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S. Chandrasekhar and Magnetohydrodynamics

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1. Introduction

This chapter summarizes the many fundamental contributions of Chandrasekhar to the subject of hydromagnetics or magnetohydrodynamics (MHD) with particular attention to the generation, static equilibrium, and dynamical stability-instability of magnetic field in various idealized settings with conceptual application to astronomical problems. His interest in MHD seems to have arisen first in connection with the turbulence of electrically conducting fluid in the presence of a magnetic field, sparked by Heisenberg's (1948a,b) formulation of an equation for the energy spectrum function $F(k)$ of statistically isotropic homogeneous hydrodynamic turbulence. From there Chandrasekhar's attention moved to the nature of the magnetic field along the spiral arm of the Galaxy (with E. Fermi), inferred from the polarization of starlight then recently discovered by Hall (1949) and Hiltner (1949, 1951). The polarization implied a magnetic field along the galactic arm, which played a key role in understanding the confinement of cosmic rays to the Galaxy. The detection and measurement of the longitudinal Zeeman effect in the spectra of several stars (Babcock and Babcock 1955) suggested the next phase of Chandrasekhar's investigations, in which he explored the combined effects of magnetic field, internal motion, and overall rotation on the figure of a star in stationary ($\partial/\partial t = 0$) equilibrium. Chandrasekhar and his students did some of the first work in formulating the quasi-linear field equations for the pressure, fluid velocity, and magnetic field in axisymmetric gravitating bodies. From there his thinking turned to the generation of the magnetic fields of planets and stars by the convective motions of the electrically conducting fluid in their interiors.

Now the outer atmosphere of planets, stars, and galaxies are so tenuous that in most cases the atmospheres do not exert significant forces on the strong external magnetic fields of these objects, so that the external magnetic field is "force-free", i.e., the Lorentz force, given by the divergence $\partial T_{ij} / \partial x_j$ of the Maxwell stress tensor T_{ij}, is negligible. The special properties of these force-free fields provide a particularly elegant mathematical formalism in the axisymmetric case.

Subsequently the challenging problem of laboratory plasmas confined in strong magnetic fields attracted Chandrasekhar's interest and, with A. N. Kaufman and K. M. Watson, he developed a perturbation solution to the collisionless Boltzmann equation in the strong field limit, applying the solution to the stability of the magnetic pinch.

During this same period of time Chandrasekhar investigated the effect of magnetic field on the convective instability of an electrically conducting fluid in an adverse temperature gradient. The question of the onset of convection is particularly important in the theory of stellar interiors, because the strong magnetic fields of some stars must surely have effects on the location and strength of the convection and the associated heat transport.

Chandrasekhar's interest in the effect of magnetic field on the dynamical stability of a convective system led to investigations of the effect on the Rayleigh-Taylor instability and the Kelvin Helmholtz instability. In the end he organized and compiled his results in a monumental tome entitled *Hydrodynamic and Hydromagnetic Stability* (Chandrasekhar 1961).

It is interesting to note that Chandrasekhar's direct involvement in MHD spanned a period of only twelve years, from about 1949 to 1961 when *Hydrodynamic and Hydromagnetic Stability* was published. Chandrasekhar's research papers are conveniently reprinted in organized form in six volumes (Selected Papers, S. Chandrasekhar, University of Chicago Press, 1989) and his work on magnetohydrodynamics is contained in volumes 3 and 4, to which we give reference at appropriate points, indicating the volume number, the paper number, and the page number in sequence within parentheses. The diversity of Chandrasekhar's contributions to MHD can be appreciated only from a detailed catalog of his publications. The present article attempts to provide sufficient perspective and detail within a reasonable span of pages to serve as an outline of the MHD papers in volumes 3 and 4.

2. Turbulence

Heisenberg's (1948a,b) heuristic formulation of statistically isotropic homogeneous hydrodynamic turbulence reproduced the basic results of Kolmogoroff (1941a,b) in terms of the energy spectrum function $F(k)$. Heisenberg (1948a,b) constructed a simple nonlinear integral equation for $F(k)$ based on the physical mixing length concept of eddy viscosity. Analytical solution provided the form of $F(k)$ for statistically steady conditions. The result yielded the inertial range $F(k) \sim k^{-5/3}$ extending from the small wave number k_o, at which the motion is driven, down to the viscous cutoff at the large wave number $k_s \sim k_0 N_R^{3/4}$ where N_R is the characteristic Reynolds number at the large scale k^{-1}_0. For $k >> k_s$ Heisenberg’s equations provided the tail $F(k) \sim k^{-7}$, whereas in the real world the cutoff beyond k_s is more abrupt. Nonetheless, there was a general feeling of optimism that the old and important problem of hydrodynamic turbulence was at last giving way to solution. The specter of intermittency etc. had not yet come to haunt the theoretical development.

Chandrasekhar was as intrigued as anyone and showed in 1949 (3, 24, 395) how Heisenberg's integral equation for statistically stationary turbulence could be reduced to a linear first order differential equation and one quadrature by a suitable choice of variables. He used $k^3F(k)$ for the dependent variable and the square of the total vorticity $\int_0^k dk\, k^2 F(k)$ for the independent variable. He went on to treat the more difficult time-dependent free decay of an initial turbulent state.

The next paper (3, 25, 409) picks up on the symmetry of the dynamical terms in the MHD equations to interchanging the velocity υ_j and the reduced magnetic field $b_j = B_j /(4\pi\rho)^{1/2}$ in an incompressible fluid. The symmetry is vividly displayed in terms of the Elsasser variables

$$U_i = v_i + b_j, \quad V_j = v_j - b_j,$$

for which the momentum and induction equations take the form

$$\begin{aligned} \frac{\partial U_j}{\partial t} + V_j \frac{\partial U_j}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \frac{1}{2}\nu \ \nabla^2 (U_j + V_j) + \frac{1}{2}\eta \nabla^2 (U_j - V_j) , \\ \frac{\partial V_j}{\partial t} + U_j \frac{\partial U_j}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \frac{1}{2}\nu \ \nabla^2 (U_j + V_j) - \frac{1}{2}\eta \nabla^2 (U_j - V_j) . \end{aligned}$$

The quantity P represents the total pressure

$$P = p + \frac{1}{2}\, \rho b_j b_j .$$

Chandrasekhar proceeded to apply the theory of invariants (Robertson 1940 and (3, 29, 442)) exploited earlier by Batchelor (1950) in connection with hydrodynamic turbulence, to the form of the double and triple correlations of υ_j and b_j. He worked out the relations between the scalar functions (coefficients) in the invariant forms for the correlations, obtaining the generalization of the hydrodynamic Von Karman-Howarth equation to MHD, and two additional relations.

The symmetry of the MHD equations in υ_j and b_j is complemented by identical forms of the induction equation

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{b}) + \eta \nabla^2 \mathbf{b}$$

and the vorticity equation in hydrodynamics,

$$\frac{\partial \omega}{\partial t} = \nabla \times (\mathbf{v} \times \omega) + \nu \nabla^2 \omega,$$

where $\omega = \nabla\times \nu$. This raises the question of whether there is a useful analogy between **b** and ω. Chandrasekhar explored the relation by writing $\mathbf{b} = \nabla\times \mathbf{a}$ in terms of the vector potential **a**. Then any analogy between ω and **b** appears as an analogy between **v** and **a**. Again the application of the theory of invariants provided forms for the double and triple correlations as well as equations relating the various scalar coefficients. But in neither formulation does one obtain enough equations to close the system without introducing additional and arbitrary assumptions. The failure to close is a result of the well known fact that the nonlinear terms in the MHD equations, like the hydrodynamic equations, provide the *n*th order correlation in terms of the (*n* + l)th order correlations, indicating that there is physics in the equations of (*n* + l)th order that is not contained up to *n*th order.

Chandrasekhar went on to show that MHD turbulence permits the construction of expressions analogous to the Lotsiansky invariant of hydrodynamic turbulence, based on similar assumptions as to the asymptotic rate of decline of correlations in u_j and in b_j between positions separated by large distance *r*.

In stationary MHD turbulence, sustained by the continual addition of kinetic energy at large scales, the scalar coefficients satisfy simpler relations and a direct analogy to the vorticity correlation $< \omega_j (\mathbf{r}) \omega_j (\mathbf{r} + \zeta) >$ is established.

So the double and triple correlations in MHD turbulence are interrelated much as in hydrodynamic turbulence. But, as already noted, the mathematics does not provide a closed system. Some physically motivated form of truncation of the equations is necessary.

We know much more about hydrodynamic and MHD turbulence now, 45 years later, thanks to the work of many the oreticians(cf. Kraichnan 1965), but a comprehensive deductive dynamical theory of turbulence still eludes the best efforts.

3. Galactic magnetic field

In the late forties the origin of cosmic rays was a problem of central interest beginning with their identification as (largely) protons by Schein, Jesse, and Wollan (1941). This led to the question of whether cosmic rays are a local phenomenon confined to the solar system by the dipole magnetic field of the Sun, or a non-local phenomenon presumably galactic in extent. Ideas of local confinement were based on a hypothetical highly symmetric solar magnetic dipole with a strength of 50 gauss at the poles of the Sun, suggested by the early work of Hale (1913). A dipole field declining as r^{-3} extrapolates from 50 gauss at the surface of the Sun to 5×10^{-6} gauss at 1 a.u, with $4 \times 10_7$ gauss cm beyond. This is sufficient to deflect a proton of 6 GeV through 180°, from which it follows that a solar dipole field might, in principle, temporarily trap protons of 6 GeV, but not much more. On the other hand, it is observed that cosmic rays arrive at the surface of Earth at the geomagnetic equator, after having penetrated through 10^8 gauss cm in the geomagnetic field. Such particles, with energies in excess of 10 Ge V, would not be trapped by the solar magnetic dipole. There was no observed break at 6 GeV in the energy spectrum of the cosmic rays. The cosmic ray intensely varied smoothly with geomagnetic latitude from the equator to the poles. So it appeared that cosmic rays are a galactic phenomenon.

Hiltner's (1949, 1951) studies of the polarization of starlight indicated a magnetic field of at least several microgauss along the local spiral arm. Unfortunately, it was not possible to deduce the strength of the galactic field from the observed polarization without having the precise composition and structure of the spinning interstellar dust grains that provide the polarization. However, Fermi (1949) suggested that cosmic rays are accelerated primarily by bouncing back and forth along the galactic field between reflections from moving magnetic gas clouds. So the structure and dynamics of the galactic magnetic field thrust itself upon the physics community as an important question. In the paper (3, 34, 529) Enrico Fermi and Chandrasekhar addressed the problem of the field strength from the observed dynamical properties of the galactic arm. The polarization studies (Hiltner 1949, 1951) suggested that the rms deflection of the magnetic field is about 0.2 radians. This deflection is presumably dynamical, representing transverse Alfvén waves for which the magnetic amplitude ΔB is related to the transverse amplitude v by $\Delta B=\pm(4\pi\rho)^{½}\ v$ for an interstellar gas density $\rho\ {\sim}2\times10^{-24}$gm/cm. An rms isotropic turbulent velocity of 5 km/sec suggested $5/\sqrt{3}\ \cong 3$ km/sec in the direction transverse to the mean field and to the line of sight, from which they obtained an estimate $B \sim 7\times10^{-6}$ gauss.

An alternative value was constructed by estimating the total pressure necessary to support the spiral arm against gravitational collapse. Representing the spiral arm by a circular cylinder of radius R and uniform total mean density ρ_t, they showed that the total pressure on the axis of the cylinder would be $\pi G_{\rho\rho_t}R^2$ where G is the gravitational constant. Then, if half of the total pressure is kinetic, equal to $\frac{1}{3}\ \rho v^2$ and the other half magnetic, equal to $B^2/8\pi$, they obtained 6×10^{-6} gauss, in good agreement with the dynamical result of 7×10^{-6} gauss.

These estimates are about twice the estimates today. The more detailed observational studies since that time suggest that ΔB is more nearly equal to B than to the $0.2B$ assumed in their paper, and the spiral arm is better approximated by a ribbon than a circular cylinder, with a half thickness of 100 pc rather than a radius of 250 pc.

In any case, their effort established the correct order of magnitude, which was more than enough to confine the galactic cosmic rays. The cyclotron radius of a 10 GeV proton moving perpendicular to a magnetic field of 3×10^{-6} gauss is 10^{13} cm or slightly less than 1 a.u., to be compared with the half thickness of the field, of the order of 100 pc = 3×10^{20} cm = 2×10^{7} a.u. To put it differently, a field of 3×10^{-6} gauss in a gaseous galactic disk of half thickness 100 pc represents 10^{15} gauss cm whereas the deflection of a 10 GeV proton through 180° requires only 3×10^{7} gauss cm. From the large-scale dynamical point of view, the cosmic rays, which form a tenuous relativistic gas, exert a pressure of about 0.5×10^{-12} dynes/cm^2, comparable to the pressure of a magnetic field of about $3-4\times10^{-6}$ gauss.

Fermi and Chandrasekhar wrote a companion paper (3, 35, 532) on the effect of strong magnetic fields within a star. They used the scalar viral equation

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2T + 3(\gamma - 1)U + M + \Omega,$$

where I is the trace of the moment of inertia tensor, T is the total internal kinetic energy, U is the internal thermal energy, and γ is the ratio of specific heats. The total magnetic energy is denoted by M and the total gravitational energy is Ω The net expansive effect of T, U, and M is obvious here, noting that neither the internal motions T nor the magnetic field M is statistically isotropic. The only negative term on the right-hand side of the scalar viral equation is the gravitational potential energy Ω. They note in passing that equilibrium, obtained by equating the right-hand side to zero, limits the rms field to $1 - 2 \times 10^8$ gauss within a main sequence star, but no more than a few kilogauss for some expanded giant stars. Then treating radial pulsations they pointed out the unbounded increase of the period as the rms field approaches this limiting value. They speculate that such strong magnetic fields may account for the long oscillation periods of some of the giant magnetic stars.

Now, the magnetic fields inside most main sequence stars are nowhere near the theoretical critical values of the order of 10^8 gauss or more. Magnetic buoyancy would bring any such fields to the surface in 10^7 years or less, even if so strong a primordial magnetic field were compressed into the star in the first place. In fact we know from the recent work of Boruta (1996) that the field in the deep interior of the Sun is no more than about 30 gauss. This limit is based on the resistive decay time of 10^{10} years for the basic dipole mode and the fact that there is no fixed dipole in excess of about 5 gauss showing at the surface of the Sun. For in order to confine a dipole field to the interior, it is necessary to superpose higher order radial modes of the dipole. Yet the higher order radial modes decay with periods of 2×10^9 years or less. Since the Sun is about 4.5×10^9 years old, the higher order modes would have decayed away by now, exposing the basic dipole to observation at the surface.

However, Chandrasekhar and Fermi pointed out some newly discovered young giant magnetic stars showing an rms surface field of 2000 gauss and a theoretical maximum internal rms magnetic field of about 3000 gauss. Clearly the magnetic field has a profound influence on the form and behavior of such stars.

They went on in the paper to treat the equilibrium and pulsations of a circular cylinder of self gravitating fluid of infinite electrical conductivity in which there is a uniform magnetic field parallel to the axis of the cylinder. The effect of the magnetic field is to stabilize the equilibrium, increasing both the minimum wavelength and the growth time of instability.

They showed how the magnetic stresses cause the otherwise spherical form of a star to become oblate in the presence of a dipole magnetic field. Finally, they noted that the criterion for the onset of Jean's gravitational instability is unaffected by the presence of a uniform magnetic field, because the unstable mode representing motion parallel to the field is unaffected.

The next paper (3, 36, 561), with Nelson Limber, picks up on the pulsation of a star in which a large-scale magnetic field is embedded. They use the time dependent scalar viral equation again, obtaining an approximate expression $\sigma^2 I = -(3\gamma - 4)(\Omega + M)$ for the frequency σ of the oscillations. The moment of inertia I is $4\pi \int dr\, r^4 \rho(r)$. The result shows that σ is real and the star is stable only so long as $M < |\Omega|$, recalling that $\Omega < 0$. The period of oscillation $2\pi/\sigma$ increases without limit as M increases toward | Ω |, as noted in the previous paper with Fermi.

The next several papers involve the MHD equations applied to a star, or other body, with axisymmetry. That is to say, they treated the case in which the magnetic field and fluid motion are independent of azimuth φ measured around some linear axis of the star. The basic nature of the rotating star with a co-aligned magnetic field suggests this idealization as a fruitful starting point for the investigation. The simplification of the MHD equation from 3D to 2D is enormous, although the resulting quasi-linear equations are by no means elementary. So first a word about the general form of the reduction of the dynamical equations in the presence of an ignorable coordinate. The reduction begins by noting that with φ as the ignorable coordinate the axisymmetric solenoidal vector **B** can be decomposed into toroidal and poloidal components, each component represented by a single scalar function of ϖ and z (cylindrical polar coordinates, where v = $(\chi^2 + y^2)^{1/2}$ represents distance from the z – axis. In terms of the unit vector e_ϖ e_φ , e_z in the respective coordinate directions, write

$$\mathbf{B}(\varpi, z) = -\mathbf{e}_\varpi \varpi \frac{\partial P}{\partial z} + \mathbf{e}_\varphi \varpi T + \mathbf{e}_z \frac{1}{\varpi} \frac{\partial}{\partial \varpi} \varpi^2 P \tag{1}$$

in terms of the scalar function T (ϖ, z) representing the toroidal or azimuthal magnetic field and $P(\varpi , z)$ representing the poloidal or meridional magnetic field. This form guarantees that ∇ **B** = 0, thereby reducing the numbers of independent functions from three to two. The essential point for static equilibrium of a gravitating sphere of uniform density is that the Lorentz force, i.e., $\partial T_{ij} /\partial x_j$, is balanced by the gradient of the pressure plus the gravitational potential. Hence the Lorentz force $(\nabla\times \mathbf{B}) \times \mathbf{B}/4\pi$ must have vanishing curl;

$$\nabla \times (\nabla \times \mathbf{B}) \times \mathbf{B} = 0.$$

In addition the azimuthal component of the Lorentz force must vanish because there is no gravitational or pressure force to oppose it.

It is easy to show that

$$\begin{aligned}(\nabla \times \mathbf{B}) \times \mathbf{B} = & - \left(\Delta_5 P \frac{\partial}{\partial \varpi} \varpi^2 P + T \frac{\partial}{\partial \varpi} \varpi^2 T\right) \mathbf{e}_\varpi \\ & + \left(-\frac{\partial P}{\partial z} \frac{\partial \varpi^2 T}{\partial \varpi} + \frac{\partial T}{\partial z} \frac{\partial}{\partial \varpi} \varpi^2 P\right) \mathbf{e}_\varphi \\ & - \left(\varpi^2 T \frac{\partial T}{\partial z} - \varpi^2 \frac{\partial P}{\partial z} \Delta_5 P\right) \mathbf{e}_z,\end{aligned} \tag{2}$$

where Δ_5 represents the axisymmetric Laplacian in five dimensions,

$$\Delta_5 = \frac{1}{\varpi^4} \frac{\partial}{\partial \varpi} \varpi^4 \frac{\partial}{\partial \varpi} + \frac{\partial^2}{\partial z^2} .$$

Setting the φ –component equal to zero requires that

$$\varpi^2 T = F\left(\varpi^2 P\right), \tag{3}$$

where F is an arbitrary function of its argument. Setting the ϖ component of the curl of the Lorentz force equal to zero can be reduced to the Jacobian relation

$$\frac{\partial\left(\Delta_5 P, \varpi^2 P\right)}{\partial(\varpi, z)} = \varpi\frac{\partial T^2}{\partial z}. \tag{4}$$

This equation can be solved using the device that the Jacobian relation

$$\frac{\partial\left(\varpi^2 P, G\left(\varpi^2 P\right)/\varpi^2\right)}{\partial(\varpi,_z)} = \varpi\frac{\partial T^2}{\partial z} \tag{5}$$

defines the function G. This can be seen by writing out the Jacobian, which reduces to

$$2G\frac{\partial}{\partial z}\left(\varpi^2 P\right) = \frac{\partial}{\partial z}\varpi^4 T^2.$$

If we let $x = \varpi^2 P\ (\varpi, z)$ and $F = \varpi^2 T$, this can be written

$$2G(\chi)\frac{\partial\chi}{\partial z} = \frac{\partial}{\partial z}F^2, \tag{6}$$

and it follows that

$$2G(\chi) = \frac{dF^2}{d\chi}. \tag{7}$$

That is to say, G is determined directly from $F\ (\varpi^2\ \mathrm{P})$. The purpose of this maneuver is to eliminate $\varpi\partial T^2\partial z$ between equations (4) and (5), with the result written in the form

$$\frac{\partial\left(\varpi^2 P, \Delta_5 P + G/\varpi^2\right)}{\partial(\varpi, z) = 0}. \tag{8}$$

The solution is

$$\Delta_5 P + \frac{G\left(\varpi^2 P\right)}{\varpi^2} = \Phi\left(\varpi^2 P\right), \tag{9}$$

where Φ is an arbitrary function of its argument. This field equation for $P\ (\varpi_1 z)$ is a quasi linear elliptic partial differential equation. So the solutions throughout a volume V are uniquely determined by specification of some linear combination of P and ∇P on the surface S enclosing V (Courant and Hilbert 1962).

This simple example serves to illustrate the general method for obtaining the field equations for magnetostatic equilibrium with axisymmetry, which Chandrasekhar pursued at some length. For instance, the paper (3, 37, 565) with K. H. Prendergast works out the field equations and some simple examples of the most general axisymmetric magnetic field that permits static equilibrium of a star of uniform density. The paper (3, 39, 575) extends the formalism to include internal fluid motion. The general conditions deduced in this way prescribe the conditions for hydrostatic equilibrium, the law of isorotation, etc. in a self-gravitating body of uniform density. The paper (3, 45, 632) goes on to apply the general variational principle developed by L. Woltjer to the axisymmetric case, thereby obtaining seven integrals of the field and fluid velocity instead of the four that Woltjer obtained in the general case.

Chandrasekhar makes the important point that the special forms of the field and fluid required by the additional three constraining integrals are not likely to be realized in nature. Three of the seven integrals involve relations between poloidal and toroidal components of the magnetic field and of the fluid velocity. Poloidal and toroidal components tend to have independent physical origins in both the field and fluid motions, and the fluid motion driven by convective forces is not likely to be of such a form as to provide the required relation of the poloidal magnetic field to the toroidal magnetic field and toroidal velocity. Hence one does not expect a convecting magnetic star to achieve a stationary axisymmetric state. This is confirmed by the observed nonuniform distribution of magnetic activity around most stars.

Then the paper (3, 41, 609) formulates the difficult problem of the oscillations of a self-gravitating magnetic star of uniform density in which there is not only an axisymmetric magnetic field but a related fluid velocity **v** everywhere parallel to the magnetic field. Both **v** and **B** are solenoidal in this case, and Chandrasekhar treats the equipartition case $\mathbf{B} = \pm (4\pi\rho)^{1/2}\,\mathbf{v}$ in which **B** and **v** contribute only to the net pressure, the Maxwell stress (tension) of **B** being precisely offset by the Reynolds stress (compression) of **v**. Expressing both **B** and **v** in terms of their toroidal and poloidal scalar functions, the field equations again reduce to second order quasi-linear partial differential form. Then a variational principle is used to study the diverse modes of pulsation of a star with toroidal field and flow. Chandrasekhar points out that the method provides only a slow convergence of the result with increasing order of trial functions, but the convergence is sufficient to show that the characteristic pulsations correspond to Alfven waves propagating around the star. This result expresses the incompressibility of the uniform star, and the Alfvén waves may be thought of as gravity waves since the field tension is canceled to lowest order by the Reynolds stress.

The elegant mathematics of these pioneering papers on axisymmetric static and stationary equilibria of magnetic stars of uniform density ρ sets off in striking manner the much more complicated problem of the gaseous magnetic star with its strong radial stratification, convection, general absence of equilibrium because of magnetic buoyancy and convective overturning, and perpetual non-steady magnetic activity because of the tendency to form current sheets in all but the simplest field topologies (Parker 1994).

4. Generation of magnetic field

A crucial question in the physics of magnetic stars and planets (not to mention interstellar gas clouds, proto-stellar disks, galaxies, and clusters of galaxies) is the origin and maintenance of their magnetic fields. The magnetic fields are continually dissipated through the slight electrical resistivity of the planet or star and, in the case of stars, by the incessant dynamical rapid reconnection of the magnetic field caught up in the internal convection. Indeed, as already noted, even a hypothetical dipole magnetic field anchored in the stable radiative core of the Sun has a characteristic decay time estimated at 10^{10} years. For the planet Earth the decay time is estimated at $\sim 2 \times 10^4$ years for the dipole mode. The turbulent mixing of magnetic fields in the convective zones of stars may hasten the demise of magnetic fields there, unless, of course, the convection has the special properties sufficient for generating the magnetic field in the first place. In fact one can see from the magnetic cycle of the Sun, and from the comparable magnetic cycles of other stars, that the magnetic fields is created and destroyed approximately every decade by the turbulent convection. The creation and destruction can be characterized by a resistive diffusion coefficient η of the order of 10^{11} – 10^{12} cm^2/sec. The characteristic decay time is L^2/η for a field of scale L, yielding 10 years for $L \sim 10^{10}$ cm. The diffusion η is conventionally attributed to turbulent mixing of magnetic field, characterized by a mixing length τ and associated eddy velocity $u(\tau)$, so that $\eta \sim 0.1\lambda\ v(\tau)$. However, it is a difficult question as to how, or whether, the turbulence can perform the assumed mixing and dissipation without necessarily producing small-scale magnetic fields vastly greater than the mean macroscopic magnetic field. In fact the mean fields in the convective zone of the Sun are themselves comparable to the equipartition field, so it is not clear why the assumed turbulent mixing and winding of the mean field does not produce small-scale fields of such great intensity as to suppress the convective mixing. The answer seems to be that the effect of the convection is to concentrate the magnetic field into intense filaments or fibrils with the interstices essentially field-free. The individual fibrils are then free to interconnect rapidly across their small diameters.

It is interesting to return to the early days 40 years ago when the problem confronting the theoretician was to establish the limiting conditions for the generation of magnetic field by the motion of a simply connected body of electrically conducting fluid. Cowling (1934; Bachus and Chandrasekhar 1956) had shown two decades earlier that “when the magnetic field and the fluid motions are symmetric about an axis and the lines of force of the magnetic field as well as the trajectories of the fluid particles are confined to meridional planes, no stationary dynamo can exist”. In fact this anti-dynamo theorem was generally understood in a stronger form, that no magnetic field and steady fluid motion with the same topology as with axisymmetry can operate as a self-sustaining dynamo. This stronger conclusion is inferred from the inability to maintain the azimuthal current that necessarily flows through the neutral point (or points) in the poloidal field in the meridional planes. Bachus and Chandrasekhar (1956) proceeded in the paper (3, 38, 570) to provide a formal proof for the ideal axisymmetric case. The proof starts from the fact that the toroidal field necessarily vanishes at the surface of the star or planetary core, whereas the field equation for a stationary field is fully elliptic. Hence the boundary condition at the surface would require that the field vanish throughout. That is to say, no steady state axisymmetric dynamo with uniform conductivity and density exists. Subsequent experience has shown that a variety of dynamo forms exist as soon as one turns to nonsymmetric steady and unsteady flows.

Now if a steady axisymmetric fluid motion cannot sustain a magnetic field, the question arises whether such fluid motion can accelerate or retard the resistive decay of the axisymmetric field. This was taken up by Chandrasekhar in (3, 40, 587), using the established axisymmetric formalism in which each vector quantity is decomposed into its toroidal and poloidal parts. Then the individual modes are found to be expressible in terms of Gegenbaur polynomials $C_n^{3/2}(\cos\theta)$ while the radial dependence is $J_{n+3/2}(kr)/r^{3/2}$ in terms of Bessel functions of half integral order. The intermodal coupling leads to a complicated array of equations. The array is necessarily truncated to effect an asymptotic solution, and Chandrasekhar displayed the convergence of the result as successively more terms were employed. The convergence was clear for weak velocity fields, which is to be expected because such fields are close to the modes of resistive decay in a static fluid. Unfortunately when the velocity is strong enough to have a substantial effect, the convergence is not so clear. It appeared from the calculations that the decay of both poloidal and toroidal magnetic fields could be slowed by a factor of ten or more by velocity fields that deform the magnetic field so as to decrease the characteristic scale. The calculations also yielded substantially retarded decay in other cases. Chandrasekhar mentions lifetimes increased by factors of 20 or 50. Unfortunately these interesting cases of prolonged field life are among those exhibiting poor convergence. In fact a subsequent calculation by G. Bachus (1957) showed formally that no increase in characteristic decay time beyond a factor of four is possible. There is no significant prolonging of the life of a magnetic field without the dynamo effects that generate new field.

Lüst & Schlüter (1954) were the first to emphasize that strong magnetic fields in relatively tenuous gases are of such form that the Lorentz force F_j, i.e., the divergence of the Maxwell stress tensor T_{ij}, is essentially zero,

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0.$$

The reason is simply that if the gas is too tenuous to push on the magnetic field, then from Newton's third law it follows that the magnetic field does not push on the gas, $F_j = 0$. The fluid motions, if any, are channeled along the strong field, which acts as a curved conduit of nonuniform cross-section in the general case. So the force-free condition is a restriction on the field, requiring

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{10}$$

in general, where α is a scalar function of position, constant along each field line

($\mathbf{B} \cdot \nabla\alpha = 0$) but varying arbitrarily from one line to the nest. The fluid moves freely along the field without significant effect on the field.

Now theory shows that the fields cannot be force-free everywhere throughout a body of gas. The simple scalar viral equation, noted above, shows that the overall effect of the magnetic field is measured by the total magnetic energy, which is positive definite. So the magnetic field engenders expansion and the field can be in static equilibrium only if held firmly in the grip of the negative gravitational potential of a star or other gravitating body. So the Lorentz force may vanish to give a force-free field in the region outside the gravitating body, but it must be remembered that the Lorentz force cannot vanish everywhere inside the body.

Lüst & Schlüter treated the special case of an axisymmetric force-free field with α = *constant* to illustrate the properties of the forcefree field. In the paper (3, 42, 618) Chandrasekhar wrote down the general solution for that illustrative case, in terms of Gegenbauer polynomials and Bessel functions of half integral order. He went on to treat the boundary conditions at the surface of a spherical shell adjoining another shell in which the constant value of α is different. The calculations show the interesting result that the energy of the poloidal and toroidal field components are equal.

The next paper (3, 43, 623), with P.C. Kendall, extends the calculations to the resistive decay of the force-free poloidal and toroidal modes in the presence of uniform resistivity, showing that the decay preserves the force-free form of the field, a general point first made by S. Lundquist (1952). Thus no fluid motions are created as a consequence of the resistive decay.

The paper (3, 44, 627), by Chandrasekhar and L. Woltjer, takes up the question of the field configuration with the maximum magnetic energy, i.e., the maximum mean square magnetic field, for a fixed mean square current density. They pointed out that there can be no minimum mean square field for a given mean square current density because the mean square current density can be made arbitrarily large without affecting the mean square field by the simple procedure of introducing many steep gradients or shears in the magnetic field. The variational problem is easily formulated, maintaining the volume integral of $(\nabla\times\mathbf{B})^2$ constant while the integral of $(\mathbf{B})^2$ is an extremum. With Lagrangian multiplier α^2 the final result is the elliptic equation

$$\nabla^2\mathbf{B} + \alpha^2\mathbf{B} = 0$$

encompassing the force-free fields with constant α, as well as other solutions. However, it should be noted that the conditions for static equilibrium are not incorporated into the derivation. So the only equilibrium field for which the magnetic energy is maximum for a given mean square current density is the force-free field with constant α.

Note again that the magnetic field cannot be force-free everywhere. For the field must be confined by inward forces if it is not to expand to infinity. In star-like structures one would expect to find either that the field is held in the grip of the central core or otherwise confined by some hypothetical enclosing boundary outside the force-free regions, or both.

5. Collisionless plasmas

Laboratory plasma confinement is achieved by surrounding a volume of plasma with a strong magnetic field. The scales are not astronomical and indeed the characteristic scale perpendicular to the confining magnetic field may be not many times larger than the cyclotron radius of the ionic component of the plasma. The convenient approximations of MHD, treating the very large-scale behavior as the dynamics of an electrically conducting isotropic fluid, become a poor approximation. The thermal velocity distribution is generally not isotropic for a variety of reasons, e.g., the free motion of charged particles along the field as distinct from the cyclotron motion perpendicular to the field. The free particle motion along the field is reflected from regions of strong field by the invariant diamagnetic moment $\frac{1}{2}\, m w_{\perp}^{2}/B$ of the particle with mass m and velocity $w_{\perp}$ perpendicular to B. The cyclotron motion of the ions and electrons around **B** provide a drift of the guiding center (the instantaneous center of the cyclotron circular motion) perpendicular to **B** as a consequence of the curvature of the field lines (the curvature drift) and as a consequence of the variation of the field intensity in the direction perpendicular to B (the gradient drift). In view of the free interpenetration of particles from different regions along the field, where the curvature and field gradients as well as the thermal velocities may be quite different, the general dynamics of the confined plasma presents a daunting problem.

Chandrasekhar, with A.N. Kaufman and K.M. Watson, took on the problem in the two papers (4, 1, 3) and (4, 2, 39) neglecting Coulomb interactions between particles (the collisionless plasma) and working in the strong field limit so that the plasma introduces only a small perturbation of the magnetic field. Thus, the calculation omits thermalization of the ions and electrons, and is a valid representation of the plasma dynamics over periods short compared to the thermalization or collision time. Even so, the formal calculation is massive, starting with the collisionless Boltzmann equation (the Vlasov equation)

$$\frac{\partial f}{\partial t} + v_j \frac{\partial f}{\partial x_j} + \left[g_i + \frac{e}{m} \left(E_j + \frac{1}{c} \epsilon_{ijk} v_j B_k \right) \right] \frac{\partial f}{\partial v_j} = 0 \tag{11}$$

for the velocity distribution functions $f(x_j, v_k, t)$ of the individual ions and electrons in the presence of a gravitational acceleration and the electric and magnetic fields E_ι and B_j, respectively. They wrote $v_j = V_j + w_i$ where

$$V = \int d^3 v_k f v_i \tag{12}$$

is the local mean velocity and w_j is the thermal velocity. The collisionless Boltzmann equation was then written in a variety of forms, e.g., equation (22) of (4, 1,

3) which was cast in the form[1]

$$\frac{\partial}{\partial t}\left(\sum_{+-} mNV_i + \frac{B^2}{4\pi c^2}\Psi_i\right) = \frac{\partial}{\partial x_i}\left(T_{ij} + P_{ij} - \sum_{+-} mNV_iV_j\right) + \rho g_i. \tag{13}$$

Here $\Psi_i = c\varepsilon_{ijk}\, E_j\, B_k\, /\, B^2$ is the so called electric drift velocity, *Tij* is the Maxwell stress tensor

$$T_{ij} = -\delta_{ij}\frac{E^2 + B^2}{8\pi} + \frac{E_iE_j + B_iB_j}{4\pi},$$

reducing to

$$T_{ij} \cong -\delta_{ij}\frac{B^2}{8\pi} + \frac{B_iB_j}{4\pi} \tag{14}$$

for $v_i << c$. P_{ij} is the total pressure tensor (ions and electrons)

$$P_{ij} \equiv \sum_{+-} m \int d^3w_k f w_i w_j, \tag{15}$$

and the summation is over both electrons and ions. The electric drift velocity Ψ_j derives from the Poynting flux $c\varepsilon_{ijk}\, E_j\, B_k\, /4\pi$ .Its contribution to the momentum density on the left-hand side is of the order of the magnetic energy density divided by the rest energy density of the particles. This is not small in the limit of tenuous plasma, of course, but it is generally small when the gas is dense enough that the Alfvén speed is small compared to *c*.

The time dependent Boltzmann equation is treated for small perturbations about a stationary state. The electromagnetic field perturbations are expressed in terms of the Lagrangian displacement of the artificial velocity U_j defined by the equation of motion

$$\frac{\partial U_i}{\partial t} = \frac{e}{m}\left(E'_i + \frac{1}{c}\epsilon_{ijk}U_jB^0_k\right), \tag{16}$$

where B^0_k represents the stationary field and the prime denotes the perturbation, with

$$B_j = B^0_j + B'_j, E_i = E^0_i + E'_i. \tag{17}$$

The calculation proceeds from there to work out the general conditions for the stationary fields B^0_j ,E^0_j , treating the particle motion essentially in the guiding center approximation, as well as developing the macroscopic boundary conditions at a discontinuity. The second paper (4, 2, 39) works out the pressure drift, which is a combination of the gradient drift of the individual particles and the net local particle cyclotron motion in the presence of a nonvanishing cyclotron radius and a plasma pressure gradient. The paper goes on to describe the general plasma conditions in a variety of special conditions.

The final paper (4, 3, 64), with A. N. Kaufman and K. M. Watson, treats

[1]A factor 1/*c* is missing from the term $\varepsilon_{ijk}E_jB_k$ in equation (22).

the stability of the laboratory magnetic pinch. Rosenbluth (1957) had previously treated the problem using the particle orbits in the guiding center approximation in place of the Boltzmann equation. The more detailed study from the Boltzmann equation gives a slightly different criterion for marginal stability, but the principal results for stabilizing the pinch are confirmed. The immense complexity of both the calculation and the ultimate stability criteria for the various modes are best studied from the original paper. No attempt to summarize the results can be made without a detailed description of the formalism.

The invariants of the guiding center motion of a charged particle in a strong magnetic field are described in a subsequent paper (4, 4, 85) by Chandrasekhar, which the reader may find useful to have in mind when studying the three papers just mentioned. The strongest invariant is the diamagnetic moment μ of the cyclotron motion of the particle (ion or electron) around the field. If $\omega_\perp$ denotes the particle velocity perpendicular to the field, we have $\mu = \frac{1}{2} M w_\perp^2/B$. The invariance of μ can be violated only by changes in the field over scales comparable to or smaller than the cyclotron radius $Mw_\perp c/eB$ or over times less than the cyclotron period $2\pi\, Mc/eB$.

The longitudinal invariant is $\oint ds \cdot W_\parallel$, where $W_\parallel$ is the particle velocity parallel to the magnetic field. The integration over length ds along the field is carried out from one mirror point (where the particle is reflected from a region of increasing B) to the other. The invariance of this quantity is preserved for changes in the field that take place over characteristic times that are large compared to the bounce time of the particle between mirror points. The concept and validity of the invariants of various orders are discussed at length in this paper.

The reader who is not already familiar with the guiding center orbit theory of particle motions and with the associated invariants may find the small book on plasma physics (Chandrasekhar and Trehan 1960) a useful place to begin. The book goes on to give a simplified and lucid treatment of the stability of the pinch before taking up plasma oscillations and transport phenomena in the collisionless plasma.

6. Magnetic fields and convective instability

Fluids are subject to a variety of dynamical instabilities. A static fluid undergoes convective overturning if heated from below or cooled from above. In general an adverse vertical density stratification may be caused by a temperature or compositional gradient, producing a Rayleigh-Taylor instability and the associated overturning of the fluid. The presence of a directed radiation field and a spatially varying opacity may induce unstable temperature and density distributions. The relative motion of two contiguous volumes of fluid produces a Kelvin-Helmholtz instability at the interface. These instabilities all arise from the interplay of fluid pressure, gravitational acceleration, and Reynolds stress $R_{ij} = -\rho \upsilon_i \upsilon_j$. The Reynolds stress is a compressive force $\rho\upsilon^2$ in the direction *of* υ_i, causing buckling of the stream lines to produce the Kelvin-Helmholtz instability.

The presence of a magnetic field in an electrically conducting fluid adds the Maxwell stress, represented by T_{ij}, described by equation (14). In particular the magnetic field introduces an isotropic pressure $B^2/8\pi$ (B in gauss) and a tension $B^2/4\pi$ along the field. The tension in the magnetic field tends to stabilize waves with phase along the magnetic field, as distinct from the Reynolds stress compression which de-stabilizes waves with phase along the velocity field. The magnetic pressure tends to expand a compressible (gaseous) fluid providing buoyancy in the presence of a gravitational field. The buoyancy of the magnetic field contributes a form of the Rayleigh-Taylor instability. Then, of course, in a rotating body the fluid velocity gives rise to a Coriolis force $2v \times \Omega$ whereas the magnetic field **B** produces no comparable effect. The tension in the field strives merely to make everything rotate with the same angular velocity along each field line.

It is evident from these brief remarks that the subject of hydrodynamic stability and instability takes on new dimensions in the presence of electrical conductivity and a magnetic field. Clearly a methodical recalculation of the classical hydrodynamic instabilities was in order, with the expectation of new instabilities as well as the suppression of familiar hydrodynamic instabilities by the tension in the field. Chandrasekhar's lifelong interest in stars led to a concern with thermal convection, so the general magnetohydrodynamical theory of convection was an obvious challenge. The customary starting point is a fluid of uniform density except for a small thermal expansion coefficient which provides the buoyant forces that drive the convection. The slight thermal density change has no sensible effect on the inertia of the fluid (the Boussinesq approximation). The classical Bernard problem of convection was studied a century earlier by Rayleigh, and by many others since. The reader is referred to Chandrasekhar's (1961) comprehensive monograph for a detailed discussion of the historical development of the theory of thermal convection. The application of convection to stellar structure immediately introduces the theoretical problem of convection in a rotating system. This suggests convection in the presence of both rotation and magnetic field with no particular special relative orientation of the gravitational acceleration g, the angular velocity and the magnetic field **B**.

To begin with the simpler cases, then, Chandrasekhar (1953; Chandrasekhar and Elbert 1955) investigated the effect of rotation on the dynamics of thermal convection. The results are concisely summarized in Chandrasekhar's Rumford Medal Lecture in 1957 (4, 8, 163), where he begins by noting that the rotation strongly constrains the fluid motion. The effect is stated by the Taylor-Proudman theorem that *all slow motions* (for which the nonlinear terms can be neglected) *in a rotating inviscid fluid are necessarily two dimensional,* being invariant in the direction of the uniform angular velocity of the body of fluid. It follows that an inviscid fluid is stable against convective overturning by an adverse temperature gradient in the direction of the angular velocity, no matter how strong the temperature gradient. The introduction of viscosity, on the other hand, vitiates the Taylor-Proudman theorem and provides convective instability in a suitably strong temperature gradient. In a rotating system the convective instability may appear as an overstability, in which the motion is oscillatory (as in a stable system) but the amplitude of the oscillations grows exponentially with time. The system is overstable at small Prandtl numbers ν/κ, (where ν is the kinematic viscosity and k is the thermometric conductivity) and unstable at large Prandtl number.

In the Rumford Lecture, Chandrasekhar (1952, 1954a) pointed out that the introduction of a magnetic field parallel to gravity and angular velocity tends to stabilize the electrically conducting fluid for the simple reason that the vertical magnetic field inhibits any variation of the horizontal fluid velocity with height, pushing the system back toward the Taylor-Proudman condition. If we supposed that the layer of fluid is capped above and below by rigid infinitely conducting boundaries, instead of free boundaries, the field is line tied at the boundaries so that the field inhibits all motion, of course. For instance, in applications to sunspots the field lines are largely free to be moved about at the upper end of the sunspot field (at the visible surface) being tied only at the distant opposite end of the bipolar field configuration. The field is tied into the convective motions at the bottom end where the lines are subject to some unknown pattern of circulation. With such strong magnetic fields the convective motions are largely constrained to vertical oscillations along the field. The general effect is to inhibit convective heat transport, thereby producing a cool region at the visible surface. One can imagine the endless variety of circumstances that arise in the presence of the three independent vectors g, Ω, and $\mathbf{B}$, together with the Prandtl number, Rayleigh number, and magnetic Reynolds number (cf. Chandrasekhar 1954b, 1956). Chandrasekhar pointed out the somewhat different and conflicting roles of Ω and $\mathbf{B}$ with the possible overstability from both Ω and $\mathbf{B}$ in certain parameter ranges and instability in other ranges. The combination (discussed at some length in chapter V of Chandrasekhar 1961) provides a number of distinct circumstances. In the paper (4, 9, 192) the overstability is addressed from the energy or thermodynamic point of view. The purely mathematical aspects of the theory of hydrodynamic and hydromagnetic (MHD) instability are treated in the paper (4, 11, 207) on characteristic value problems and the paper (4, 12, 221) on adjoint differential systems and variational principles. There is extensive discussion to be found at several places in the monograph (Chandrasekhar 1961).

The foregoing labors were all theoretical, of course, involving a variety of mathematical techniques and enormous algebraic undertakings. It is interesting to note, then, that at the same time an experimental effort was launched at the University of Chicago to test the theoretical predictions. The project was initiated under the auspices of Professor S.K. Allison who was Director of the Institute for Nuclear Studies (now the Enrico Fermi Institute). Professor D. Fultz carried through a number of experiments of convection in rotating systems – the rotating dishpan experiments (Fultz and Nakagawa 1955; Nakagawa and Frenzen 1955). Dr.Y. Nakagawa carried on the effort with the addition of uniform magnetic fields, up to about 8000 gauss between the pole pieces of a 36 inch cyclotron magnetic. The cyclotron had been decommissioned some time earlier and the magnetic yoke and pole pieces were reconditioned and put to use again. Nakagawa used mercury in depths of a few centimeters, achieving magnetic Reynolds numbers rather less than one. Chandrasekhar worked closely with the experimenters and communicated several of the experimental papers for publication in the Proceedings of the Royal Society. Nakagawa (1957) showed the close agreement of theory and experiment in the presence of magnetic field **B** . A year later he exhibited results of combined Ω and **B** (Nakagawa 1959) generally confirming the validity of the theoretical predictions.

7. Magnetic fields and dynamical instability

Chandrasekhar's contributions to the effect of magnetic field on the dynamical instability of Couette flows, the Rayleigh-Taylor instability in adverse density gradients, and the Kelvin-Helmholtz instability between fluids with relative tangential velocity are summarized in the aforementioned monograph (Chandrasekhar 1961). The stationary flow between concentric cylinders in relative rotation is an example of Couette flow. The fluid velocity is entirely azimuthal and a function only of distance ϖ from the axis of rotation. Under steady conditions the torque (in the axial direction) transmitted by the viscosity is independent of ϖ, from which it is readily shown that $\upsilon(\varpi) \sim 1/\varpi$ in the presence of a uniform viscosity. Rayleigh pointed out a century ago that Couette flow is stable if the angular momentum density $\rho\varpi\upsilon(\varpi)$ increases outward and unstable if it decreases outward. We note that for uniform density and viscosity the angular momentum density is independent of radius, providing neutral stability. On the other hand, if viscosity is neglected, then any variation of υ with radius is possible, providing both stable and unstable Couette flow. The dynamical effects can be strikingly different in different cases, and the interested reader is referred to Chandrasekhar's monograph. Chapter IX of the monograph takes up the stability for a conducting fluid with a uniform magnetic field parallel to the axis of rotation, an azimuthal magnetic field (parallel to the azimuthal velocity υ), and a combination of axial and azimuthal fields, with and without viscosity. The magnetic tension tends to stabilize the system, of course, and the detailed effects are different in each special case.

The Rayleigh-Taylor instability of superposed fluids arises when the upper fluid is denser so that gravitational potential energy is released by interchanging or overturning fluid. The effects of vertical magnetic field and of horizontal magnetic field are treated in chapter X, with the tension in the magnetic field inhibiting the onset of instability. Short wavelengths are most strongly inhibited by a vertical magnetic field so that the growth rate does not increase without bound with increasing wave number, as it does in the inviscid non-conducting case. The inhibition declines to zero in the limit of long wavelengths, of course. The stabilizing effect of a horizontal magnetic field is equivalent to the effect of surface tension.

Finally, the influence of a magnetic field on the Kelvin-Helmholtz instability is treated in chapter XI, with similar results. The tension in the field tends to stabilize any waves with phase extending along the field, with the consequence that the velocity difference between the two relatively moving semi-infinite regions of fluid must exceed the Alfvén speed to produce instability. For fluids of different densities ρ_1 and ρ_2 the final result is slightly more complicated because there is no single Alfvén speed, but the principle is the same, that the system is stable when the tension in the field exceeds the Reynolds compressive stress. The magnetic field perpendicular to the direction of flow has no effect on the unstable waves with wave vector parallel to the flow.

8. Concluding remarks

In conclusion one can only remark on the vast and various contributions that Chandrasekhar has made to magnetohydrodynamics. The present article is only the briefest summary of the many different problems elucidated by Chandrasekhar's theoretical studies. The importance of his contributions can be comprehended at the most primitive level by noting that his monograph on Hydrodynamic and Hydromagnetic Stability (Chandrasekhar 1961) has sold n copies with In n ~ 11. The monograph has been reprinted now by Dover Publications of New York. It must be appreciated that the monograph covers only a modest part of Chandrasekhar's contributions to hydromagnetics or MHD. The publication by Dover is not without practical significance to the scientific community and it was not without personal significance to Chandrasekhar who recognized the important scientific role of Dover Publications in reprinting landmark books after they have passed out of print on the regular market. This point is best made by relating an experience of some 35 years ago. I was a junior faculty member of the Physics Department at the University of Chicago. One morning, walking to my office I met Chandrasekhar coming the other way. He was in good spirits, and as we met he said, "Well, Parker, I have been immortalized." To my puzzled look he added "Dover has decided to publish my Radiative Transfer." And as we all know Dover went on to publish several of his monographs, which make excellent textbooks to this day.

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