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A relativistic core–envelope model on pseudospheroidal space-time

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Abstract. A core–envelope model for superdense matter distribution with the feature – core consisting of anisotropic fluid distribution and envelope with isotropic fluid distribution is reported on the background of pseudospheroidal space-time. The physical plausibility of the model is examined analytically and numerically.

Keywords. General relativity; anisotropic fluid sphere; core–envelope models.

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1. Introduction

Theoretical investigations of Ruderman [1] and Canuto [2] on compact stars having densities much higher than nuclear densities led to the conclusion that matter may be anisotropic at the central region of the distribution. Maharaj and Maartens [3] have obtained models of spherical anisotropic distributions with constant density. Gokhroo and Mehra [4] have extended this model to include anisotropic distributions with variable density. Recently, Dev and Gleiser [5] have given a number of exact solutions for various forms of the equation of state connecting the radial and tangential pressures.

When matter density of spherical objects is much higher than nuclear density, it is difficult to have a definite description of matter in the form of an equation of state. The uncertainty about the equation of state of matter beyond nuclear regime led to the consideration of a complementary approach [6–8], called core–envelope models. In this approach, a relativistic stellar configuration is made up of two regions: a core, surrounded by an envelope – containing matter distribution with different physical features. A detailed analysis of such models have been discussed by Hartle [8], and Iyer and Vishveshwara [9]. Core–envelope models with both pressure and density being continuous along the core boundary have been given by Negi, Pande and Durgapal [10]. A common feature of the core–envelope models discussed in literature is that their core and envelope regions contain distributions of perfect fluid in equilibrium. In this paper we have considered core–envelope models with the feature – core consisting of fluid distribution with anisotropic pressure and envelope consisting of fluid with isotropic pressure.

2. Space-time metric and field equations

We begin with a static spherically symmetric distribution of matter in equilibrium with space-time metric given by

$$\mathrm{d}s^2 = -\frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 + \mathrm{e}^{\nu(r)}\mathrm{d}t^2, \tag{1}$$

where K and R are geometric parameters.

The t = constant sections of this space-time have the geometry of a 3-pseudospheroid with Cartesian equation:

$$\frac{w^2}{b^2} - \frac{x^2+y^2+z^2}{R^2} = 1, \tag{2}$$

immersed in a four-dimensional Euclidean space with metric:

$$\mathrm{d}\sigma^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \mathrm{d}w^2. \tag{3}$$

The geometric parameters K, R and b are related by

$$K = 1 + \frac{b^2}{R^2} > 1. \tag{4}$$

This can be easily seen by introducing transformations:

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta, \quad w = b\sqrt{1+\frac{r^2}{R^2}}. \tag{5}$$

The space-time metric (1) is regular everywhere as $K > 1$. When $K = 1$, the three space of metric (1) is flat, and when $K = 0$ it degenerates into open hyperboloid. It has been found that the space-time metric (1) is suitable to describe spherical distributions of superdense matter in equilibrium which are stable under radial pulsations [11].

The Einstein's field equation is

$$\mathbf{R}_{ij} - \frac{1}{2}\mathbf{R}g_{ij} = -\frac{8\pi G}{c^2}T_{ij}, \tag{6}$$

where g_{ij}, $\mathbf{R}_{ij}$ and $\mathbf{R}$ are the metric tensor, Ricci tensor and scalar curvature respectively; and T_{ij} is the energy–momentum tensor. Following [3], the energy–momentum tensor is given by

$$T_{ij} = \left(\rho + \frac{p}{c^2}\right)u_iu_j - \left(\frac{p}{c^2}\right)g_{ij} + \pi_{ij}, \quad u_iu^i = 1, \tag{7}$$

where ρ, p and u_i respectively denote matter density, isotropic pressure, and unit four-velocity vector. The anisotropic stress tensor is given by

$$\pi_{ij} = \sqrt{3}\left(\frac{S}{c^2}\right)\left[c_i c_j - \frac{1}{3}\left(u_i u_j - g_{ij}\right)\right]. \tag{8}$$

For radially symmetric anisotropic fluid distribution of matter, $S = S(r)$ denotes the magnitude of the anisotropic stress tensor and

$$c^i = \left(-\mathrm{e}^{-\lambda/2}, 0, 0, 0\right) \tag{9}$$

is a space-like radial vector. For equilibrium models

$$u = \left(0, 0, 0, \mathrm{e}^{\nu/2}\right). \tag{10}$$

The energy–momentum tensor (7) has non-vanishing components

$$T_1^1 = -\frac{1}{c^2}\left(p + \frac{2S}{\sqrt{3}}\right), \quad T_2^2 = T_3^3 = -\frac{1}{c^2}\left(p - \frac{S}{\sqrt{3}}\right), \quad T_4^4 = \rho. \tag{11}$$

The pressure along the radial direction

$$p_{\mathrm{r}} = p + \frac{2S}{\sqrt{3}} \tag{12}$$

will be different from the pressure along the tangential direction

$$p_{\perp} = p - \frac{S}{\sqrt{3}}. \tag{13}$$

The magnitude of anisotropic stress tensor is given by

$$S = \frac{p_{\mathrm{r}} - p_{\perp}}{\sqrt{3}}. \tag{14}$$

The field equation corresponding to metric (1) is given by a set of three equations:

$$\frac{8\pi G}{c^4}\left(p + \frac{2S}{\sqrt{3}}\right) = \left[\left(1 + \frac{r^2}{R^2}\right)\frac{\nu'}{2} - \frac{K-1}{R^2}\right]\left(1 + K\frac{r^2}{R^2}\right)^{-1}, \tag{15}$$

$$\begin{aligned}\frac{8\pi G}{c^4}S = &-\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'}{2r}\right)\left(1 + \frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-1} + \frac{(K-1)\,r}{R^2}\\ &\times\left(\frac{\nu'}{2} + \frac{1}{r}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-2} - \frac{(K-1)}{R^2}\left(1 + K\frac{r^2}{R^2}\right)^{-1},\end{aligned} \tag{16}$$

$$\frac{8\pi G}{c^2}\rho = \frac{3(K-1)}{R^2}\left(1 + \frac{K}{3}\frac{r^2}{R^2}\right)\left(1 + K\frac{r^2}{R^2}\right)^{-2}. \tag{17}$$

Thus the ansatz of assigning pseudospheroidal geometry to the physical space of interior space-time of relativistic fluid distribution comprising of core–envelope regions of matter in equilibrium provides the density profile (17) which is positive throughout the regions. Further, since

$$\frac{1}{c^2}\frac{\mathrm{d}\rho}{\mathrm{d}r} = -\frac{5K(5K-1)}{4\pi G R^4}\left(1+\frac{K}{5}\frac{r^2}{R^2}\right)\left(1+K\frac{r^2}{R^2}\right)^{-3} < 0, \tag{18}$$

the matter density is decreasing radially outward.

We consider a star with anisotropic core having radial pressure p_{r} and transverse pressure $p_{\perp}$. At the core boundary $r = b$, the two pressures coincide and the envelope contains isotropic fluid distribution. The radial pressure decreases in the enveloping region and it becomes zero at the surface, say $r = a$, where a is the radius of the star under consideration.

We consider the core up to radius $r = b$, throughout which $S(r) \neq 0$. The radius of the star is taken as a and we divide it into two parts:

(i) $0 \leq r \leq b$ as the core of the star described by an anisotropic fluid distribution.
(ii) $b \leq r \leq a$ as the outer envelope of the core which can be described by an isotropic fluid distribution.

3. The core of the star

The core of the star model is characterized by the anisotropic distribution of matter. Therefore throughout the core region $0 \leq r \leq b$, the radial pressure p_{r} is different from the tangential pressure $p_{\perp}$ and hence $S(r) \neq 0$. To obtain a solution for (16), we introduce new variables z and ψ defined by

$$z = \sqrt{1+\frac{r^2}{R^2}}, \quad \psi = \frac{\mathrm{e}^{\nu/2}}{(1-K+Kz^2)^{1/4}} \tag{19}$$

in terms of which eq. (16) assumes the form

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}z^2} + \left[\frac{2K\,(2K-1)\left(1-K+Kz^2\right) - 5K^2z^2}{4\left(1-K+Kz^2\right)^2} + \frac{8\sqrt{3}\pi G R^2 S\left(1-K+Kz^2\right)}{c^4\left(z^2-1\right)}\right]\psi = 0. \tag{20}$$

Since the nature of the anisotropy parameter S is not known precisely in superdense matter distribution it is not possible to solve eq. (20). So we have the freedom to make *ad hoc* assumptions on S which facilitate integration of (20) without sacrificing simplicity and regularity of S throughout its region of validity.

We prescribe the anisotropy parameter as

$$S = -\frac{c^4\left(z^2-1\right)\left[2K\,(2K-1)\left(1-K+Kz^2\right) - 5K^2z^2\right]}{32\pi G\sqrt{3}R^2\left(1-K+Kz^2\right)^3}. \tag{21}$$

We note from eq. (21) that S vanishes for $z = 1$ and hence regular at the origin. It also vanishes at

$$z = \sqrt{\frac{(K-1)(2K-1)}{2K(K-1.75)}}. \tag{22}$$

In fact, this property of S facilitates the construction of core–envelope models with anisotropic core and isotropic envelope.

The prescription of S that is given by (21) reduces the coefficient of ψ in (20) to zero and the resulting equation admits the general solution

$$\psi = Cz + D, \tag{23a}$$

where C and D are constants of integration, leading to the simple solution

$$\mathrm{e}^{\nu/2} = \left(1 + K\frac{r^2}{R^2}\right)^{1/4}\left(C\sqrt{1+\frac{r^2}{R^2}} + D\right). \tag{23b}$$

The space-time metric of the core region $0 \le r \le b$ is explicitly written as

$$\begin{aligned}\mathrm{d}s^2 = &-\frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\phi^2 \\ &+\sqrt{1+K\frac{r^2}{R^2}}\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)^2\mathrm{d}t^2.\end{aligned} \tag{24}$$

The radial pressure p_{r} and anisotropy parameter $S(r)$ are now given by

$$\frac{8\pi G}{c^4}p_{\mathrm{r}} = \frac{\begin{array}{c}C\sqrt{1+\frac{r^2}{R^2}}\left[3+2K\left(\frac{r^2}{R^2}\right)+K(2-K)\frac{r^2}{R^2}\right] \\ +D\left[1+K(2-K)\frac{r^2}{R^2}\right]\end{array}}{R^2\left(1+K\frac{r^2}{R^2}\right)^2\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)}, \tag{25}$$

$$\frac{8\pi G}{c^4}\sqrt{3}S = -\frac{\left(\frac{r^2}{R^2}\right)\left[2K(2K-1)\left(1+K\frac{r^2}{R^2}\right) - 5K^2\left(1+\frac{r^2}{R^2}\right)\right]}{4R^2\left(1+K\frac{r^2}{R^2}\right)^3}. \tag{26}$$

If we take $K = 2$, the matter density ρ, radial pressure p_{r} and the anisotropy parameter $S(r)$ take the simple form

$$\frac{8\pi G}{c^2}\rho = \frac{\left(3+2\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)^2}, \tag{27}$$

$$\frac{8\pi G}{c^4}p_{\mathrm{r}} = \frac{C\sqrt{1+\frac{r^2}{R^2}}\left(3+4\frac{r^2}{R^2}\right)+D}{R^2\left(1+2\frac{r^2}{R^2}\right)^2\left(C\sqrt{1+\frac{r^2}{R^2}}+D\right)}, \tag{28}$$

$$\frac{8\pi\sqrt{3}G}{c^4}S = \frac{\left(\frac{r^2}{R^2}\right)\left(2-\frac{r^2}{R^2}\right)}{R^2\left(1+2\frac{r^2}{R^2}\right)^3}. \tag{29}$$

The anisotropic parameter given by (29) has the following desired features:

(i) $S(r)$ vanishes at the centre, ensuring the regularity of the distribution.
(ii) $S(r)$ increases with r in the neighbourhood of the centre, reaches a maximum value, and subsequently decreases as r increases.
(iii) The form of $S(r)$ is suitable to describe anisotropic matter distribution in the core region for which both radial and tangential pressures become equal at a suitably chosen core boundary $r = b = \sqrt{2}R$, where $S(r)$ vanishes.

4. The envelope of the star

The envelope of the star model is characterized by the isotropic distribution of matter. Hence throughout the enveloping region $b \le r \le a$, the radial pressure p_{r} equals the tangential pressure $p_{\perp}$ and hence $S(r) = 0$ in this region. Then eq. (16) reduces to

$$\begin{aligned}&-\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'}{2r}\right)\left(1+\frac{r^2}{R^2}\right)\left(1+K\frac{r^2}{R^2}\right)\\&+\frac{(K-1)\,r}{R^2}\left(\frac{\nu'}{2}+\frac{1}{r}\right)-\frac{(K-1)}{R^2}\left(1+K\frac{r^2}{R^2}\right)=0.\end{aligned} \tag{30}$$

If we choose variables z and F defined by

$$z=\sqrt{1+\frac{r^2}{R^2}},\quad F=\mathrm{e}^{\nu/2}, \tag{31}$$

eq. (30) takes the form

$$\left(1-K+Kz^2\right)\frac{\mathrm{d}^2F}{\mathrm{d}z^2}-Kz\frac{\mathrm{d}F}{\mathrm{d}z}+K\,(K-1)\,F=0. \tag{32}$$

Equation (32) admits closed form solution for all values of K. In particular, when $K = 2$, it takes the form

$$F=\mathrm{e}^{\nu/2}=A\sqrt{1+\frac{r^2}{R^2}}+B\left[\sqrt{1+\frac{r^2}{R^2}}L(r)-\frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right], \tag{33}$$

where A and B are constants of integration and

$$L(r)=\sqrt{1+\frac{r^2}{R^2}}\ln\left(\sqrt{2}\sqrt{1+\frac{r^2}{R^2}}+\sqrt{1+2\frac{r^2}{R^2}}\right). \tag{34}$$

The space-time metric of the enveloping region $b \le r \le a$ is explicitly given by

$$\mathrm{d}s^2 = -\frac{1+2\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 + \left\{A\sqrt{1+\frac{r^2}{R^2}} + B\left[\sqrt{1+\frac{r^2}{R^2}}L(r) - \frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right]\right\}^2 \mathrm{d}t^2. \tag{35}$$

The density and isotropic pressure of the distribution are given by

$$\frac{8\pi G}{c^2}\rho = \frac{3+2\frac{r^2}{R^2}}{R^2\left(1+2\frac{r^2}{R^2}\right)^2}, \tag{36}$$

$$\frac{8\pi G}{c^4}p = \frac{A\sqrt{1+\frac{r^2}{R^2}} + B\left[\sqrt{1+\frac{r^2}{R^2}}L(r) + \frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right]}{R^2\left(1+2\frac{r^2}{R^2}\right)\left\{A\sqrt{1+\frac{r^2}{R^2}} + B\left[\sqrt{1+\frac{r^2}{R^2}}L(r) - \frac{1}{\sqrt{2}}\sqrt{1+2\frac{r^2}{R^2}}\right]\right\}}. \tag{37}$$

The constants A and B appearing in (35) and (37) are to be determined by matching (35) with the Schwarzchild exterior metric

$$\mathrm{d}s^2 = -\left(1-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 + \left(1-\frac{2m}{r}\right)\mathrm{d}t^2 \tag{38}$$

across the boundary $r = a$ of the star, where $p(r) = 0$. The constants A and B have expressions:

$$A = \frac{\sqrt{1+\frac{a^2}{R^2}}L(a) + \frac{1}{\sqrt{2}}\sqrt{1+2\frac{a^2}{R^2}}}{\sqrt{2}\sqrt{1+2\frac{a^2}{R^2}}}, \tag{39}$$

$$B = -\frac{\sqrt{1+\frac{a^2}{R^2}}}{\sqrt{2}\sqrt{1+2\frac{a^2}{R^2}}}. \tag{40}$$

Though we can obtain core–envelope models for any value of K, we have considered the particular case $K = 2$ for two reasons. Firstly, we found in §3 that a simple form for p_{r} is possible for $K = 2$. Secondly, the isotropic fluid distribution with space-time metric (35) has been studied extensively by Tikekar and Thomas [11] and found that such space-times are suitable to represent superdense spherical distributions of matter stable under radial pulsations.

5. Physical plausibility

The core–envelope model obtained for $K = 2$ is a physically viable model if it complies with the following requirements in the core and enveloping regions.

(i) $\rho > 0$, $\frac{\mathrm{d}\rho}{\mathrm{d}r} < 0$ for $0 \le r \le a$,

(ii) $p_{\mathrm{r}} > 0$, $p_{\perp} > 0$, $\frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}r} < 0$ for $0 \le r \le b$,

(iii) $\rho - p_{\mathrm{r}} - 2p_{\perp} \ge 0$ for $0 \le r \le b$,

(iv) $\frac{1}{c^2}\frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}\rho} < 1$, $\frac{1}{c^2}\frac{\mathrm{d}p_{\perp}}{\mathrm{d}\rho} < 1$ for $0 \le r \le b$,

(v) $p > 0$, $\frac{1}{c^2}\frac{\mathrm{d}p}{\mathrm{d}\rho} < 1$ and $\rho - 3p > 0$ for $b \le r \le a$.

The expressions (27) and (18) clearly indicate the fulfillment of (i). The perfect fluid in the enveloping region is chosen to comply with the demands of (v) [11].

We expect the fulfillment of the following conditions at the boundary $r = b$ separating the core from the enveloping region.

(a) The space-time metric in the core region should smoothly match with the space-time metric in the enveloping region.

(b) The anisotropy parameter S should vanish and the anisotropic pressure of the fluid in the core region should continuously join with the pressure in the enveloping region.

The vanishing of anisotropy parameter $S(r)$ at $r = b$ determines the radius of the core $b = \sqrt{2}R$ from eq. (29).

The conditions (a) and (b) determine the arbitrary constants C and D as

$$C = 5^{-(5/4)}\left\{2A + \left[2\ln\left(\sqrt{5}+\sqrt{6}\right) + \sqrt{7.5}\right]B\right\}, \tag{41}$$

$$D = 5^{-(5/4)}\left\{3\sqrt{3}A + \left[3\sqrt{3}\ln\left(\sqrt{5}+\sqrt{6}\right) - 8\sqrt{2.5}\right]B\right\}, \tag{42}$$

where A and B are given by (39) and (40), respectively.

Substituting for C and D in (28), we get

$$\frac{8\pi G}{c^4}p_{\mathrm{r}} = \frac{(2A + 5.827B)\,z\left(4z^2 - 1\right) + (5.196A - 4.624B)}{R^2\left(2z^2-1\right)^2\left[(2A + 5.827B)\,z + (5.196A - 4.624B)\right]}. \tag{43}$$

Examining the positivity of radial pressure p_{r}, tangential pressure $p_{\perp}\left(= p_{\mathrm{r}} - \sqrt{3}S\right)$, energy condition and causality requirements in general, it is highly tedious due to the complexity of the expressions involved. However we have examined the above requirements numerically for certain specific models of this class.

6. Discussion

The scheme given by Tikekar [12], for estimating the mass and size of the fluid spheres on the background of spheroidal space-times can be used to determine the mass and size of the fluid distribution consisting of core and envelope.

Following this scheme we choose $\rho(a) = 2\times10^{14}$ g cm^{-3} as the density of matter at the boundary $r = a$ of the configuration and introduce a density variation parameter λ given by

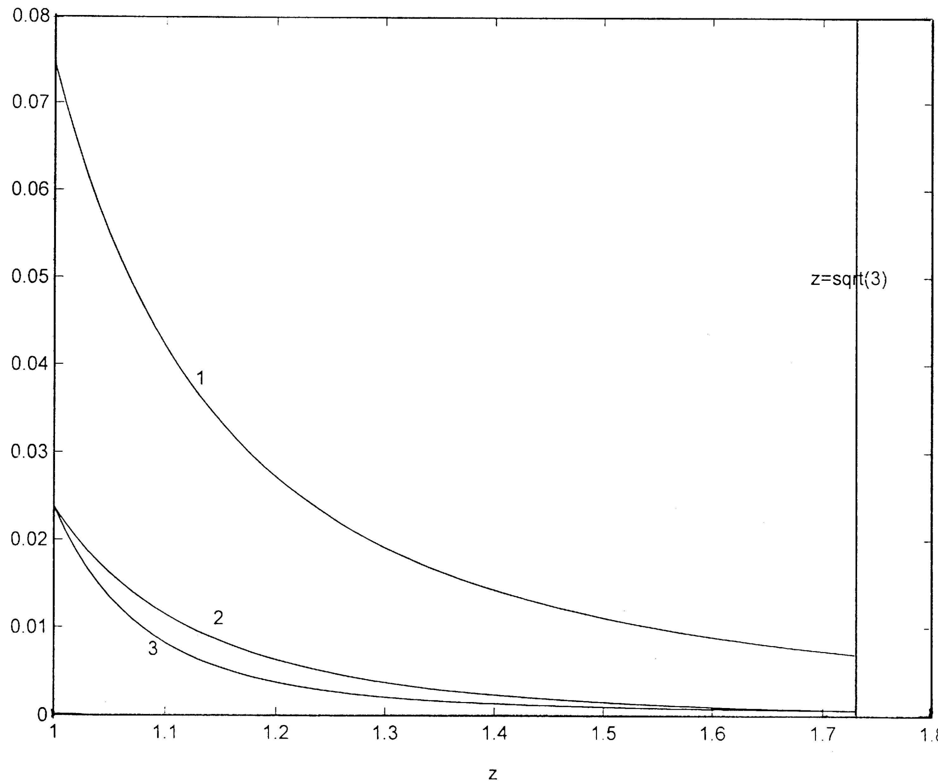

Figure 1. The graphs of $\bar{\rho} = \kappa\rho$, $\bar{p}_{\mathrm{r}} = (\kappa/c^2)p_{\mathrm{r}}$ and $\bar{p}_{\perp} = (\kappa/c^2)p_{\perp}$ in the core region, $1 \le z \le \sqrt{3}$, against $z = \sqrt{1 + (r^2/R^2)}$, where $\kappa = 8\pi G/c^2$. These graphs are denoted by 1, 2 and 3 respectively.

$$\lambda = \frac{\rho(a)}{\rho(0)} = \frac{1 + \frac{2}{3}\frac{a^2}{R^2}}{\left(1 + 2\frac{a^2}{R^2}\right)^2}. \tag{44}$$

Since ρ is a decreasing function of r, $\lambda < 1$.

Equation (44) determines the value of a^2/R^2 in terms of λ by the equation

$$\frac{a^2}{R^2} = \frac{1 - 6\lambda + \sqrt{24\lambda + 1}}{12\lambda}. \tag{45}$$

Equation (27) implies that the matter density at the centre is explicitly related with the curvature parameter R as

$$8\pi\rho(0) = 8\pi\frac{\rho(a)}{\lambda} = \frac{3}{R^2}. \tag{46}$$

Equation (46) determines R in terms of $\rho(a)$ and λ. The size of the configuration can be obtained from (45) in terms of surface density $\rho(a)$ and density variation parameter λ.

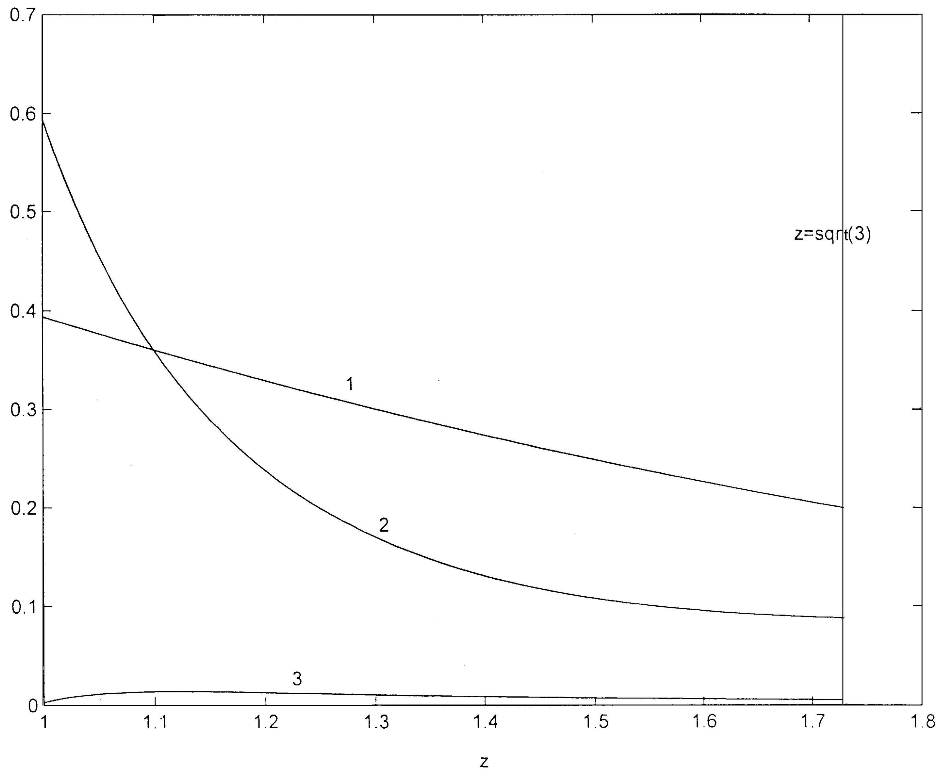

Figure 2. The graphs of $(1/c^2)(\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho)$, $(1/c^2)(\mathrm{d}p_{\perp}/\mathrm{d}\rho)$, and $\kappa(\rho - (1/c^2)$ $p_{\mathrm{r}} - (2/c^2)p_{\perp})$ in the core region, $1 \leq z \leq \sqrt{3}$, against $z = \sqrt{1 + (r^2/R^2)}$, where $\kappa = 8\pi G/c^2$. These graphs are denoted by 1, 2 and 3 respectively.

We choose $\lambda = 0.05$ and $\rho(a) = 2 \times 10^{14}$ g cm^{-3}. Equation (46) then implies that $R = 6.347$ km, $a/R = 1.907$ and $a = 12.107$ km. The core boundary is determined using $b = \sqrt{2}R$. It follows that $b = 8.82$ km. The matter density at the core boundary $\rho(b) = 3.75 \times 10^{14}$ g cm^{-3}. The constants A, B, C and D appearing in the metrics and that follow from (39), (40), (41) and (42) have values

$$A = 0.501, \quad B = -0.184, \quad C = -0.00936, \quad D = 0.462.$$

In figure 1, we have graphically shown the variation of $\bar{\rho} = \kappa\rho$, $\bar{p}_{\mathrm{r}} = (\kappa/c^2)p_{\mathrm{r}}$ and $\bar{p}_{\perp} = (\kappa/c^2)p_{\perp}$ in the core region against $z = \sqrt{1 + (r^2/R^2)}$ and in figure 2, we have shown the variations of $(1/c^2)(\mathrm{d}p_{\mathrm{r}}/\mathrm{d}\rho)$, $(1/c^2)(\mathrm{d}p_{\perp}/\mathrm{d}\rho)$ and $\kappa\left(\rho - (1/c^2)p_{\mathrm{r}} - (2/c^2)p_{\perp}\right)$ against $z = \sqrt{1 + (r^2/R^2)}$, where $\kappa = 8\pi G/c^2$.

This analysis indicates the physical viability of the specific model of this class. Accordingly, it is suggestive that this class is rich enough to describe physically plausible core–envelope models with the following salient features:

(i) The core region contains a distribution of anisotropic fluid and is surrounded by an envelope of perfect fluid at rest.

(ii) The density profile is continuous throughout, even at the core boundary.
(iii) The radial and tangential pressures are continuous throughout the core region and continuously join across the core boundary with isotropic pressure of the fluid in the envelope.

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