

Continuous Separation of Glucose and Fructose at High Concentration Using Two-Section Simulated Moving Bed Process

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Abstract—A two-section simulated moving bed (SMB) modified from a three-section SMB [Barker et al., 1975] was applied to the separation of an aqueous mixture of glucose and fructose at high concentration up to 500 kg/m³. Dowex 50W-X12 resin of Ca⁺⁺ form was used as an adsorbent and water as an isocratic eluent. The equilibrium isotherms in terms of a quadratic expression and a plug flow model with mass transfer effect were used to predict both the products and on-concentrations in the two-section SMB process. The two-section SMB process suggested in this work was successful in obtaining high fructose corn syrup (fructose with 55-90% w/w) at the high concentration of 500 kg/m³.

Key words: Two Section SMB, Chromatography, Glucose, Fructose, Separation

INTRODUCTION

SMB process has been widely used for various industrial applications such as hydrocarbon and sugar purification. Recently, it was successfully adapted to enantioseparation [Michael and Jochen, 2001; Won et al., 2001]. The possibility of applying SMB for the separation of 2,6- and 2,7-dimethylnaphthalene as a feed stock for polyethylenephthalate was shown by numerical simulation based on the single column chromatography experimental data [Kim et al., 2001]. SMB process came from moving bed process. A typical apparatus for the moving bed process [Barker and Crittcher, 1960] was for a gas chromatography application to separate the azeotropic mixture of benzene (b.p. 80.1 °C) and cyclohexane (b.p. 80.7 °C). Later his moving bed apparatus was modified to a moving column system for use with ternary hydrocarbon mixtures by introduction of a side arm containing a fresh flowing stream of packing between the bottom stripper and the feed inlet [Barker and Huntington, 1966]. Although the moving bed system has been successful for the laboratory and some large scale systems, it has been found to suffer from difficulties in achieving control of solids on a very large scale, mass transfer efficiency loss due to the uneven packing, attrition of the expansive packing, and the low mobile phase velocity limited by the bed fluidization velocity. To overcome the problems in the scale-up of a moving bed system, the simulated moving bed (SMB) process was developed.

In the SMB process, the shifting of the feed and the product ports in the direction of fluid flow can simulate the movement of solids in the opposite direction, and the countercurrent contact of solid and liquid leads to a high mass transfer driving force. There had been two main approaches. Four section SMB of Sorbex Process developed by UOP [Broughton, 1968] consisted of a single column divided into a number of compartments, where the inlet and outlets in each compartment were controlled by using a master multiport valve. In this run mode, the flow rates of the withdrawn streams

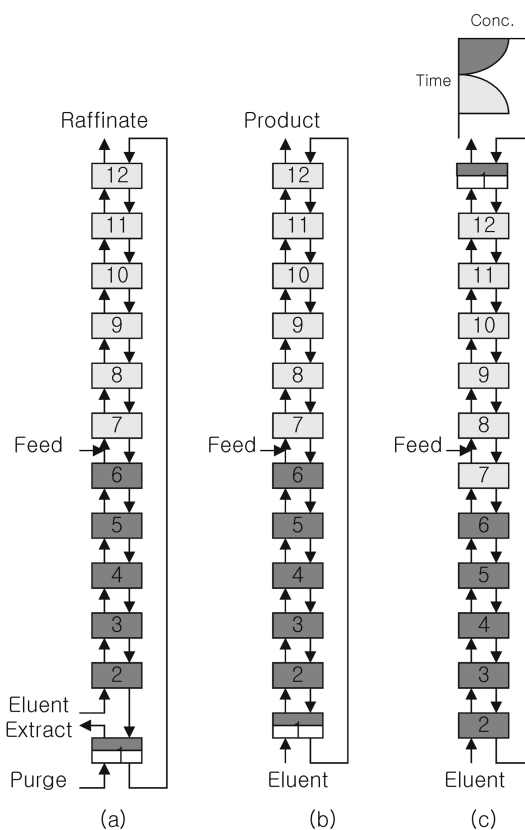


Fig. 1. Schematic explanation of SMB operation.

(a) three section SMB and (b) and (c) two section SMB at the end and start of one switch interval.

(extract and raffinate) should be controlled accurately to maintain the flow and pressure stability. In three section SMB [Fig. 1(a)], column configuration was made of three zones - an isolated purge, pre-feed, and post-feed section. The role of isolated purge column is to elute the slowly moving component within a periodic time interval. This system can operate with or without recycling and utilize

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all of the bed at any time. Four section SMB operates isocratically, while in three section SMB nonisocratic is possible due to an isolated purge section. A useful and comprehensive review of these processes has been given [Ruthven and Ching, 1989] and the advantages and disadvantages of two types of SMB well explained [Ganetsos and Barker, 1993].

In a previous work [Lee and Lee, 1992], various purge rates were tested with the same eluent and feed rate in three section SMB. In these experiments, it was successfully operated without a high purge rate and the sum of eluent and feed rate was enough to purge the retained component within a switch time. When columns are shifted, the less adsorbed component moves through the purge column regenerated in the previous time interval and comes out from the later part of one switch interval. If the column configuration of SMB can be modified as shown in Fig. 1(b) and (c), products come out in the order of slow and fast moving components due to the difference of exit time of each component. In this operation mode, it can be named "Two Section SMB" because there are only two different flow rate sections; pre-feed and post-feed section.

The object of this paper is to investigate operation mode of two section SMB in the nonlinear isotherm region and compare its performance with that of three section SMB.

EXPERIMENT

The equipment was made up of a set of twelve 1-cm diameter stainless steel columns with a packed length of 30 cm (DOWEX 50 W 12X in its Ca^{++} form) linked alternately top and bottom to form a closed loop. The upper and lower bed supports consisted of two layers of fine mesh stainless steel screen. Two distributors with one inlet and twelve outlets were used to control feed and eluent inlet. All of columns were surrounded by a constant temperature enclosure. The countercurrent movement of two section SMB is simulated by sequencing a system of inlet and outlet port functions around 12 columns. In Fig. 1(b), the feed enters column 7 and the less strongly adsorbed glucose moves with the eluent which enters the system at column 1. The fructose, the strongly adsorbed component due to the formation of a chemical complex with Ca^{++} , is preferentially retained by the resin. After one switch time, the position of the inlet and outlet port is advanced by one column as in Fig. 1(c). This simulation has the same effect of the movement of fructose with the stationary phase. Due to the difference of component velocity in the column and the column rearrangement, the strongly adsorbed fructose comes out first and then the less strongly adsorbed glucose. At the end of 12 such switches, one cycle is completed.

Prior to the run the SMB unit was fully purged by distilled water, and at time zero the feed solution, containing 500 g/L of each sugar, was introduced at column 7 as indicated in Fig. 1(b). Liquid samples were taken from the draw-off point of each column and analyzed by HPLC (Waters Associated Co.) by using an "Aminex 87C" Column (Bio-Rad Co.). Elution was performed isothermally at 85 °C with constant fluid velocity of 0.4 cc/min. Under these conditions the retention times for the glucose and fructose peak were 4.3 and 5.5 min, respectively. All runs of two types of SMB were carried out at 50 °C to reduce the viscosity and maintain the pressure drop within the acceptable limits.

RESULTS AND DISCUSSION

Two section SMB operation is achieved by advancing the eluent and feed port at fixed time intervals by one column in the direction of fluid flow. The solid phase of the column is counter-currently moved in the direction of mobile phase flow at a fixed rate; that is, the slow-moving component can be made to travel with the solid phase and the fast-moving component with the mobile phase. In the SMB process, each adsorption column can be considered to be a fixed bed except at the moment of moving each inlet and outlet point. Mass balance equations with the initial and boundary conditions for each component and column can be given by Eqs. (1)-(7).

$$\frac{\partial c_i}{\partial t} = -v_i \frac{\partial c_i}{\partial z} - \frac{1-\epsilon}{\epsilon} k (q_i^* - q_i) \quad (1)$$

$$\frac{\partial q_i}{\partial t} = k (q_i^* - q_i) \quad (2)$$

$$\frac{\partial c_i}{\partial z} = 0, \text{ at } z = L \quad (3)$$

$$c = q = 0, \text{ at } t = 0 \quad (4)$$

where the subscript i is the column number of 1 to 12.

When feed enters column 7, the following conditions are written for the inlet points.

At the eluent inlet point

$$c_{1,0} = 0 \quad (5)$$

At the feed inlet point

$$v_7 c_{7,0} = v_6 c_{6,1} + v_f c_f \quad (6)$$

At the inlet points of other columns

$$c_{j,0} = c_{j-1,1} \quad (j=2, \dots, 6, 8, \dots, 12) \quad (7)$$

where the first subscript is the column number and the second subscript denotes the inlet (0) and outlet (1) of the liquid stream in each column.

Blue dextran, which because of its size (molecular weight=2,000,000) does not penetrate the resin, was used as a tracer to estimate the void fraction, ϵ , and the value obtained from its mean retention time was 0.372. The effective overall mass transfer coefficient k was determined so that the calculated breakthrough curve may fit to the experimental one, measured with feed streams containing either glucose or fructose (between 10 to 50% w/v of either glucose or fructose). The mean k values for each concentration of glucose and fructose were calculated. Generally, the diffusion coefficient may be influenced by the concentration, fluid velocity, and the presence of other components. But in this system these influences were not considered to be severe because the gross features of the dynamic behavior of the system are determined by the equilibrium relationship.

To investigate the concentration dependence of the distribution coefficient for aqueous solutions of glucose and fructose at high concentrations on ion exchange resin, a series of step chromatographic tests were carried out by using four columns from the SMB unit. The distribution coefficient may be determined from the fundamental elution equation.

$$V_R = V_m + V_s K \tag{8}$$

When the distribution coefficient depends on the concentration, Eq. (8) is not valid and must be replaced by Eq. (9), which is a more general elution equation.

$$V_R = V_m + V_s \frac{dq^*}{dc} \tag{9}$$

For a linear system $q^*/c = K = dq^*/dc$ and Eq. (9) reduces to Eq. (8). However, if the system is nonlinear it is obviously incorrect to apply Eq. (8) at high concentration since Eqs. (8) and (9) are no longer equivalent. The nonlinearity and coupling between the equilibria for glucose and fructose can be expressed in terms of a general quadratic expression.

$$K_G = K_G(c_G, c_F) \equiv \frac{q_G^*}{c_G} = K_{G0} + A_1 c_G + B_1 c_F$$

$$K_F = K_F(c_G, c_F) \equiv \frac{q_F^*}{c_F} = K_{F0} + A_2 c_G + B_2 c_F \tag{10}$$

The coefficients K_{G0} , K_{F0} , A_1 , B_2 could be obtained from a series of step chromatographic tests and the cross coefficients B_1 and A_2 have to be found by matching the experimental profiles with K_{G0} , K_{F0} , A_1 and B_2 fixed. These values are summarized in Table 1 with the effective overall mass transfer coefficients.

The numerical solution of the transient model of each fixed bed was performed through the orthogonal collocation method which reduces the original partial differential equations to a set of ordinary ones [Villadsen and Michelson, 1980; Finlayson, 1980]. These differential equations were integrated through a fourth order Runge-Kutta method [Gerald and Wheatley, 1984], and selection of 10 collocation points at each fixed bed gave sufficient precision to calculate the theoretical profiles. When the columns were advanced at the end of a switch interval, concentration profiles corresponding to collocation points of each column were shifted by one column in the opposite direction of fluid flow according to the column movement. With this procedure, the pseudo-equilibrium concentration profile can be calculated. After six cycles, a pseudo-equilibrium state can be reached.

The essential requirement to achieve a good separation is that the flow rates in two sections (pre-feed and post feed section) must be adjusted in such a way as to achieve a net flow of the more strongly adsorbed species toward the pre-feed section, and a net flow of the less strongly adsorbed species toward the post-feed section as in Fig. 1(b) and (c). From the fact that fructose is the slow-moving component and glucose is the fast-moving component in the cation exchanged resin, the flow ratio of the downward flow in the ad-

Table 1. Parameters used in this work

	Glucose	Fructose
Limiting adsorption equilibrium constant	0.123	0.310
Concentration-dependent equilibrium constant	$K_G = 0.1226 + 0.0007c_G + 0.0044c_F$	$K_F = 0.3083 + 0.0025c_G + 0.0006c_F$
Effective overall mass transfer coefficient, [min ⁻¹]	3.005	2.710

sorbed phase to the upward flow in the mobile phase [$\gamma = (1 - \epsilon)Ku/\epsilon v$] can be conveniently specified for a linear system.

$$\begin{aligned} \text{pre-feed section} & \quad \gamma_F > 1.0, \gamma_G < 1.0 \\ \text{post-feed section} & \quad \gamma_F > 1.0, \gamma_G < 1.0 \end{aligned} \tag{11}$$

The operating conditions of the SMB process in the linear isotherm range can be obtained by properly drawing the operating line at the linear equilibrium line in a McCabe-Thiele diagram [Ching and Ruthven, 1985]. But when the equilibrium constant depends on the concentration and has a coupling effect between equilibria of both components, the exact equilibrium line in McCabe-Thiele diagram cannot be drawn without the information of on-concentration profiles and thus the operating line cannot be drawn either. The conditions of flow rates can be obtained by using the following relations with fixed switch time per unit column length, margin of α from Eq. (12).

$$\begin{aligned} u &= L/\tau \\ S &= A(1 - \epsilon)u \\ E' &= E + A\epsilon u \\ \text{pre-feed section} & \quad E/S = K_G \alpha \\ \text{post-feed section} & \quad E/S + F/S = K_F/\alpha \end{aligned} \tag{12}$$

Considering the maximum concentration of each sugar in the non-

Table 2. Experimental conditions

Run mode	Run no.	Eluent rate [cc/min]	Feed rate [cc/min]	Purge rate [cc/min]	Switch time [min]	Column length [cm]
Three Section SMB	1	3.78	0.66	4.44	3	30
	2	3.78	0.66	4.44	6	60
	3	3.78	0.66	4.44	9	90
Two Section SMB	4	3.78	0.66	-	3	30
	5	3.78	0.66	-	6	60
	6	3.78	0.66	-	9	90

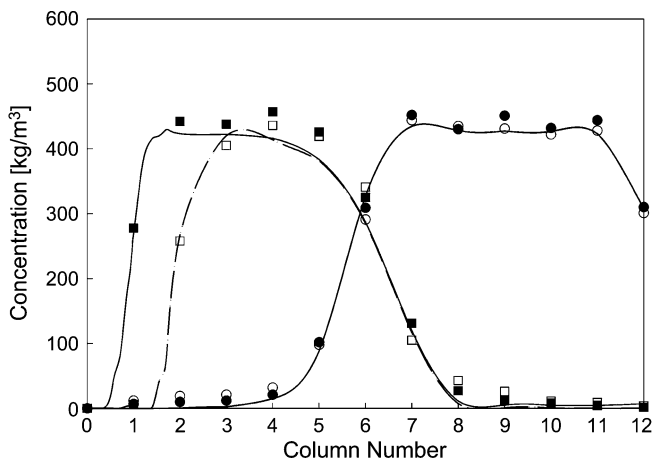


Fig. 2. Comparison of experimental and theoretical concentration profiles at the end of a switch time, total of 12 columns, each 30 cm.

Three section SMB: Run 1, cal; ... exp; fru (□), glu (○)
Two section SMB: Run 4, cal; - exp; fru (■), glu (●)

linearity of equilibrium constants, details of six sets of the experimental conditions are given in Table 2. These experimental conditions were set for the comparison of the performance of three and two section SMB.

Representative on-concentration profiles are shown in Fig. 2 with the theoretical ones. In Runs 1 and 4, a total of 12 columns, each 30 cm, was used. It is evident that the theoretical curves provide a good representation of the experimental profiles. Some discrepancies between the profiles stemmed from the fact that no allowance was made in the calculation of the solute hold-up in the valves and inter-column transfer tubes. In these runs, due to the similar zone of mass transfer, similar on-concentration profiles could be obtained.

In the simulated moving bed system, it has been asked how many columns are needed to simulate the moving bed process. Some authors [Liapis and Rippen, 1979] showed that most of the benefit of counter-current flow could be achieved by a rather modest degree of subdivision of the bed.

In the four section SMB, the mean concentration profile averaged over the switch interval is more important than the profile at the mid-time of the switch interval [Lu and Ching, 1997]. When the bed was divided into very small subsections, one would have a perfect analogue of countercurrent flow. But if the subsection were decreased under the same total column length, the above two concentrations would differ somewhat. Because of mass transfer effects and the movement of concentration peaks with the eluent flow, the product profiles were nonlinearly decreased in the extract and increased in the raffinate during the switch interval. On the other hand, in the three and two section SMB, the product profiles were not continuously obtained with a switch time; the effect of subsection was how many ones were needed to get a high purity of product. In Runs 2 and 5, a total of 6, each 60 cm, was used, respectively, to study this effect with the same flow conditions of Run 1 and the experimental and calculated results are shown in Fig. 3. In this operation mode, the column configuration of three section SMB was 1, 2 and 3 subsections in each purge, pre-feed, and post-feed section and that of two section SMB was 3 and 3 in pre- and post-feed

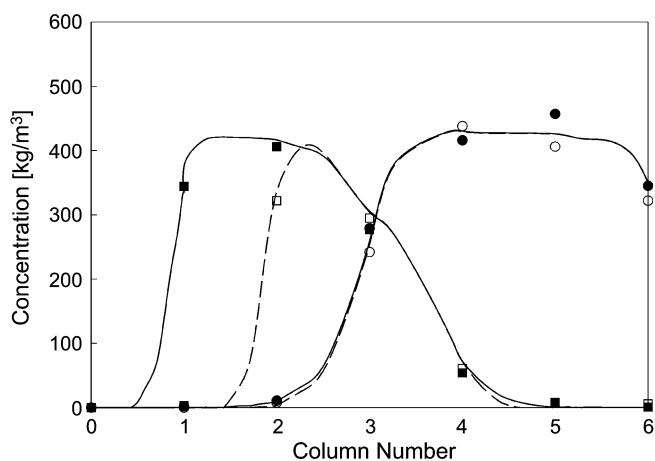


Fig. 3. Comparison of experimental and theoretical concentration profiles at the end of a switch time, total of 6 columns, each 60 cm.

Three section SMB: Run 2, cal; ... exp; fru (□), glu (○)
Two section SMB: Run 5, cal; - exp; fru (■), glu (●)

section. With the reduction of pre-feed zone in three section SMB, the on-column concentration profiles were more overlapped. But in the two section SMB, each two subsection was sufficient to sep-

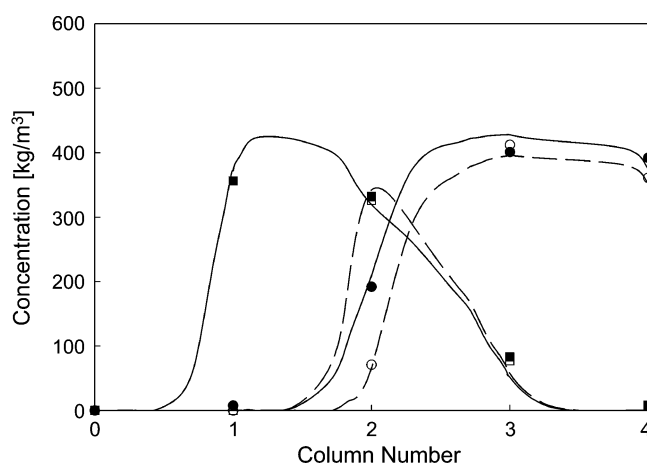


Fig. 4. Comparison of experimental and theoretical concentration profiles at the end of a switch time, total of 4 columns, each 90 cm.

Three section SMB: Run 3, cal; ... exp; fru (□), glu (○)
Two section SMB: Run 6, cal; - exp; fru (■), glu (●)

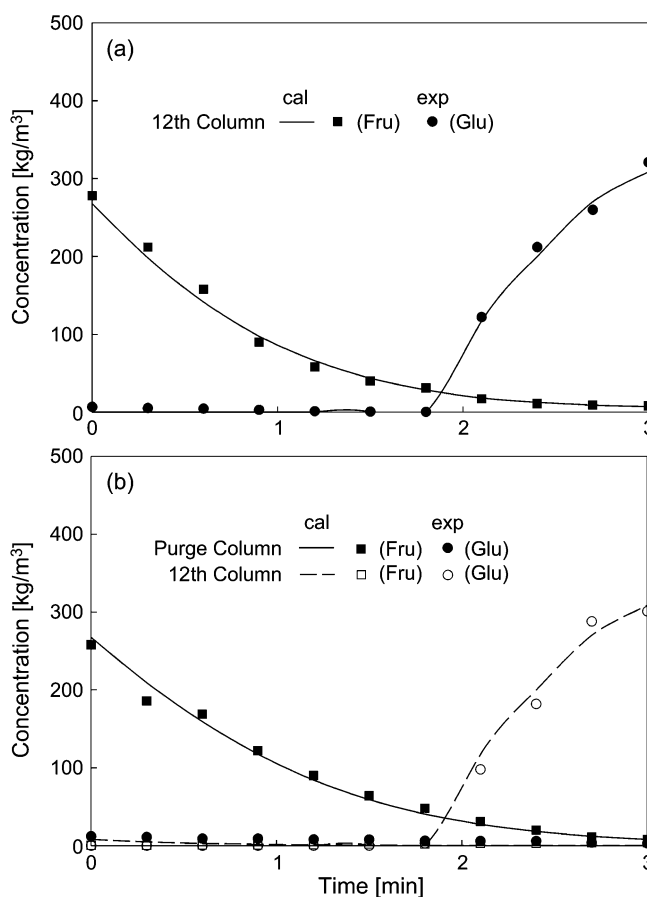


Fig. 5. Experimental and theoretical concentration profiles of fructose- and glucose-rich product.

(a) two section SMB (Run 4) and (b) three section SMB (Run 1).

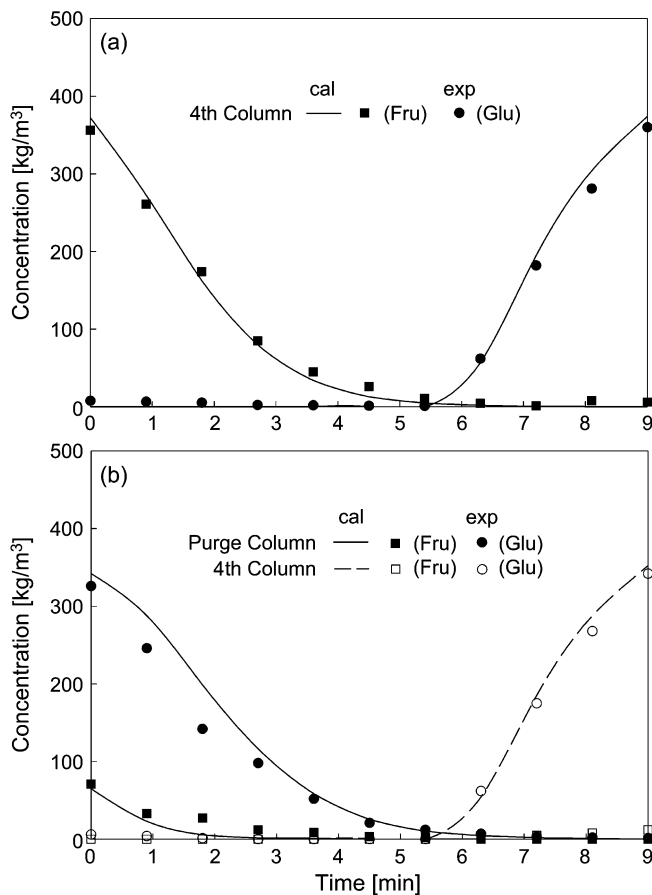


Fig. 6. Experimental and theoretical concentration profiles of fructose- and glucose-rich product.
 (a) two section SMB (Run 6) and (b) three section SMB (Run 3).

arate the feed stock due to the effective usage of bed. When the subsection was more reduced, this effect was obviously apparent. In Runs 3 and 6, a total of 4, each 90 cm was used in each operation mode. As shown in Fig. 4, with the column configuration of 1, 1, 2 in each zone of three-section SMB, it could not give a sufficient bed length to separate the feed components continuously. In the ‘‘Sarex’’ type of process for glucose and fructose separation, wherein product purity requirements are modest, most units appear to be designed with either one or two subsections per section [Hidajat and Ching, 1986]. With respect to this, Run 6 can correspond to the operation mode of four section SMB with one subsection. In Figs. 5 and 6, the experimental concentration profiles of products are shown with the theoretical values. In (a) of all figures, all profiles are for two section SMB and two products come out of the last column of each column configuration. For three section SMB, fructose-rich products come out of a purge column and glucose-rich products the last column. These profiles are represented in the same figure as in (b). Here, the critical dilution was arbitrarily defined as that value where the product concentration was about 10% of the feed concentration. In all experiments, the fructose-rich products above the critical dilution were collected and the glucose-rich products were collected from the start of its exit. The experimental results of this collection method are summarized in Tables 3 and 4, respectively. For

Table 3. Experimental results in the fructose-rich product

Run mode	Run no.	Product collection period [min]	Average concentration [kg/m ³]	Purity [%]	Recovery [%]
Three	1	0 to 1.5	145.6 (146.7)	93.95 (100)	97.95 (98.69)
Section	2	0 to 2.4	167.8 (191.5)	92.00 (99.51)	90.27 (103.09)
SMB	3	0 to 3.6	168.8 (200.8)	85.77 (92.71)	90.82 (108.04)
Two	4	0 to 1.5	135.4 (131.7)	97.54 (100)	91.09 (88.58)
Section	5	0 to 2.4	156.9 (171.4)	95.13 (99.99)	86.16 (92.24)
SMB	6	0 to 3.6	180.1 (179.5)	97.48 (100)	96.94 (96.59)

Values in parentheses are the theoretical results.

Table 4. Experimental results in the glucose-rich product

Run mode	Run no.	Product collection time [min]	Average concentration [kg/m ³]	Purity [%]	Recovery [%]
Three	1	1.8 to 3	144.0 (185.3)	97.89 (99.92)	96.87 (99.70)
Section	2	3.6 to 6	191.2 (185.6)	92.60 (100)	103.09 (99.86)
SMB	3	5.4 to 9	169.0 (175.5)	96.85 (100)	90.95 (94.46)
Two	4	1.8 to 3	188.7 (185.2)	93.05 (92.85)	101.53 (99.70)
Section	5	3.6 to 6	178.3 (185.7)	96.09 (98.07)	95.89 (99.92)
SMB	6	5.4 to 9	176.4 (185.3)	96.98 (99.35)	94.92 (99.76)

Values in parentheses are the theoretical results.

the calculation of purity and recovery in two section SMB, the following relations were used:

$$\begin{aligned}
 \text{purity of fructose} &= \frac{\int_0^t c_{Fru} dt}{\int_0^t (c_{Fru} + c_{Glu}) dt} \\
 \text{purity of glucose} &= \frac{\int_{t_s}^t c_{Glu} dt}{\int_{t_s}^t (c_{Fru} + c_{Glu}) dt} \\
 \text{recovery of fructose} &= \frac{v_{Fru} \int_0^t c_{Fru} dt}{v_F c_F \tau} \\
 \text{recovery of glucose} &= \frac{v_{Glu} \int_{t_s}^t c_{Glu} dt}{v_F c_F \tau} \quad (13)
 \end{aligned}$$

Here t_s means the start time of glucose exit and t_e is the end time of critical concentration collected.

CONCLUSION

In this work, a new operation mode of a two section simulated moving bed (SMB) was suggested through a modification of that of three section SMB. With the application of the continuous separation of glucose and fructose at high concentration up to 500 kg/m³, the performance of two and three section SMB process was compared. Although general conclusions can be drawn from the limited range of experimental and theoretical results, this unit has the same performance as the three section SMB process in obtaining high fructose corn syrup (55 to 90 w/v% of fructose). In the view of apparatus and operating cost, it is more economic than three section SMB. Especially with the run mode of reduced subsections, it used the bed more effectively so that it could obtain fructose-rich product with higher purity rather than three section SMB.

ACKNOWLEDGEMENT

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NOMENCLATURE

A	: cross sectional area of column [cm ²]
c	: fluid phase concentration [kg m ⁻³]
c _F	: feed concentration [kg m ⁻³]
E	: eluent flow rate [cc min ⁻¹]
E'	: actual eluent flow rate [cc min ⁻¹]
F	: feed flow rate [cc min ⁻¹]
K	: adsorption equilibrium constant
k	: effective overall mass transfer coefficient [min ⁻¹]
L	: length of adsorbed bed [cm]
q	: sorbate concentration in the adsorbed phase [kg m ⁻³]
q*	: sorbate concentration at equilibrium with c [kg m ⁻³]
S	: hypothetical adsorbent recirculation rate in equivalent counter-current system (=A(1-ε)u) [cc min ⁻¹]
SMB	: simulated moving bed
t	: time [min]
u	: hypothetical solid velocity (=L/τ) [cm min ⁻¹]
V _m	: total volume of the mobile phase [m ³]
V _R	: retention volume of the component [m ³]
V _s	: volume of the stationary phase [m ³]
v	: interstitial fluid phase velocity [cm min ⁻¹]
v _f	: feed velocity [cm min ⁻¹]

Greek Letters

α	: margin defined in Eq. (12)
γ	: dimensionless parameter (= (1-ε)Ku/εv)
ε	: void fraction of packed bed
τ	: switch time [min]

Subscripts

F	: fructose
G	: glucose

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