California, San Diego (1983).
- Kim, H., Kim, S. Y. and Sohn, Y. S., "A Study on the Prediction of ROLLOVER Phenomena from the Stratified LNG Storage Tank," *Energy Eng. J.*, 3(1), 95 (1994).
- Sarsten, J. A., "LNG Stratification and Rollover," *Pipeline and Gas J.*, 37 (Sept. 1972).
- Turner, J. S., "The Coupled Turbulent Transports of Salt and Heat across a Sharp Density Interface," *Int. J. Heat Mass Transfer*, 8, 759 (1965).
- Turner, J. S., "Buoyancy Effects in Fluids," Cambridge University Press (1973).
- Turner, J. S., "Double-Diffusive Phenomena," Ann. Rev. Fluid Mech., 6, 37 (1974).
- Turner, J. S. and Huppert, H. E., "Double-Diffusive Convection," J. Fluid Mech., 106, 299 (1981).
- Veronis, G., "On Finite Amplitude Instability in Thermohaline Convection," J. Mar. Res., 23, 1 (1965).