SOLUTION OF THE PROBLEM OF THE STRESS STATE OF NONCIRCULAR CYLINDRICAL SHELLS OF VARIABLE THICKNESS

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A procedure is proposed for solving two-dimensional boundary-value problems on the stress-strain state of open and closed noncircular cylindrical shells of variable thickness under surface loads. The solution is based on the use of the spline-collocation method along the directrix and the method of discrete othogonalization along the generatrix. Examples of solutions for ellipsoidal shells of variable thickness are given.

The problem of the stress-strain state of noncircular cylindrical shells of variable thickness is described by a system of partial differential equations with variable coefficients and corresponding boundary conditions [3, 13]. This class of problems can be examined in a simplified formulation on the basis of the Mushtari–Donnel–Vlasov equations [1, 4, 12]. In many cases, the use of various approximate and numerical methods to solve such problems does not make it possible to satisfy the boundary conditions with sufficient accuracy and obtain the desired solution [2, 6, 9].

In this investigation, we solve problems of the given type on the basis of spline approximation in one coordinate direction and the numerical method of discrete othogonalization in the other coordinate direction.

This approach to solving boundary-value problems of shell theory was proposed in [5, 7, 8]. Some results that have been obtained from solving problems for circular and noncircular cylindrical shells of constant thickness can be found in [5, 11].

Here, we will examine the class of problems concerning the stress-strain state of noncircular isotropic thin cylindrical shells with a thickness that changes along the generatrix and directrix. The shells are subjected to an arbitrary surface load. The solution is obtained on the basis of the Mushtari–Donnel–Vlasov equations [1, 12]. The shells may be closed or open along the generatrix. Accordingly, the boundary conditions are assigned either on the curvilinear edges or over the entire contour.

The initial equations that describe the deformation of this class of shells in the coordinate system s, t — where s and t are arc length along the generatrix and directrix, respectively — are written in the following form [1, 4, 12]:

the expressions for the strains

$$\varepsilon_s = \frac{\partial u}{\partial s}, \quad \varepsilon_t = \frac{\partial v}{\partial t} + \frac{w}{R}, \quad \varepsilon_{st} = \frac{\partial u}{\partial t} + \frac{\partial v}{\partial s},$$

$$\kappa_s = -\frac{\partial^2 w}{\partial s^2}, \quad \kappa_t = -\frac{\partial^2 w}{\partial t^2}, \quad \kappa_{st} = -\frac{\partial^2 w}{\partial s \partial t}; \tag{1}$$

the equations of equilibrium

$$\frac{\partial N_S}{\partial s} + \frac{\partial S}{\partial t} = 0, \qquad \frac{\partial N_s}{\partial s} + \frac{\partial N_t}{\partial t} = 0,$$

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$$\frac{\partial Q_s}{\partial s}+\frac{\partial Q_t}{\partial t}-\frac{1}{R}N_t+q_\gamma=0\,, \tag{2}$$

$$\frac{\partial M_s}{\partial s}+\frac{\partial H}{\partial t}-Q_s=0\,,\qquad \frac{\partial M_t}{\partial t}+\frac{\partial H}{\partial s}-Q_t=0\,,$$

the elasticity relations

$$N_s=D_N(\varepsilon_s+\nu\,\varepsilon_t)\,,\quad N_t=D_N(\varepsilon_t+\nu\,\varepsilon_s)\,,\quad S=\frac{1-\nu}{2}D_N\varepsilon_{st}\,,$$

$$M_s=D_M(\kappa_s+\nu\,\kappa_t)\,,\quad M_t=D_M(\kappa_t+\nu\,\kappa_s)\,,\quad H=(1-\nu)D_M\kappa_{st}\,, \tag{3}$$

where the radius of curvature of the directrix $R=R(t)$, while the stiffnesses

$$D_N=\frac{Eh(s,t)}{1-\nu^2}\,,\quad D_M=\frac{Eh^3(s,t)}{12(1-\nu^2)}\,.$$

In Eqs. (1)–(3), u, v, and w are the displacements along the generatrix, the directrix, and a normal to the middle surface; ε_s, ε_t, ε_{st}, κ_s, κ_t, and κ_{st} are the shear and bending strains; N_s, N_t, S, Q_s, and Q_t are forces; M_s, M_t, and H are moments; $h=h(s,t)$ is the thickness of the shell; E and ν are the elastic modulus and Poisson's ratio; $q_\gamma=q_\gamma(s,t)$ is the surface load.

After certain transformations, Eqs. (1)–(3) lead to a resolvent system of equations in displacements

$$D_N\left\{\frac{\partial}{\partial s}\left[\frac{\partial u}{\partial s}+\nu\left(\frac{\partial \mathrm{v}}{\partial t}+\frac{w}{R}\right)\right]+\frac{1-\nu}{2}\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial t}+\frac{\partial \mathrm{v}}{\partial s}\right)\right\}+\frac{\partial D_N}{\partial s}\left[\frac{\partial u}{\partial s}+\nu\left(\frac{\partial \mathrm{v}}{\partial t}+\frac{w}{R}\right)\right]+\frac{1-\nu}{2}\frac{\partial D_N}{\partial t}\left(\frac{\partial u}{\partial t}+\frac{\partial \mathrm{v}}{\partial s}\right)=0\,,$$

$$D_N\left[\frac{\partial}{\partial t}\left(\frac{\partial \mathrm{v}}{\partial t}+\frac{w}{R}+\nu\frac{\partial u}{\partial s}\right)+\frac{1-\nu}{2}\frac{\partial}{\partial s}\left(\frac{\partial u}{\partial t}+\frac{\partial \mathrm{v}}{\partial s}\right)\right]+\frac{\partial D_N}{\partial t}\left(\frac{\partial \mathrm{v}}{\partial t}+\frac{w}{R}+\nu\frac{\partial u}{\partial s}\right)+\frac{1-\nu}{2}\frac{\partial D_N}{\partial s}\left(\frac{\partial u}{\partial t}+\frac{\partial \mathrm{v}}{\partial s}\right)=0\,, \tag{4}$$

$$\frac{\partial}{\partial s}\left[D_M\frac{\partial \Delta w}{\partial s}+\frac{\partial D_M}{\partial t}(1-\nu)\frac{\partial^2 w}{\partial s\,\partial t}+\frac{\partial D_M}{\partial s}\left(\frac{\partial^2 w}{\partial s^2}+\nu\frac{\partial^2 w}{\partial t^2}\right)\right]+$$

$$+\frac{\partial}{\partial t}\left[D_M\frac{\partial \Delta w}{\partial t}+\frac{\partial D_M}{\partial s}(1-\nu)\frac{\partial^2 w}{\partial s\,\partial t}+\frac{\partial D_M}{\partial t}\left(\frac{\partial^2 w}{\partial t^2}+\nu\frac{\partial^2 w}{\partial s^2}\right)\right]+\frac{D_N}{R}\left(\frac{\partial \mathrm{v}}{\partial t}+\frac{w}{R}+\nu\frac{\partial u}{\partial s}\right)=q_\gamma\,,\quad 0\le s\le L,\,0\le t\le 2\pi\,.$$

In the case of shells that are closed along the directrix, the boundary conditions are assigned on the curvilinear edges. In the case of shells that are open along the directrix, the boundary conditions are assigned on the curvilinear and straight edges. The boundary conditions can be formulated in displacements or in mixed form. Arbitrary boundary conditions are assigned on the curvilinear edges, while the following boundary conditions are assigned on the straight edges in the case of open cylindrical shells: conditions corresponding to pinned or fixed support on both edges, specifically:

$$u=\mathrm{v}=w=\vartheta_t=0\ \text{ at }\ t=t_1\,,\ t=t_2 \tag{5}$$

or

$$u=\mathrm{v}=w=M_t=0\ \text{ at }\ t=t_1\,,\ t=t_2\,; \tag{6}$$

conditions corresponding to pinned support of one end (6) and fixed support of the other end (5). Generally speaking, other types of boundary conditions can also be assigned on the straight edges.

Using certain transformations, we write resolvent system of differential equations (4) in the form:

$$\frac{\partial^2 u}{\partial s^2}=a_{11}\frac{\partial^2 u}{\partial t^2}+a_{12}\frac{\partial u}{\partial t}+a_{13}\frac{\partial u}{\partial s}+a_{14}\frac{\partial \mathrm{v}}{\partial t}+a_{15}\frac{\partial^2 \mathrm{v}}{\partial s\,\partial t}+a_{16}\frac{\partial \mathrm{v}}{\partial s}+a_{17}w+a_{18}\frac{\partial w}{\partial s},$$

$$\frac{\partial^2 \mathrm{v}}{\partial s^2}=a_{21}\frac{\partial^2 u}{\partial t}+a_{22}\frac{\partial u}{\partial s}+a_{23}\frac{\partial^2 u}{\partial s\,\partial t}+a_{24}\frac{\partial^2 \mathrm{v}}{\partial t^2}+a_{25}\frac{\partial \mathrm{v}}{\partial s}+a_{26}\frac{\partial \mathrm{v}}{\partial s}+a_{27}w+a_{28}\frac{\partial w}{\partial t},$$

$$\frac{\partial^2 w}{\partial s^4}=a_{31}\frac{\partial u}{\partial s}+a_{32}\frac{\partial \mathrm{v}}{\partial t}+a_{33}w+a_{34}\frac{\partial^2 w}{\partial t^2}+a_{35}\frac{\partial^3 w}{\partial t^3}+a_{36}\frac{\partial^4 w}{\partial t^4}+a_{37}\frac{\partial^2 w}{\partial s\,\partial t}+$$

$$+a_{38}\frac{\partial^3 w}{\partial s\,\partial t^2}+a_{39}\frac{\partial^4 w}{\partial s^2\,\partial t^2}+a_{3,10}\frac{\partial^3 w}{\partial s^2\,\partial t}+a_{3,11}\frac{\partial^2 w}{\partial s^2}+a_{3,12}\frac{\partial^3 w}{\partial s^3}+\frac{q_\gamma}{D_M}, \tag{7}$$

where the coefficients a_{ij} ($i = 1, 2, 3$; $j = 1, 2, ..., 12$) are expressed through the mechanical characteristics and are functions of the coordinates s and t.

The solution of the boundary-value problem for system of partial differential equations (7), with the corresponding boundary conditions on the boundary, will be sought in the form

$$u(s,t)=\sum_{i=0}^{N} u_i(s)\,\varphi_i(t), \quad \mathrm{v}(s,t)=\sum_{i=0}^{N} \mathrm{v}_i(s)\,\varphi_i(t), \tag{8}$$

$$w(s,t)=\sum_{i=0}^{N} w_i(s)\,\psi_i(t),$$

where $u_i(s)$, $\mathrm{v}_i(s)$, $w_i(s)$, ($i=\overline{0,N}$) are unknown functions; $\varphi_i(t)$ and $\psi_i(t)$ are linear combinations of B-splines of degree three and five, respectively [8, 10]. These spline combinations exactly satisfy the boundary conditions on the straight contours for open shells and the periodicity conditions for closed shells.

Shown below are the expressions for third- and fifth-degree B-splines on an expanded grid which is numbered on the basis of the middle node of the carrier

$$B_3^i(t)=\frac{1}{6}\begin{cases}0, & t<t_{i-2},\\ z^3, & t_{i-2}\le t<t_{i-1},\\ 1+3z+3z^2(1-z), & t_{i-1}\le t<t_i,\\ 3z^3-6z^2+4, & t_i\le t<t_{i+1},\\ (1-z)^3, & t_{i+1}\le t<t_{i+2},\\ 0, & t\ge t_{i+2},\end{cases} \tag{9}$$

$$B_5^i(t)=\frac{1}{120}\begin{cases}0, & t<t_{i-3},\\ z^5, & t_{i-3}\le t<t_{i-2},\\ -5z^5+5z^4-10z^3-10z^2+5z+1, & t_{i-2}\le t<t_{i-1},\\ 10z^5-20z^4-20z^3+20z^2+50z+26, & t_{i-1}\le t<t_i,\\ -10z^5+30z^4-60z^2+66, & t_i\le t<t_{i+1},\\ 5z^5-20z^4+20z^3+20z^2-50z+26, & t_{i+1}\le t<t_{i+2},\\ (1-z)^5, & t_{i+2}\le t<t_{i+3},\\ 0, & t\ge t_{i+3},\end{cases} \tag{10}$$

where $z=(t-t_k)/h$, ($t_{k+1}-t_k=h=\text{const}$) on the interval $[t_k, t_{k+1}]$;

$$k=\overline{i-(m+1)/2,\, i+(m+1)/2-1}; \quad i=\overline{-(m+1)/2+1,\, N+(m+1)/2-1}; \quad m=3,5.$$

In particular, we have the following relations when the edge $t = t_1$ is fixed and the edge $t = t_2$ is pinned

$$\varphi_0(t)=-4B_3^{-1}(t)+B_3^0(t), \quad \varphi_1(t)=B_3^{-1}(t)-\frac{1}{2}B_3^0(t)+B_3^1(t),$$

$$\varphi_i(t)=B_3^i(t), \quad (i=\overline{2,N-2}), \quad \varphi_{N-1}(t)=B_3^{N-1}(t)-\frac{1}{2}B_3^N(t)+B_3^{N+1}(t),$$

$$\varphi_N(t)=B_3^N(t)-4B_3^{N+1}(t), \quad \psi_0(t)=\frac{165}{4}B_5^{-2}-\frac{33}{8}B_5^{-1}(t)+B_5^0(t), \tag{11}$$

$$\psi_1(t)=B_5^{-1}(t)-\frac{26}{33}B_5^0(t)+B_5^1(t), \quad \psi_2(t)=B_5^{-2}(t)-\frac{1}{33}B_5^0(t)+B_5^2(t),$$

$$\psi_i(t)=B_5^i(t), \ (i=\overline{3,N-3}), \quad \psi_{N-2}(t)=B_5^{N-2}(t)-B_5^{N+2}(t),$$

$$\psi_{N-1}(t)=B_5^{N-1}(t)-B_5^{N+1}(t), \quad \psi_N(t)=B_5^N(t)-3B_5^{N+1}(t)+12B_5^{N+2}(t).$$

Equations (11) can easily be used to construct expressions for $\varphi_i(t)$ and $\psi_i(t)$ when both edges are fixed or pinned. Similar procedures are used to construct expressions in the form of linear combinations of B-splines for other boundary conditions or symmetry conditions [5, 8, 10].

Choosing the functions $\varphi_i(t)$ and $\psi_i(t)$ so as to satisfy the boundary conditions or symmetry conditions on the straight edges, we insert Eqs. (8) into differential equations (7) and require that they be satisfied at the collocation points $t=t_k$ $(k=\overline{0,N})$, i.e., on straight line $N + 1$. After completing several transformations, we obtain a system of ordinary differential equations of the order $8(N + 1)$

$$\frac{d\bar{Z}}{ds}=A(s)\bar{Z}+\bar{f}(s), \quad (0\le s\le L). \tag{12}$$

Here,

$$\bar{Z}=\{\bar{z}_1,\bar{z}_2,\dots,\bar{z}_8\}^T=\{\bar{u},\bar{u}',\bar{v},\bar{v}',\bar{w},\bar{w}',\bar{w}'',\bar{w}'''\}^T,$$

where $\bar{z}_m=\{z_{m_0},z_{m_1},\dots,z_{m_N}\}^T$, $(m=\overline{1,8})$.

We can use the boundary conditions assigned on the edges $s = 0$ and $s = L$ to formulate boundary conditions for system (12). In the general case, the boundary conditions for that system will have the form

$$B_1\bar{Z}(0)=\bar{b}_1, \ B_2\bar{Z}(L)=\bar{b}_2. \tag{13}$$

We use the stable method of discrete orthogonalization [3, 9] to solve the boundary-value problem for system (12) with boundary conditions (13). Inserting the values found for the functions $u_i(s)$, $v_i(s)$, and $w_i(s)$, $(i=\overline{0,N})$ into Eqs. (8), we obtain the solution of the initial problem for the displacements and use that solution to calculate all of the factors of the stress-strain state of the shell. Some results obtained from an evaluation of the accuracy of this approach were presented in [5, 8, 11].

Let us now present the results of the solution of problems on the basis of the given approach. We will examine the problem of the stress-strain state of an isotropic cylindrical shell with an elliptic cross section. The shell is subjected to a uniformly distributed normal surface load q_γ. We write the parametric equations of the cross section of the shell in the form $x=b\cos\psi$, $z=a\sin\psi$ $(0\le\psi\le 2\pi)$, where a and b are the minor and major semi-axes of the ellipse, respectively. The perimeter of the middle section of the shell remains constant and equal to the perimeter of a circle of radius R, i.e., we have the equality

$$\pi(a+b)\left(1+\frac{\Delta^2}{4}+\frac{\Delta^4}{64}+\frac{\Delta^6}{256}+\ldots\right)=2\pi R, \tag{14}$$

where

$$\Delta=\frac{b-a}{b+a}, \quad a=\frac{R}{f}(1-\Delta), \quad b=\frac{R}{f}(1+\Delta),$$

$$f=1+\frac{\Delta^2}{4}+\frac{\Delta^4}{64}+\frac{\Delta^6}{256}+\ldots, \quad \frac{a}{b}=\frac{1-\Delta}{1+\Delta};$$

$$\frac{d}{dt}=\frac{1}{\gamma(\psi)}\frac{d}{d\psi}, \quad \gamma(\psi)=\sqrt{\left(\frac{dx}{d\psi}\right)^2+\left(\frac{dz}{d\psi}\right)^2}.$$

We will study the stress-strain state of the ellipsoidal cylindrical shell for three variants:
variant 1 — an open shell ($-\pi/2 \le \psi \le \pi/2$), the curvilinear and straight edges of which are fixed, i.e., the following boundary conditions are satisfied:

$$u=\mathrm{v}=w=\frac{\partial w}{\partial s}=0 \ \text{ at } \ s=0, \ s=L,$$

$$u=\mathrm{v}=w=\frac{\partial w}{\partial t}=0 \ \text{ at } \ \psi=-\frac{\pi}{2}, \ \psi=\frac{\pi}{2}; \tag{15}$$

variant 2 — an open shell ($-\pi/2 \le \psi \le \pi/2$) with fixed curvilinear edges and pinned straight edges, i.e., the following boundary conditions are satisfied:

$$u=\mathrm{v}=w=\frac{\partial w}{\partial s}=0 \ \text{ at } \ s=0, \ s=L,$$

$$u=\mathrm{v}=w=M_t=0 \ \text{ at } \ \psi=-\frac{\pi}{2}, \ \psi=\frac{\pi}{2}; \tag{16}$$

variant 3 — a closed shell with fixed curvilinear edges, i.e., the following boundary conditions are satisfied:

$$u=\mathrm{v}=w=\frac{\partial w}{\partial s}=0 \ \text{ at } \ s=0, \ s=L. \tag{17}$$

We can solve the problem by examining part of the shell:

$$0 \le s \le L/2, \quad 0 \le \psi \le \pi/2,$$

specifying symmetry conditions with $s = L/2$, $\psi = 0$, and $\psi = \pi/2$.

The problem was solved using the following data: $R = 20$; $L = 60$; $q_\gamma = q_0 = \text{const}$, $h = h_0(1+\alpha|\sin\psi|)$, $h_0 = 0.5$, $\nu = 0.3$; Δ and α were given different values, which are shown in the tables. Twelve terms were taken into account in series (8).

Table 1 shows the maximum deflections w in the section $s = L/2$ at $\psi = 0.7\,(\pi/2)$ for variants 1 and 2 and $\psi = \pi/2$ for variant 3. The table also shows the forces N_t at $\psi = 0$ for variants 1 and 2 and $\psi = \pi/2$ for variant 3, as well as the moments M_t at $\psi = \pi/2$ for variants 1 and 3 and $\psi = 0.8\,(\pi/2)$ for variant 2. The tabular data illustrates how the deflections, forces, and moments change in relation to shell thickness along the directrix for the three variants when the value of Δ is also changed.

Table 2 show the distribution of the deflections, forces, and moments along the generatrix for a closed shell with $s = L/2$ and the same parameters as in the previous case. The distributions are shown for different values of α when $\Delta = 0.005$. The goal here was to make the distribution of the deflection w along the generatrix of the elliptic cylindrical shell as uniform

TABLE 1

Δ	n	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.25$	$\alpha = 0.5$
$w/\frac{10^4 q_0}{E}$					
0.1	1	0.4440	0.3985	0.3484	0.2891
	2	0.4600	0.4213	0.3723	0.3084
	3	0.5363	0.4845	0.4233	0.3498
0.2	1	0.9409	0.8441	0.7227	0.5683
	2	0.9673	0.8894	0.7875	0.6511
	3	1.246	1.120	0.9718	0.7956
$N_t/10^3 q_0$					
0.1	1	2.256	2.169	2.032	1.806
	2	2.121	2.062	1.967	1.805
	3	−1.770	−1.755	−1.738	−1.719
0.2	1	4.418	4.294	4.072	3.659
	2	4.183	4.108	3.965	3.677
	3	−3.661	−3.609	−3.547	−3.470
$M_t/10^3 q_0$					
0.1	1	−1.633	−1.858	−2.205	−2.801
	2	0.6614	0.7377	0.8548	1.056
	3	0.2542	0.2783	0.3168	0.3882
0.2	1	−3.913	−4.391	−5.127	−6.382
	2	1.690	1.856	2.130	2.583
	3	0.7512	0.8084	0.8998	1.070

TABLE 2

$\psi/\frac{\pi}{2}$	$\Delta=0$	$\Delta=0.005$	$\Delta=0.005$	$\Delta=0.005$	$\Delta=0.005$
	$\alpha=0$	$\alpha=0$	$\alpha=0.25$	$\alpha=0.50$	$\alpha=0.83$
$w/\frac{10^3 q_0}{E}$					
0	0.7334	0.5492	0.5322	0.5150	0.4934
0.2	0.7334	0.5841	0.5548	0.5281	0.4968
0.4	0.7334	0.6761	0.6087	0.5551	0.4988
0.6	0.7334	0.7911	0.6694	0.5812	0.4961
0.8	0.7334	0.8853	0.7155	0.5995	0.4929
1	0.7334	0.9215	0.7327	0.6061	0.4918
$N_t/10^2 q_0$					
0	0.2000	1.081	1.170	1.261	1.374
0.2	0.2000	0.9129	0.8439	0.7825	0.7086
0.4	0.2000	0.4723	0.3590	0.2555	0.1330
0.6	0.2000	−0.0722	−0.1406	−0.1988	−0.2634
0.8	0.2000	−0.5133	−0.5119	−0.5029	−0.4860
1	0.2000	−0.6820	−0.6493	−0.6106	−0.5582
$M_t/10\,q_0$					
0	0	−0.8367	−0.5589	−0.3460	−0.1224
0.2	0	−0.6832	−0.4657	−0.2431	0.0601
0.4	0	−0.2765	−0.1084	0.0456	0.2013
0.6	0	0.2487	0.3026	0.2566	0.0101
0.8	0	0.6939	0.6537	0.4396	−0.1447
1	0	0.8710	0.8012	0.5320	−0.1640

as possible by suitably choosing the parameters characterizing the variable thickness of the shell. It follows from the table that the distribution is most uniform when $\alpha = 0.83$. The results show how the forces N_t and moments M_t are redistributed along the directrix as a result of a change in thickness.

Thus, the results shown in Tables 1 and 2 from the solution of the given problems demonstrate the effect of a change in thickness in noncircular cylindrical shells on the stress-strain state.

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