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An investigation of the influence of the pairing correlations on the properties of the isobar analog resonances in $A = 208$ nuclei

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Abstract. Within the quasi-particle random phase approximation (QRPA), the method of the self-consistent determination of the isovector effective interaction which restores a broken isotopic symmetry for the nuclear part of the Hamiltonian is given. The effect of the pairing correlations between nucleons on the following quantities were investigated for the $A = 208$ nuclei: energies of the isobar analog 0^+ states, the isospin admixtures in the ground state of the even–even nuclei, and the differential cross-section for the ^{208}Pb(^{3}He,t)^{208}Bi reaction at $E(^3\text{He}) = 450$ MeV. Both couplings of the excitation branches with $T_z = T_0 \pm 1$, and the analog state with isovector monopole resonance (IVMR) in the quasi-particle representation were taken into account in our calculations. As a result of these calculations, it was seen that the pairing correlations between nucleons have no considerable effect on the $T = 23$ isospin admixture in the ground state of the ^{208}Pb nucleus, and they cause partially an increase in the isospin impurity of the isobar analog resonance (IAR). It was also established that these correlations have changed the isospin structure of the IAR states, and shifted the energies of the IVMR states to the higher values.

Keywords. Shell model; Hartree–Fock and random-phase approximations; beta decay.

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1. Introduction

The isospin admixtures of the nuclear states are very important in nuclear physics because they play a considerable role in the determination of the effective vector coupling constant based on the super-allowed Fermi transitions, and in the description of the energies and widths of the IAR and isospin multiplets [1–4]. In order to calculate these admixtures in nuclei, various models [5–12] have been proposed.

One of these models is the two-liquid hydrodynamic model described in ref. [5]. It is used to estimate the energies of the collective isovector monopole excitation with isospin $T_z = T_0 \pm 1$. The quantitative estimates using the shell model [6–8] are approximately an order of magnitude larger than the estimate of Bohr and Mottelson [9].

Moreover, theoretical calculations on the isospin admixtures for proton-rich nuclei have been performed using Hartree–Fock (HF) + random phase approximation (RPA) or Tamm–Dankoff approximation (TDA) [10–12].

Most of these studies do not treat the effective interaction between nucleons in a self-consistent way, i.e., the shell model potential and the effective interaction are chosen independently. In the calculations of the isospin admixture, the fulfillment of the isospin invariance of the Hamiltonian (without Coulomb interaction) which arises from the property that nuclear forces are charge-independent is very important, but it is generally violated by the isovector term of the average field. The same violation also holds in the presence of the static pairing correlations between nucleons. All these problems can be avoided by choosing the effective interactions in such a way that they are consistent with the average field, but not randomly chosen [13–16].

In ref. [16], the Coulomb isospin impurity in the $A = 208$ nuclei has been investigated using Pyatov–Salamov method [17] without the pairing correlations between nucleons. In ref. [18], both Fermi and Gamov–Teller strength is calculated for a number of nuclei in a wide excitation energy interval using the partially self-consistent pn-quasi-particle continuum random phase approximation (QCRPA).

In most of these studies, either the pairing correlations between nucleons are not considered [16] or the isospin admixture effects are not investigated even in the presence of these correlations [18]. Moreover, it has been stated in ref. [11] that the pairing correlations between nucleons could be of primary importance for medium and heavy nuclei.

In this paper, the self-consistent calculations of the isobar analog 0^+ states in the $A = 208$ nuclei and the Coulomb isospin mixing effects including the pairing correlations between nucleons have been carried out using the QRPA method. Our aim is to investigate the effect of these correlations on the energies of the IAR state, the isospin admixture in the ground state of the even–even nuclei, and the differential cross-section for the ^{208}Pb(^{3}He,t)^{208}Bi reaction at $E(^3\mathrm{He}) = 450$ MeV.

2. Hamiltonian

The same procedures followed for the particle space in ref. [15] will be repeated for the quasi-particle space in our study. Let us now consider a system of nucleons in a spherical symmetric average field interacting via pairing forces. The corresponding single quasi-particle (SQP) Hamiltonian is given by

$$\hat{H}_{\mathrm{SQP}} = \sum_{\tau,j,m} \varepsilon_j^{(\tau)} \alpha_{jm}^{\dagger}(\tau)\alpha_{jm}(\tau) \qquad (\tau = \mathrm{n},\mathrm{p}), \tag{1}$$

where $\varepsilon_j^{(\tau)}$ is the single quasi-particle energy of the nucleons with angular momentum j, and $\alpha_{jm}^\dagger(\tau)(\alpha_{jm}(\tau))$ is the quasi-particle creation (annihilation) operator. In addition to the Coulomb potential term in the Hamiltonian given in eq. (1), there are isovector terms which also break the isospin invariance, i.e.,

$$[\hat{H}_{\rm SQP} - V_{\rm C}, \hat{T}^\rho] \neq 0, \tag{2}$$

where $V_{\rm C}$ is the Coulomb potential and it is given by the expression

$$V_{\rm C} = \sum_{i=1}^{A} v_{\rm c}(r_i)\left(\frac{1}{2} - t_z^i\right), \quad t_z^i = \begin{cases} 1/2, & \text{for neutrons;} \\ -1/2, & \text{for protons,} \end{cases}$$

with the radial part of the Coulomb potential

$$v_{\rm c}(r) = \frac{e^2(Z-1)}{Z}\int \frac{\rho_{\rm p}(r')}{|\vec{r}-\vec{r}\,'|}{\rm d}\vec{r}\,'. \tag{3}$$

Here, $\rho_{\rm p}(r')$ corresponds to the proton density distribution in the ground state. The isospin operators $\hat{T}^\rho$ are defined in the following way [19]:

$$\hat{T}^\rho = \frac{1}{2}[\hat{T}_+ + \rho\hat{T}_-] = \begin{cases} \hat{T}_x, & \rho = +1 \\ i\hat{T}_y, & \rho = -1 \end{cases}, \quad \hat{T}_\pm = \sum_{k=1}^{A} \hat{t}_\pm^k \,, \tag{4}$$

where $\hat{t}_+^k(\hat{t}_-^k)$ is the raising (lowering) isospin operators. The restoration of this isospin violation for the isovector term is very important to have a correct description of the IAR state in the odd–odd nuclei and the isospin admixtures with $T_z = T_0 \pm 1$ in the ground state of the even–even nuclei. For this purpose, the effective interaction term $(\hat{h})$ is added to the Hamiltonian given in eq. (2) in such a way that the nuclear part of the Hamiltonian should be commutative with the component of the total isotopic spin, $\hat{T}^\rho$, i.e.,

$$[\hat{H}_{\rm SQP} + \hat{h} - V_{\rm C}, \hat{T}^\rho] = 0. \tag{5}$$

The effective interaction $(\hat{h})$ is defined as

$$\hat{h} = \sum_{\rho=\pm1} \frac{1}{4\gamma_\rho}[\hat{H}_{\rm SQP} - V_{\rm C}, \hat{T}^\rho]^\dagger[\hat{H}_{\rm SQP} - V_{\rm C}, \hat{T}^\rho], \tag{6}$$

where γ_ρ is an average of the double commutator in the ground state

$$\gamma_\rho = \frac{\rho}{2}\langle 0|[[\hat{H}_{\rm SQP} - V_{\rm C}, \hat{T}^\rho], \hat{T}^\rho]|0\rangle, \tag{7}$$

and it is, up to a sign, identical with the nuclear symmetry energy [15]. The form of the effective interaction in eq. (6) allows us to treat the Coulomb isospin mixing effects in a simple and self-consistent way.

3. Isobaric states

We shall use the eigenstates of the single quasi-particle Hamiltonian as a basis and consider the isobaric 0^+ excitations in odd–odd nuclei generated from the correlated ground state of the parent even–even nucleus by the charge-exchange forces. The basis set of the neutron–proton quasi-particle pair creation and annihilation operators is defined as

$$\hat{A}^{\dagger}_{j_{\rm n}j_{\rm p}} = \frac{1}{\sqrt{2j_{\rm n}+1}} \sum_m (-1)^{j_{\rm n}-m} \alpha^{\dagger}_{j_{\rm p}m} \alpha^{\dagger}_{j_{\rm n},-m}, \quad \hat{A}_{j_{\rm n}j_{\rm p}} = \left(\hat{A}^{\dagger}_{j_{\rm n}j_{\rm p}}\right)^{\dagger}. \tag{8}$$

The bosonic commutation rules of these operators in the quasi-boson approximation are given by

$$[\hat{A}_{j_{\rm n}j_p}, \hat{A}^{\dagger}_{j_{\rm n'}} j_{\rm p}] \approx \delta_{j_{\rm n}j_{\rm n'}} \delta_{j_{\rm p}j_{\rm p'}}, \quad [\hat{A}_{j_{\rm n}j_{\rm p}}, \hat{A}_{j_{\rm n'}j_{\rm p'}}] = 0. \tag{9}$$

In the quasi-particle space, the effective interaction ($\hat{h}$) and the average of the double commutator (γ_ρ) can be written as

$$\hat{h} = -\sum_{j_{\rm n},j_{\rm p},j_{\rm n'},j_{\rm p'},\rho} \frac{1}{4\chi_\rho} E^{\rho}_{j_{\rm n}j_{\rm p}} E^{\rho}_{j_{\rm n'}j_{\rm p'}} \left(\hat{A}_{j_{\rm n}j_{\rm p}} - \rho \hat{A}^{\dagger}_{j_{\rm n}j_{\rm p}}\right) \left(\hat{A}^{\dagger}_{j_{\rm n'}j_{\rm p'}} - \rho \hat{A}_{j_{\rm n'}j_{\rm p'}}\right),$$

$$\chi_\rho = -\gamma_\rho = \sum_{j_{\rm n},j_{\rm p}} b^{\rho}_{j_{\rm n}j_{\rm p}} E^{\rho}_{j_{\rm n}j_{\rm p}}, \quad b^{\rho}_{j_{\rm n}j_{\rm p}} = \frac{1}{2}(\bar{b}_{j_{\rm n}j_{\rm p}} + \rho b_{j_{\rm n}j_{\rm p}}), \tag{10}$$

with

$$\begin{aligned}
E^{\rho}_{j_{\rm n}j_{\rm p}} &= \left\{ \varepsilon_{j_{\rm n}j_{\rm p}} b^{\rho}_{j_{\rm n}j_{\rm p}} + \frac{1}{2}(\bar{\varphi}_{j_{\rm n}j_{\rm p}} - \rho \varphi_{j_{\rm n}j_{\rm p}}) \right\}, \\
\varepsilon_{j_{\rm n}j_{\rm p}} &= \varepsilon_{j_{\rm n}} + \varepsilon_{j_{\rm p}}, \\
b_{j_{\rm n}j_{\rm p}} &= \sqrt{2j_{\rm p}+1} \left\langle j_{\rm p} \left\| \hat{t}_- \right\| j_{\rm n} \right\rangle u_{j_{\rm p}} v_{j_{\rm n}}, \\
\bar{b}_{j_{\rm n}j_{\rm p}} &= \sqrt{2j_{\rm p}+1} \left\langle j_{\rm p} \left\| \hat{t}_- \right\| j_{\rm n} \right\rangle u_{j_{\rm n}} v_{j_{\rm p}}, \\
\varphi_{j_{\rm n}j_{\rm p}} &= \sqrt{2j_{\rm p}+1} \left\langle j_{\rm p} \left\| \hat{t}_- v_{\rm c}(r) \right\| j_{\rm n} \right\rangle u_{j_{\rm p}} v_{j_{\rm n}}, \\
\bar{\varphi}_{j_{\rm n}j_{\rm p}} &= \sqrt{2j_{\rm p}+1} \left\langle j_{\rm p} \left\| \hat{t}_- v_{\rm c}(r) \right\| j_{\rm n} \right\rangle u_{j_{\rm n}} v_{j_{\rm p}}, \\
\varepsilon_{j_\tau} &= \left[C_\tau^2 + (E_{j_\tau} - \lambda_\tau)^2\right]^{1/2}, \\
u_{j_\tau} &= \left[\frac{1}{2}\left(1 + \frac{E_{j_\tau} - \lambda_\tau}{\varepsilon_{j_\tau}}\right)\right]^{1/2}, \\
v_{j_\tau} &= \left[\frac{1}{2}\left(1 - \frac{E_{j_\tau} - \lambda_\tau}{\varepsilon_{j_\tau}}\right)\right]^{1/2},
\end{aligned} \tag{11}$$

where C_τ, E_{j_τ}, λ_τ correspond to the pairing correlation parameter, the single particle energies of nucleons, and the chemical potential, respectively. The $v_{j_\tau}(u_{j_\tau})$'s are the occupation (unoccupation) amplitudes which are obtained in BCS calculation.

In QRPA, the collective 0^+ states are considered as one-phonon excitations described by

$$|\psi_i\rangle = \hat{Q}_i^\dagger|0\rangle = \sum_{j_\mathrm{n},j_\mathrm{p}} (r^i_{j_\mathrm{n}j_\mathrm{p}} \hat{A}^\dagger_{j_\mathrm{n}j_\mathrm{p}} - s^i_{j_\mathrm{n}j_\mathrm{p}} \hat{A}_{j_\mathrm{n}j_\mathrm{p}})|0\rangle, \tag{12}$$

where $\hat{Q}_i^\dagger$ is the phonon creation operator, $|0\rangle$ is the phonon vacuum ($\hat{Q}_i|0\rangle = 0$) corresponding to the ground state of the even–even nucleus; $r^i_{j_\mathrm{n}j_\mathrm{p}}$, and $s^i_{j_\mathrm{n}j_\mathrm{p}}$ are the neutron–proton quasi-particle pair amplitudes. Assuming that the phonon operators obey the commutation relations given below

$$\langle 0|[\hat{Q}_i, \hat{Q}_j^\dagger]|0\rangle = \delta_{ij}, \quad \langle 0|[\hat{Q}_i, \hat{Q}_j]|0\rangle = 0, \tag{13}$$

we obtain the following orthonormalization condition for the amplitudes:

$$\sum_{j_\mathrm{n},j_\mathrm{p}} (r^i_{j_\mathrm{n}j_\mathrm{p}} r^{i'}_{j_\mathrm{n}j_\mathrm{p}} - s^i_{j_\mathrm{n}j_\mathrm{p}} s^{i'}_{j_\mathrm{n}j_\mathrm{p}}) = \delta_{ii'}. \tag{14}$$

The eigenvalues and the eigenfunctions of the restored Hamiltonian can be obtained by solving the equation of motion in QRPA,

$$\left[\hat{H}_\mathrm{SQP} + \hat{h}, \hat{Q}_i^\dagger\right]|0\rangle = w_i \hat{Q}_i^\dagger|0\rangle, \tag{15}$$

where the w_i's are the energies of the isobaric 0^+ states. Employing the conventional procedure of QRPA, we obtain the dispersion equation for the excitation energy of the isobaric 0^+ states as

$$\left[\chi_{+1} - \sum_{j_\mathrm{n},j_\mathrm{p}} \frac{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} (E^{+1}_{j_\mathrm{n}j_\mathrm{p}})^2}{\varepsilon^2_{j_\mathrm{n}j_\mathrm{p}} - w_i^2}\right] \left[\chi_{-1} - \sum_{j_\mathrm{n},j_\mathrm{p}} \frac{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} (E^{-1}_{j_\mathrm{n}j_\mathrm{p}})^2}{\varepsilon^2_{j_\mathrm{n}j_\mathrm{p}} - w_i^2}\right] - w_i^2 \sum_{j_\mathrm{n},j_\mathrm{p}} \left[\frac{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} E^{+1}_{j_\mathrm{n}j_\mathrm{p}} E^{-1}_{j_\mathrm{n}j_\mathrm{p}}}{\varepsilon^2_{j_\mathrm{n}j_\mathrm{p}} - w_i^2}\right]^2 = 0. \tag{16}$$

The neutron–proton quasi-particle pair amplitudes are analytically expressed in the following form:

$$\begin{aligned} r^i_{j_\mathrm{n}j_\mathrm{p}} &= \frac{1}{\sqrt{Z(w_i)}} \frac{E^{+1}_{j_\mathrm{n}j_\mathrm{p}} + L(w_i) E^{-1}_{j_\mathrm{n}j_\mathrm{p}}}{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} - w_i}, \\ s^i_{j_\mathrm{n}j_\mathrm{p}} &= \frac{1}{\sqrt{Z(w_i)}} \frac{E^{+1}_{j_\mathrm{n}j_\mathrm{p}} - L(w_i) E^{-1}_{j_\mathrm{n}j_\mathrm{p}}}{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} + w_i}, \end{aligned} \tag{17}$$

where

$$L(w_i) = \frac{\chi_{+1} - \sum_{j_\mathrm{n},j_\mathrm{p}} \frac{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} \left(E^{+1}_{j_\mathrm{n}j_\mathrm{p}}\right)^2}{\varepsilon^2_{j_\mathrm{n}j_\mathrm{p}} - w_i^2}}{w_i \sum_{j_\mathrm{n},j_\mathrm{p}} \frac{\varepsilon_{j_\mathrm{n}j_\mathrm{p}} E^{+1}_{j_\mathrm{n}j_\mathrm{p}} E^{-1}_{j_\mathrm{n}j_\mathrm{p}}}{\varepsilon^2_{j_\mathrm{n}j_\mathrm{p}} - w_i^2}},$$

and $Z(w_i)$ is determined from the condition (14).

The solutions of eq. (16) contain the IAR state. This is easily seen for the case of the constant Coulomb potential

$$\left\langle j_{\rm p} \left\| \hat{t}_{-} v_{\rm c}(r) \right\| j_{\rm n} \right\rangle = \Delta E_{\rm C} \left\langle j_{\rm p} \left\| \hat{t}_{-} \right\| j_{\rm n} \right\rangle, \ \Delta E_{\rm C} = \text{constant}. \tag{18}$$

Equation (16) now contains the solution for $w_k = \Delta E_{\rm C}$ corresponding to the average energy of the single quasi-particle Coulomb shift of the (N, Z) and $(N-1, Z+1)$ nuclei. From eqs (12), (14) and (17), it follows that

$$\hat{Q}_k^{\dagger} |0\rangle_{w_k = \Delta E_{\rm C}} = \frac{1}{\sqrt{2T_0}} \hat{T}_{-} |0\rangle, \tag{19}$$

i.e., this solution describes the IAR state. Because eq. (13) is fulfilled, the isospin is exactly conserved in all states.

4. Fermi beta transitions

It has been shown in ref. [15] that two independent isobaric excited 0^+ states exist when no pairing correlations between nucleons are considered: the states with $T_z = T_0 - 1$ which also include the IAR state in the $(N-1, Z+1)$ nucleus, and the excited states with $T_z = T_0 + 1$ occurring in the $(N+1, Z-1)$ nucleus. Furthermore, it must be noted that these two branches depend on the mother nucleus due to the $\beta^{\pm}$ transition matrix elements which obey the Fermi sum rule.

In the presence of the pairing correlations, the two branches mentioned above are no longer independent of each other and occur in the $(N-1, Z+1)$ and $(N+1, Z-1)$ nuclei, since the particle number conservation is violated. Moreover, the total probability of the $\beta^{\pm}$ transitions from the parent (N, Z) nucleus to the $(N-1, Z+1)$ and $(N+1, Z-1)$ nuclei increases.

The 0^+ states in the neighbor odd–odd nuclei are characterized by the Fermi transition matrix elements between these states and the ground state of the even–even nuclei. Using the wave functions in eq. (12), the corresponding matrix elements can be written as follows:

a) For the transitions $(N, Z) \Longrightarrow (N-1, Z+1)$,

$$M_{\beta^-}^i = \langle 0 | [\hat{Q}_i, \hat{T}_{-}] | 0 \rangle = \sum_{j_{\rm n}, j_{\rm p}} (r_{j_{\rm n} j_{\rm p}}^i b_{j_{\rm n} j_{\rm p}} + s_{j_{\rm n} j_{\rm p}}^i \bar{b}_{j_{\rm n} j_{\rm p}}). \tag{20}$$

b) For the transitions $(N, Z) \Longrightarrow (N+1, Z-1)$,

$$M_{\beta^+}^i = \langle 0 | [\hat{Q}_i, \hat{T}_{+}] | 0 \rangle = \sum_{j_{\rm n}, j_{\rm p}} (r_{j_{\rm n} j_{\rm p}}^i \bar{b}_{j_{\rm n} j_{\rm p}} + s_{j_{\rm n} j_{\rm p}}^i b_{j_{\rm n} j_{\rm p}}). \tag{21}$$

It is possible to show that the transitions in question obey the Fermi sum rule given below.

$$S^{(-)} - S^{(+)} = 2T_0 = N - Z, \tag{22}$$

where

$$S^{(\pm)} = \sum_i |M^i_{\beta\pm}|^2.$$

The matrix element for the analog state obtained in eq. (18) is non-vanishing in the case of the constant Coulomb shift, and it is stated by the expression $M^{\rm IAR}_{\beta^-} = \sqrt{2T_0}$. This matrix element exhausts the full strength of the Fermi transition.

5. Isospin admixture of the states

When we investigate the isospin structure of the ground state of the considered nuclei, we see that the isospin impurity of the ground states and the IAR state is related to the $M_{\beta\pm}$ matrix elements [15]. Expanding the ground state wave function in the pure isospin components $|T, T_z\rangle$, we obtain

$$|0\rangle = a|T_0, T_0\rangle + b|T_0 + 1, T_0\rangle, \quad a^2 + b^2 = 1. \tag{23}$$

Using the expansions in eqs (20) and (21), we can calculate the expectation value of the square of the isospin in the ground state of the parent nucleus:

$$\langle 0|\hat{T}^2|0\rangle = T_0(T_0 + 1) + \sum_i |M^i_{\beta+}|^2. \tag{24}$$

On the other hand, from eq. (23), we have

$$\langle 0|\hat{T}^2|0\rangle = T_0(T_0 + 1) + 2b^2(T_0 + 1). \tag{25}$$

Comparing eqs (24) and (25), we see that the $T_0 + 1$ isospin admixture in the ground state of the parent nucleus is determined by the sum of the squares of the β^+ transition matrix elements from the isobaric states of the nucleus:

$$b^2 = [2(T_0 + 1)]^{-1} \sum_i |M^i_{\beta+}|^2. \tag{26}$$

Using the operator $\hat{T}_-$, we can generate a collective analog state in the $(N-1, Z+1)$ nucleus:

$$|A\rangle = \{\langle 0|\hat{T}_+\hat{T}_-|0\rangle\}^{-1/2}\hat{T}_-|0\rangle. \tag{27}$$

This state is not generally an eigenstate of the Hamiltonian $(\hat{H}_{\rm SQP} + \hat{h})$. It is distributed over the spectrum of the isobaric states $(\hat{Q}^\dagger_i|0\rangle)$ which contain the T_0-1, T_0, $T_0 + 1$ and $T_0 + 2$ isospin admixtures. Thus for the IAR state [15], we have

$$\begin{aligned}\hat{Q}^\dagger_i|0\rangle = {} & \gamma_{\rm IAR}|T_0 - 1, T_0 - 1\rangle + \alpha_{\rm IAR}|T_0, T_0 - 1\rangle \\ & +\beta_{\rm IAR}|T_0 + 1, T_0 - 1\rangle + \Delta_{\rm IAR}|T_0 + 2, T_0 - 1\rangle.\end{aligned} \tag{28}$$

Neglecting the small $T_0 + 2$ isospin admixtures in the IAR state, we obtain [15]

$$\alpha_{\mathrm{IAR}} = \frac{a\,\sqrt{T_0/2}M_{\beta^-}^{\mathrm{IAR}}}{T_0 + b^2(T_0+1)},$$

$$\beta_{\mathrm{IAR}} = \frac{b\,\sqrt{(2T_0+1)/2}M_{\beta^-}^{\mathrm{IAR}}}{T_0 + b^2(T_0+1)},$$

$$\gamma_{\mathrm{IAR}}^2 = 1 - \alpha_{\mathrm{IAR}}^2 - \beta_{\mathrm{IAR}}^2. \tag{29}$$

The presence of the T_0+1 isospin admixtures for the IAR state considerably affects the calculated isospin-mixing effects in the super-allowed Fermi transition. The isospin-mixing effects are usually expressed as a correction factor δ_{c} in the matrix element of a super-allowed transition [1,2,4,8]:

$$|M_{\beta^+}^{\mathrm{IAR}}|^2 = 2T_0(1-\delta_{\mathrm{c}}). \tag{30}$$

6. Differential cross-section for the ^{208}Pb(^{3}He,t)^{208}Bi reactions

The experimental studies [20–22] have shown that a possible candidate for the excitation of the Gamow–Teller (GT) and the IAR state can be the (^{3}He,t) reaction. There may be several reasons for this. First of all, the (^{3}He,t) reaction has recently been successfully measured with high resolution using an ion optics technique 'dispersion matching' at the A1200 facility in NSCL. This success at NSCL opened a new possibility of investigating the spin–isospin resonances on the β^+ side via the (^{3}He,t) reaction at intermediate energies.

Secondly, Fujiwara *et al* [20] have compared the ratio of the cross-section at zero degree in (^{3}He,t) for transitions to the ground state of ^{12}N to the 3.51 MeV $3/2^-$ state in ^{13}N. They have obtained a reasonable value for this ratio at $E(^3\mathrm{He}) = 450$ MeV which is in agreement with the (^{3}He,t) values at higher energies. This result indicates that the single-step process is already predominant at 450 MeV, similar to the high energy case. Thus, they have accepted that the assumption that the (^{3}He,t) differential cross-section at zero degrees is proportional to the GT β decay strengths is reasonable. This approximate proportionality between GT strength and 0° cross-section at energies >100 MeV contributed significantly to the understanding of the GT strength in nuclei [21,22]. Therefore, the 0° cross-sections are very important in understanding the nature of the GT and IAR strengths. Since we are interested in the properties of the IAR state, we will present here only the 0° cross-sections for the excitation of the IAR state in the following form [22,23]:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}W}\right)_F (q\approx 0,\ \theta=0) = \left(\frac{\mu}{\pi\hbar^2}\right)^2 \left(\frac{k_f}{k_i}\right) N_F J_F^2 B(F), \tag{31}$$

where J_F is the volume integral of the central part of the effective interaction, N_F is the distortion factor which may be approximated by the function $\exp(-xA^{1/3})$, μ and k denote the reduced mass and wave number in the center of mass system, respectively. The value of x is given in ref. [20], and $B(F) = |M_{\beta^-}^{\mathrm{IAR}}|^2$ corresponds to the reduced matrix elements of the β transition to the IAR state.

7. Results and discussion

In this section, the numerical calculations for the isospin admixtures in the ground state of the ^{208}Pb nucleus and the 0° differential cross-section of the IAR state excited via the (^{3}He,t) reactions at $E(^{3}\mathrm{He}) = 450$ MeV are performed by considering the pairing correlations between nucleons and including the effective interaction term in a self-consistent way. Our numerical results obtained for the following cases are compared:

- *Case* 1: The single particle (SP) calculations (without effective interaction and pairing correlations).
- *Case* 2: The RPA calculations (the effective interaction ($\hat{h}$) without pairing correlations is considered).
- *Case* 3: The SQP calculations (the pairing correlations without the effective interaction ($\hat{h}$) are considered).
- *Case* 4: The QRPA calculations (both pairing correlations and the effective interaction ($\hat{h}$) are considered).

In numerical calculations, the Woods–Saxon potential with Chepurnov parametrization [24] was used. The basis contained all discrete and quasi-stationary states, and all the neutron and proton transitions changing the radial quantum number by Δn=0,1,2 were included. The values of the pair correlation functions, C_{n} and C_{p}, were taken from ref. [25]. For the volume integral in eq. (31), we take the value of $J_F = 53 \pm 5$ MeV·fm^3 [20]. The left-hand side of the sum rule given in eq. (22) is fulfilled with ≈1% accuracy.

Let us first examine the effect of the pairing correlations between nucleons on the admixture of the $|T = 23, T_z = 22\rangle$ isospin state in the ground state of the ^{208}Pb nucleus which arises from the Coulomb interaction between protons. It can be seen from eq. (26) that the $T_0 + 1$ isospin admixtures (b^2) can be determined by the total strength of the β^+ transition from the ground state of the ^{208}Pb nucleus to the excited states of the neighbor ^{208}Tl nucleus ($S^{(+)}$). The numerical values for the $T_0 + 1$ isospin admixture and the total β^+ transition strength calculated from different models are presented in table 1. It is obviously seen from this table that although the calculated SQP values for $S^{(+)}$ and b^2 are eight to nine times larger than the SP values, their RPA and QRPA values are very close to each other. Some explanation can be given to these results in view of the investigation of the excited 0^+ states spectrum in the ^{208}Tl nucleus shown in figures 1–4. In these figures, the energies (w_i) of the excited 0^+ states calculated from the ground state of the ^{208}Pb nucleus and the log(ft) values for the β^+ (left-hand side) and β^- (right-hand side) transitions to these excited states are presented. The bold lines at the bottom part of these figures correspond to the ground state of the investigated nucleus.

If we compare the spectra of the excited 0^+ states obtained in the SP and SQP models (see figures 1 and 3), it will be seen that the density of the excited 0^+ states and the β^+ transition rate increase when the pairing interactions between nucleons are taken into account. The reason for this increase can be attributed to the existence of the proton–neutron transitions with $\Delta n = 0$. In both spectra, there is a homogenous distribution for the β^+ transition rate. The RPA and QRPA

Table 1. The numerical values for the $T_0 + 1$ isospin admixture (b^2) and the total β^+ transition strength ($S^{(+)}$) calculated from different models.

	SP	RPA	SQP	QRPA
$S^{(+)}$	0.0223	0.153	0.182	0.152
b^2 (%)	0.0500	0.333	0.408	0.332

spectra for the excited 0^+ states shown in figures 2 and 4 indicate that there is no considerable change in the β^+ transition rate. The reason for the nearness of the RPA and QRPA values of the $T_0 + 1$ isospin admixture (b^2) and the total β^+ transition strength ($S^{(+)}$) can be given to the closeness of the β^+ transition rate values in these spectra. This means that the pairing correlations between nucleons with the inclusion of the effective interaction have no significant effect on the $T = 23$ isospin admixture in the ground state of the ^{208}Pb nucleus.

Let us now investigate the excited 0^+ states in ^{208}Bi, characterized by the β^- transitions from the ground state of the parent ^{208}Pb nucleus. The calculation results have been presented on the right-hand sides of figures 1–4. These excited states can be conditionally divided into three energy regions: the microscopic region (0–9 MeV), the region of Coulomb shift energy between the ^{208}Pb and ^{208}Bi isotopes (11–18 MeV), and the IVMR region (25–28 MeV). The SP calculation results for the w_i excitation energies and the log(ft) values of the β^- transition are presented in figure 1. As seen from this figure, the excited 0^+ states exist only in the microscopic

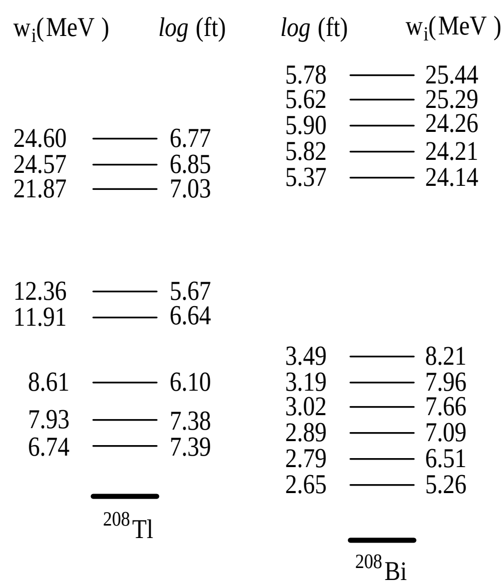

Figure 1. The SP calculations of the discrete spectrum of isobaric 0^+ excitations in ^{208}Tl and ^{208}Bi nuclei.

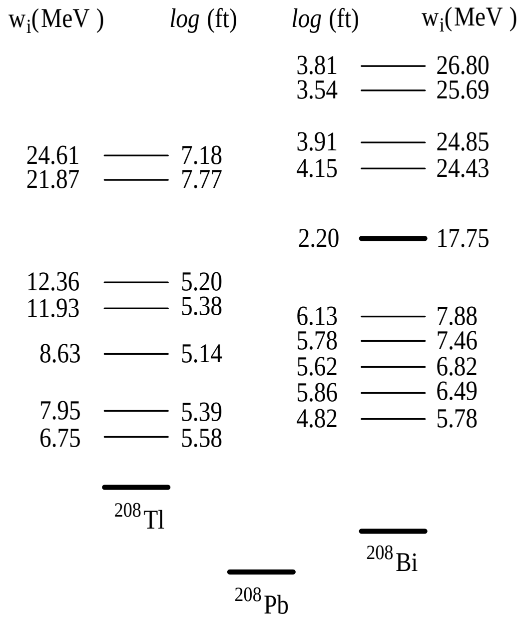

Figure 2. The RPA calculations of the discrete spectrum of isobaric 0^+ excitations in ^{208}Tl and ^{208}Bi nuclei.

and the IVMR regions. The excited 0^+ states in microscopic region are composed of the neutron particle–proton hole pair with $\Delta n = 0$, and the corresponding states are: $(3p^{\rm n}_{1/2}3p^{\rm p}_{1/2})_{0^+}$, $(3p^{\rm n}_{3/2}3p^{\rm p}_{3/2})_{0^+}$, $(2f^{\rm n}_{5/2}2f^{\rm p}_{5/2})_{0^+}$, $(2f^{\rm n}_{7/2}2f^{\rm p}_{7/2})_{0^+}$, $(1h^{\rm n}_{9/2}1h^{\rm p}_{9/2})_{0^+}$ and $(1i^{\rm n}_{13/2}1i^{\rm p}_{13/2})_{0^+}$. The square of the matrix element for the β^- transition from the ground state of the ^{208}Pb nucleus to the excited 0^+ states in the ^{208}Bi nucleus is almost equal to $2j+1$. Here, j is the total angular momentum of the nucleon involved in the transition. The total strength of the β^- transition to these six states exhausts 99.827% of the Fermi sum rule. Single particle calculations demonstrate that there is no state in the region of the Coulomb energy shift which corresponds to the IAR state. The total β^- transition strength of the excited 0^+ states in the IVMR region exhausts 0.173% of the Fermi sum rule.

The corresponding RPA calculation results are given in figure 2. As seen from this figure, due to the presence of the effective interaction, the distribution of the β^- transition probability changes. An IAR state occurs in the region of the Coulomb shift with an excitation energy of $w_{\rm IAR} = 17.75$ MeV, and this state exhausts 88.81% of the Fermi sum rule. All the excited 0^+ states in the microscopic region exhaust 1.58% of Fermi sum rule (note that the corresponding single particle value of this quantity is 99.827%). The total β^- transition strength of the excited 0^+ states in the IVMR region exhausts 9.61% of the Fermi sum rule. It can obviously be seen that the isospin impurity of the IAR state is due to the excited 0^+ states which are associated with IVMR existed in the energy region of 24.43–26.80 MeV. When the microscopic structure of the IAR state is studied, it has been seen that

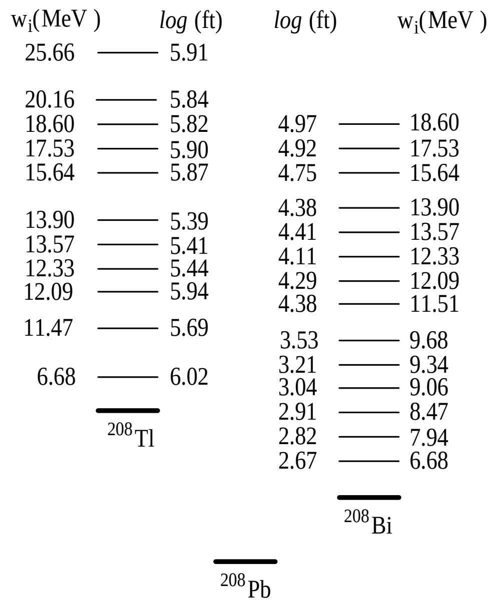

Figure 3. The SQP calculations of the discrete spectrum of isobaric 0^+ excitations in ^{208}Tl and ^{208}Bi nuclei.

this state is composed of the superposition of the six 0^+ states mentioned above. These six states constitute 99% of the norm of the IAR wave function.

Similar SQP calculation results are shown in figure 3. In this case, the significant contribution (95.68%) to the Fermi sum rule comes from the excited 0^+ states lying in the microscopic energy region, and the remaining part of the total β^- transition strength is distributed over the states in the Coulomb energy shift region.

The corresponding QRPA calculations are given in figure 4. When the effective interaction ($\hat{h}$) is taken into account, the redistribution of the excited 0^+ states exhausting the total β^- transition strength comes out. As stated in the RPA case (see figure 2), three energy regions (microscopic, Coulomb energy shift, and IVMR) also exist here. An IAR state with an excitation energy of $w_{\mathrm{IAR}} = 17.86$ MeV among the few excited states in the Coulomb shift region occurs. This state exhausts 89.92% of the Fermi sum rule, and it is also composed of the six 0^+ states mentioned before. The IVMR region lying in the energy interval of 25.64–28.0 MeV exhausts 7.28% of the total β^- transition strength (9.61% in the RPA case). The corresponding value in the microscopic region is 0.06%. When we compare figures 2 and 4, we see that the pairing correlations between nucleons cause a shift in the energies of the IAR state and the states in the IVMR region by 0.1 MeV and 1.2 MeV, respectively.

Let us briefly mention about the structure of the IAR state. The calculated values in different models for γ^2, α^2 and β^2 which characterize the $T_0 - 1$, T_0, and $T_0 + 1$ isospin components of the IAR state are given in table 2. As seen from this table, there is no considerable difference in the β^2 values. This is the natural consequence of the proportionality of this quantity to the $T_0 + 1$ isospin admixture (b^2) (see

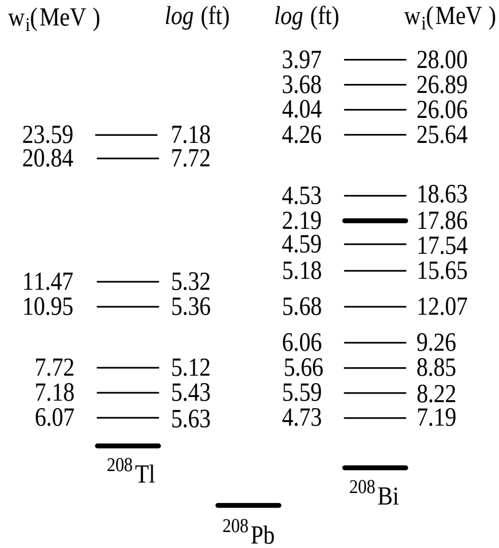

Figure 4. The QRPA calculations of the discrete spectrum of isobaric 0^+ excitations in ^{208}Tl and ^{208}Bi nuclei.

Table 2. The characteristic quantities of the IAR state in the ^{208}Bi nucleus calculated by different models.

	α^2	β^2	γ^2	δ_c (%)	$\frac{d\sigma}{d\Omega}$ (mb/sr)
RPA	89.50	0.611	9.89	9.52	9.28
QRPA	90.85	0.619	8.53	8.21	9.24

eq. (29)). The increase of the α^2 values in the QRPA calculation means that the isospin impurity of the IAR state increases with the consideration of the pairing interactions between nucleons. It is an expected result from eq. (29) that there will be a decrease in the γ^2 values. Let us note that the increase of the isospin impurity for the IAR state leads to a decrease in the δ_c value. This can be obviously seen from the fourth column of table 2. Finally, the 0° differential cross-section of the ^{208}Pb(^{3}He,t)^{208}Bi reaction at $E(^3\text{He}) = 450$ MeV for the excitation of the IAR state has been calculated. As seen from the last column of table 2, the RPA and QRPA values for the 0° differential cross-section are very close to each other. Moreover, it has been observed that these calculated values are in good agreement with the corresponding experimental [20] value (9.2±2 mb/sr).

8. Conclusion

We have investigated the influence of the pairing correlations between nucleons on the properties of the IAR state with the consideration of the effective interaction term ($\hat{h}$) obtained from Pyatov–Salamov method [17]. As a result of our calculations, the following conclusions can be drawn. Pairing correlations between nucleons

- have no significant effect on the $T = 23$ isospin admixture in the ground state of the ^{208}Pb nucleus,
- lead to a smooth splitting of the IAR state and a small decrease in the isospin mixing effects of the IAR state [$(\delta_c)_{\rm RPA} = 9.578\%$ and $(\delta_c)_{\rm QRPA} = 8.212\%$],
- have not affected the $T_0 + 1$ isospin admixtures of the IAR state but decreased slightly the $T_0 - 1$ isospin admixtures [$\gamma^2_{\rm RPA} = 9.89\%$ and $\gamma^2_{\rm QRPA} = 8.53\%$],
- shifted up the energies of the states in the IVMR region by an amount of 1.2 MeV.

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