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Non-static plane symmetric cosmological model in Wesson's theory

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Abstract. The problem of non-static plane symmetric perfect fluid distribution in Wesson's [1] scale invariant theory of gravitation with a time-dependent gauge function is investigated. The false vacuum model of the universe is constructed and some physical properties of the model are discussed.

Keywords. Non-static; plane symmetric; gauge function; space-time.

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1. Introduction

Wesson [1] proposed a simple formulation of scale invariant theory of gravitation incorporating the gauge function $\beta(x^i)$, where x^i are coordinates in the four-dimensional space-time and the tensor field identified with Riemannian metric tensor g_{ij}. One can interpret g_{ij} as a gravitational potential tensor, which determines the interaction between the matter (or other fields) and the gravitation. It is pointed out that, this theory agrees with general relativity up to the accuracy of the observations made up to now. Dirac [2,3], Hoyle and Narlikar [4] and Canuto *et al* [5] have earlier studied several aspects of the scale invariant theory of gravitation. But, Wesson's [1] formulation of scale invariant theory of gravitation is so far the best theory to describe all interactions between the matter and the gravitation.

Mohanty and Daud [6] have already studied the cosmological model governed by vacuum field equations when the space-time is described by homogeneous and anisotropic Bianchi type-I metric with gauge function. In that paper, they have shown that the model reduce to Kasner model [7] when cosmological constant is zero, but for non-zero cosmological constant, the model isotropize as in Einstein theory. Moreover, Mohanty and Mishra [8] have studied the feasibility of Bianchi type-VIII and IX space-times combinedly in this theory with a time-dependent gauge function and a matter field in the form of perfect fluid. In that paper, they have constructed the radiating model of the universe for the feasible Bianchi type-VIII space-time. It is evident from the literature that the investigation in this direction is not yet complete and there is a need for further work, which may unravel some of the hidden secrets of the theory.

In this paper, an attempt is made to study the compatibility of non-static plane symmetric space-time with a matter field in the form of perfect fluid in scale invariant theory with the Dirac gauge function $\beta = \beta(ct)$. In §2, the field equations of scale invariant theory are set up. In §3, the explicit exact solution is obtained. In §4, some physical properties of the model are discussed and concluding remarks are given in §5.

2. Wesson's field equations

The field equations for scale invariant theory [1] with Dirac gauge function are

$$G_{ij} + 2\frac{\beta_{;ij}}{\beta} - 4\frac{\beta_{,i}\beta_{,j}}{\beta^2} + \left(g^{ab}\frac{\beta_{,a}\beta_{,b}}{\beta^2} - 2g^{ab}\frac{\beta_{;ab}}{\beta}\right)g_{ij} + \Lambda_0\beta^2 g_{ij} = -\kappa T_{ij} \quad (2.1)$$

with

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij}. \quad (2.2)$$

Here, G_{ij} is the conventional tensor involving g_{ij}, T_{ij} the energy momentum tensor, R_{ij} the Ricci tensor and R the Ricci scalar. Comma and semicolon respectively denote partial and conventional covariant differentiation using g_{ij}. The cosmological term Λg_{ij} of Einstein theory is now transformed to $\Lambda_0\beta^2 g_{ij}$ in scale invariant theory with a dimensionless constant Λ_0. G and κ are respectively the Newtonian and Wesson's gravitational parameter.

The line element for the plane symmetric non-static metric with a gauge function $\beta = \beta(ct)$ is

$$\mathrm{d}s_w^2 = \beta^2\mathrm{d}s_E^2 \quad (2.3)$$

with

$$\mathrm{d}s_E^2 = \mathrm{e}^{2h}(c^2\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - s^2\mathrm{d}z^2), \quad (2.4)$$

where r, θ and z are cylindrical polar coordinates and h and s are functions of time t only.

Here, the region of the space-time containing perfect fluid is considered, whose energy momentum tensor is given by

$$T_{ij}^m = (p_\mathrm{m} + \rho_\mathrm{m}c^2)U_iU_j - p_\mathrm{m}g_{ij} \quad (2.5)$$

together with

$$g_{ij}U^iU^j = 1, \quad (2.6)$$

where U^i is the four-velocity vector of the fluid. p_m and ρ_m are proper isotropic pressure and mass energy density of the fluid respectively.

The surviving components of conventional Einstein tensor (2.2) for the metric (2.4) are

$$r^2G_{11} = G_{22} \equiv \frac{r^2}{c^2}\left[2h_{44} + \frac{s_{44}}{s} + h_4^2 + 2h_4\frac{s_4}{s}\right], \quad (2.7)$$

$$G_{33} \equiv \frac{s^2}{c^2}[2h_{44} + h_4^2], \tag{2.8}$$

$$G_{44} \equiv -\left[3h_4^2 + 2h_4\frac{s_4}{s}\right]. \tag{2.9}$$

Hereafterwards the suffix 4 after a field variable denotes exact differentiation with respect to time t only.

Using the comoving coordinate frame $(0,0,0,ce^h)$, the non-vanishing components of the field eq. (2.1) for the metric (2.3) can be written in the following explicit form

$$r^2G_{11} = G_{22} \equiv -\kappa p_{\rm m}e^{2h} - \frac{1}{c^2}\left[2\frac{\beta_{44}}{\beta} + \left(2h_4 + 2\frac{s_4}{s}\right)\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h}\right], \tag{2.10}$$

$$G_{33} \equiv -\kappa p_{\rm m}s^2e^{2h} - \frac{s^2}{c^2}\left[2\frac{\beta_{44}}{\beta} + (2h_4)\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h}\right], \tag{2.11}$$

$$G_{44} \equiv -\kappa\rho_{\rm m}c^4e^{2h} + \left[\left(6h_4 + 2\frac{s_4}{s}\right)\frac{\beta_4}{\beta} - 3\frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h}\right]. \tag{2.12}$$

Equations (2.1) and (2.10)–(2.12) suggest the definition of quantities $p_{\rm v}$ (vacuum pressure) and $\rho_{\rm v}$ (vacuum density) which involve neither the Einstein tensor nor the properties of conventional matter [1]. These two quantities can be obtained as

$$2\frac{\beta_{44}}{\beta} + \left(2h_4 + 2\frac{s_4}{s}\right)\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h} = \kappa p_{\rm v}c^2e^{2h}, \tag{2.13}$$

$$2\frac{\beta_{44}}{\beta} + (2h_4)\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h} = \kappa p_{\rm v}c^2e^{2h}, \tag{2.14}$$

$$\left(6h_4 + 2\frac{s_4}{s}\right)\frac{\beta_4}{\beta} - 3\frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2c^2e^{2h} = -\kappa\rho_{\rm v}c^4e^{2h}. \tag{2.15}$$

In this case, when there is no matter and the gauge function β is a constant, one recovers the relation

$$c^2\rho_{\rm v} = -c^4\frac{\lambda_{\rm GR}}{8\pi G} = -p_{\rm v} \quad \text{i.e. } c^2\rho_{\rm v} + p_{\rm v} = 0, \tag{2.16}$$

which is the equation of state for vacuum. Here, $\lambda_{\rm GR} = \lambda_0\beta^2 =$ constant is the cosmological constant in general relativity. Also $p_{\rm v}$ being dependent on the constants $\lambda_{\rm GR}$, c and G is uniform in all directions and hence isotropic in nature.

It is evident from the aforesaid equations that $p_{\rm v}$ being isotropic is consistent only when

$$s = d_1, \quad \text{since } \frac{\beta_4}{\beta} \neq 0, \tag{2.17}$$

where d_1 is an integrating constant.

Using eq. (2.17) in eqs (2.13)–(2.15), the pressure and energy density for the vacuum case can be obtained as

$$p_{\rm v} = \frac{1}{\kappa c^2 {\rm e}^{2h}}\left[2\frac{\beta_{44}}{\beta} + 2h_4\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2 c^2 {\rm e}^{2h}\right], \tag{2.18}$$

$$\rho_{\rm v} = -\frac{1}{\kappa c^4 {\rm e}^{2h}}\left[6h_4\frac{\beta_4}{\beta} - 3\frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2 c^2 {\rm e}^{2h}\right]. \tag{2.19}$$

$p_{\rm v}$ and $\rho_{\rm v}$ refer to the properties of vacuum only. The definition of the above two quantities is natural as regards to the scale invariant theory of vacuum [1]. The total pressure $p_{\rm t}$ and total energy density $\rho_{\rm t}$ can be calculated as

$$p_{\rm t} \equiv p_{\rm m} + p_{\rm v}, \tag{2.20}$$

$$\rho_{\rm t} \equiv \rho_{\rm m} + \rho_{\rm v}. \tag{2.21}$$

Using the aforesaid definition of $p_{\rm t}$ and $\rho_{\rm t}$, the field equations in scale invariant theory i.e., eqs (2.10)–(2.12) can now be written using the components of Einstein tensor (2.7)–(2.9) and the results obtained in eqs (2.17)–(2.19) as

$$2h_{44} + h_4^2 = -\kappa p_{\rm t} c^2 {\rm e}^{2h}, \tag{2.22}$$

$$h_4^2 = \kappa\rho_{\rm t} c^4 {\rm e}^{2h}. \tag{2.23}$$

3. Solutions

Equations (2.22) and (2.23) are two equations in three unknowns, viz., h, $p_{\rm t}$ and $\rho_{\rm t}$. For complete determinacy one extra condition is needed. So, the equation of state $p_{\rm t} + \rho_{\rm t} c^2 = 0$, i.e., false vacuum model is considered.

Using the equation of state $p_{\rm t} + \rho_{\rm t} c^2 = 0$, eqs (2.22) and (2.23) yield

$$h = d_2 t + d_3. \tag{3.1}$$

Now, the total pressure $p_{\rm t}$ and total energy density $\rho_{\rm t}$ can be given as

$$p_{\rm t} = -\rho_{\rm t} c^2 = -\frac{1}{\kappa c^2}[d_2^2 {\rm e}^{-2(d_2 t + d_3)}]. \tag{3.2}$$

The pressure and energy density corresponding to vacuum case can be calculated as

$$p_{\rm v} = \frac{{\rm e}^{-2(d_2 t + d_3)}}{\kappa c^2}\left[2\frac{\beta_{44}}{\beta} + 2d_2\frac{\beta_4}{\beta} - \frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2 c^2 {\rm e}^{2(d_2 t + d_3)}\right], \tag{3.3}$$

$$\rho_{\rm v} = -\frac{{\rm e}^{-2(d_2 t + d_3)}}{\kappa c^4}\left[6d_2\frac{\beta_4}{\beta} - 3\frac{\beta_4^2}{\beta^2} - \Lambda_0\beta^2 c^2 {\rm e}^{2(d_2 t + d_3)}\right]. \tag{3.4}$$

The matter pressure $p_{\rm m}$ and energy density $\rho_{\rm m}$ can be obtained as

$$p_{\rm m} = -\frac{1}{\kappa c^2}\left[\mathrm{e}^{-2(d_2 t + d_3)}\left(d_2^2 - 2\frac{d_2}{t} + \frac{3}{t^2}\right) - \frac{\Lambda_0}{t^2}\right], \tag{3.5}$$

$$\rho_{\rm m} = \frac{1}{\kappa c^4}\left[\mathrm{e}^{-2(d_2 t + d_3)}\left(d_2^2 - 6\frac{d_2}{t} + \frac{3}{t^2}\right) - \frac{\Lambda_0}{t^2}\right]. \tag{3.6}$$

So, the false vacuum non-static plane symmetric model in scale invariant theory is given by eqs (2.17), (3.1) and (3.2). The metric in this case is

$$\mathrm{d}s_{\rm W}^2 = \frac{1}{c^2 t^2}\{\mathrm{e}^{2(d_2 t + d_3)}(c^2\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - s^2\mathrm{d}z^2)\}. \tag{3.7}$$

4. Some physical properties of the model

The scalar expansion of the model can be calculated as

$$\Theta = U^i_{;i} = \frac{3d_2}{c\mathrm{e}^{(d_2 t + d_3)}},$$

from which it is evident that $\Theta \to 0$ as $t \to \infty$, i.e., the universe is expanding with increase of time and the rate of expansion is slow with increase in time.

It is observed that

$$\frac{\rho_{\rm m}}{\Theta^2} \to \infty \quad \text{as } t \to 0 \quad \text{and} \quad \frac{\rho_{\rm m}}{\Theta^2} \to \text{ constant as } t \to \infty$$

which confirms the homogeneity nature of the space-time.

The shear scalar $\sigma = 0$, which indicates that, the shape of the universe remains unchanged during evolution. Also the space-time is isotropized during evolution in scale invariant theory. The vorticity W of the model vanishes, which indicates that U^i is hypersurface orthogonal. As the acceleration is found to be zero, the matter particle follows a geodesic path in this theory. Further, it is observed that

$$\rho_{\rm m} \to \infty \quad \text{as } t \to 0 \quad \text{and} \quad \rho_{\rm m} \to 0 \quad \text{as } t \to \infty,$$

which indicates that there is a Big Bang-like singularity at the initial epoch.

5. Conclusions

The non-static plane symmetric cosmological model constructed here expands with increase of time and the rate of expansion is slow with increase in time. As far as the matter is concerned, the model admits Big Bang-like singularity at the initial epoch. It is also observed that, the matter density $\rho_{\rm m}$ vanishes for $\Lambda_0 = -3$, but $p_{\rm m} \neq 0$ for $d_2 = d_3 = 0$. Thus for a viable physical situation, $\Lambda_0 \neq -3$. The model appears to be steady state.

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