Combustion, Explosion, and Shock Waves, Vol. 36, No. 5, 2000

Diffusion Combustion of a Liquid Fuel Film on a Metal Substrate

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UDC 536.46

Translated from *Fizika Goreniya i Vzryva*, Vol. 36, No. 5, pp. 12–21, September–October, 2000.
Original article submitted July 13, 1999; revision submitted February 20, 2000.

Diffusion combustion of a film of a liquid fuel (n-undecane and n-butanol) deposited on the surface of a thin metal substrate is studied experimentally and theoretically. The experimental data obtained show that the mechanism determining the heating and evaporation of the combustible liquid is the heat transfer from the region of combustion products to the heating zone due to the high longitudinal thermal conductivity of the substrate. Prior to combustion, the combustible liquid may evaporate, not reaching the boiling point. A simple model that takes into account these features is proposed. The calculated velocities of flame propagation and temperature profiles are compared to experimental data. It is shown that the model gives a correct description of the dependence of the flame velocity on the substrate thickness, initial temperature, and properties of the substrate and liquid fuel.

INTRODUCTION

Flame propagation over the fuel surface is a complex physicochemical process. Experimental research is complicated by the fact that it is almost impossible to vary some governing parameters and retain other parameters constant. The gas-phase reaction is accompanied by a number of complicating parameters: pyrolysis of the fuel, formation of condensed products, heterogeneous reactions, phase transitions, etc. It is difficult to take these factors into account in simple models. Korzhavin et al. [1] considered a layered system consisting of a liquid fuel film on a thin metal substrate. In this system, it is possible to vary the parameters of the problem (evaporation heat, boiling point of the fuel, initial temperature, thermal conductivity, substrate thickness, etc.) independently within wide limits. The use of substances with well-known properties as fuels and substrates allows a comparison with the existing theories avoiding uncertainties caused by the absence of some data, for example, on the pyrolysis of combustible materials, such as paper, polymers, etc. On the other hand, such a system offers wide possibilities for modeling the combustion of actual combustible materials. The existing theories of flame propagation [2–4] do not take into account the special features of layered systems. Therefore, their updating or the development of a new theory is needed. As is shown below, accounting for the special features of a layered system within the framework of the existing theories does not yield good agreement between experimental and theoretical data.

The available experimental data on combustion of thin films of fuels do not often correspond to the notion of thermally thin systems, and their comparison with the theory is unjustified. For example, the definition of a thermally thin layer was not given in [5], where flame propagation above thermally thin strips of paper was studied. Treatment of the numerical data of [5] in accordance with the definition of thermal thinness [4] for a flame velocity $u = 0.2$–0.4 cm/sec, thickness of the fuel layer $h_{\rm f} = 0.021$ cm, and temperature diffusivity $\varkappa_{\rm f} = 0.94 \cdot 10^{-3}$ cm^2/sec for filter paper that does not give ashes shows that the system under study is not thermally thin according to the criterion of [4], since $uh_{\rm f}/\varkappa_{\rm f} = 4.5$–$9 \gg 1$, and not vice versa.

Obviously, the fact that a system belongs to

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0010-5082/00/3605-0562 $25.00

thermally thin systems should be determined on the basis of a comparison of the rates of particular thermophysical processes. Thus, in the experimental work [6], which deals with ignition by a radiation flux, this problem was solved depending on the ratio of the Fourier and Biot numbers: $\mathrm{Fo} = \varkappa_f t/h_f^2$ and $\mathrm{Bi} = \alpha h_f/\lambda_f$. Here α is the coefficient of convective heat transfer, t is the exposure time of the layer, and λ_f is the thermal conductivity of the fuel. In the case of low Biot numbers, the system was assumed to be thermally thin for $\mathrm{Fo} > 1$. This means that a flat sample is uniformly heated over its thickness if the exposure time of a layer is greater than the time of thermal relaxation in the layer $h_f^2/\varkappa_f$. The dependence of Fo on Bi took into account the sample cooling by the ambient air. In the theoretical paper of Rybanin et al. [4], where combustion of thin layers of a fuel was considered, the criterion $h_f \ll \varkappa_f/u$ was used: the layer thickness was much smaller than the thermal thickness in the combustion wave. This relation is equivalent to the criterion $h_f^2/\varkappa_f \ll \varkappa_f/u^2$, which means that the time of thermal relaxation in the layer should be much smaller than the characteristic time of heat transfer in the flame front. It seems that the condition of a thermally thin system was not satisfied in many experimental papers devoted to the problem under study, because of the low thermal diffusivity of the fuels. Therefore, without verification whether a system is thermally thin, it is incorrect to compare experimental data with calculation results for thermally thin systems.

Korzhavin et al. [1] proposed a scheme of an experiment with the use of metal substrates with a high temperature diffusivity, which satisfied the conditions of a thermally thin system. In analyzing the mechanism of flame propagation, the high longitudinal thermal conductivity of such a system should take into account the possibility of heat transfer from the region of combustion products to the preflame zone. The present work deals with theoretical and experimental investigation of propagation of the diffuse flame above thermally thin systems.

EXPERIMENTAL TECHNIQUE

The system "liquid fuel on a metal substrate" is almost unexplored. It is known, however, that flame propagation above a liquid is complicated by convection in the liquid itself. In this case, the flow of a fuel film may be caused by two factors. The first factor is the force of gravity if the system is located vertically or inclined. It is shown experimentally that the downward motion of the film may be ignored under certain conditions. The second factor is the film flow caused by the gradient of the surface-tension forces, which is conditioned by the temperature gradient in the system upon flame propagation. It will be shown below that this phenomenon does not exert a significant effect on flame propagation for thin films either.

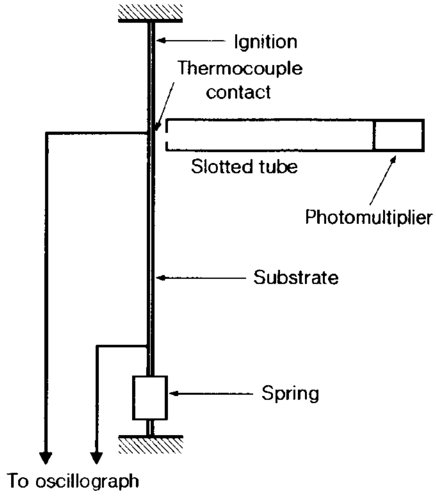

Fig. 1. Layout of the experimental facility.

In the present work, we used films of n-butanol (8 μm) and n-undecane (7 μm) as a fuel and aluminum or copper foils 7–120 μm thick as substrates. The layout of the experiment is shown in Fig. 1. For the experiment, a strip of a metal foil of width of more than 5 mm (for these dimensions, the velocity is independent of the width, and a foil 40 mm wide was usually used) and length of 0.2–2 m was fixed vertically. A thermocouple contact (copper–Constantan) was located at the point located opposite the horizontal slot of the photomultiplier. The copper element of the thermocouple was the foil strip itself. The reliability of the thermocouple contact was ensured by spring pressing. Such a device allowed us to locate the structural characteristics of the flame-front edge within 0.8 mm.

In addition to the flame glowing and substrate temperature, the flame velocity was measured. In some experiments, the fuel film was applied to both sides of the foil, and a two-sided flame was studied.

In a thermally thin system, due to the fast heat exchange between the layers, the determining role is played not only by individual parameters of system elements but also by the effective temperature diffusivity [7]:

$$\varkappa = \frac{\lambda_f h_f + \lambda_s h_s}{c_f \rho_f h_f + c_s \rho_s h_s},$$

where λ, h, c, and ρ are the thermal conductivity, thickness, heat capacity, and density; the subscripts "f" and "s" refer to the liquid fuel and solid substrate, respectively.

In a thermally thin layered system, the characteristic times of thermal relaxation in the layers of the fuel τ_f and the substrate τ_s should be much smaller than the characteristic time of heat transfer along the substrate τ_b. For systems studied in the present work, we had $h_\mathrm{f} \simeq 7$ μm, $h_\mathrm{s} = 7$–120 μm, $\varkappa_\mathrm{f} \simeq 10^{-3}$ cm^2/sec, $\varkappa_\mathrm{s} \simeq 0.2$–$1.12$ cm^2/sec, and $u \simeq 5$ cm/sec. It follows from here that $\tau_b = \varkappa_*/u^2 = 8 \cdot 10^{-3}$–$4.5 \cdot 10^{-2}$ sec, $\tau_\mathrm{s} = h_\mathrm{s}^2/\varkappa_\mathrm{s} \simeq 10^{-4}$–$10^{-6}$ sec, and $\tau_\mathrm{f} = h_\mathrm{f}^2/\varkappa_\mathrm{f} \simeq 5 \cdot 10^{-4}$ sec, i.e., τ_f and τ_s are much smaller than τ_b. Hence, the chosen layered system consisting of a film of a liquid fuel on a metal foil is really thermally thin within the entire examined range of parameters.

MATHEMATICAL MODEL

The experimental studies of combustion-wave propagation above films of liquid hydrocarbon fuels on thin substrates of copper, aluminum, and other metals [1] allowed us to formulate a physical model of the phenomenon. The combustion wave propagates along the fuel surface due to the formation of a combustible mixture near the surface upon evaporation of the liquid fuel. Evaporation of the fuel results from its heating by the heat coming along the substrate from the combustion products. The substrate is heated due to the interphase heat exchange behind the flame edge. The flame edge is located above the point on the substrate, where the temperature of the substrate and fuel is equal to the temperature at which a stoichiometric mixture is formed under equilibrium conditions. After that, the fuel film completely evaporates at a certain distance from the flame edge, which is smaller than the flame-front length. Complete evaporation of the liquid may occur at temperatures below the boiling point.

The scheme of flame propagation over the layered system considered is shown in Fig. 2. It consists of three regions wherein the gas has different temperatures. Region 1 is filled by air with a temperature T_0 equal to the gas temperature far from the combustion region. Diffuse combustion of the evaporated fuel occurs in region 2. It is assumed that this region near the substrate is filled by hot combustion products with a temperature T_b. Cooling of the combustion products caused by heat exchange with the substrate and ambient gas and also the convective ascent of the heated combustion products determine the length of this region L. In what follows, we assume that this length is roughly equal to the length of the diffuse plume. In region 3, the temperature of the gas consisting of a mixture of air and combustion products is T_0 because of the cooling of the latter. It is assumed that the gas temperature under the substrate is also equal to T_0, and the heat transfer between the gas and the "substrate–fuel" system follows the mechanism of free convection. The heat released in the course of the chemical reaction in the gas phase heats the substrate and is transferred over it due to its thermal conduction to the region, where fuel evaporation occurs.

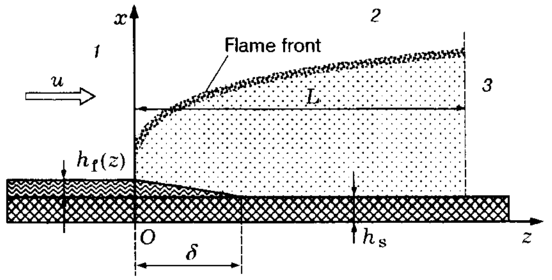

Fig. 2. Model representation of flame propagation over a layered system "fuel film–substrate": 1) heating zone; 2) zone of combustion products; 3) cooling zone.

A moving liquid film may also transfer heat [8, 9]. The mass transferred through a unit cross-sectional area of the liquid film per unit time is determined by the expression [10]

$$\dot m = \frac{\rho_\mathrm{f} h_\mathrm{f}}{2\mu}\,\sigma_T\,\frac{dT}{dz},$$

where T is the temperature, $\sigma_T = d\sigma/dT$, and σ and μ are the surface tension and dynamic viscosity of the liquid, respectively. It is seen from this formula that the transported mass of the liquid depends, in particular, on the fuel-film thickness. There are no experimental data on the dependence $h_\mathrm{f}(z)$. Therefore, it is difficult to evaluate the contribution of convective heat transfer by the liquid. Nevertheless, this estimation may be made on the basis of experimental profiles of the substrate temperature. It was experimentally shown that the temperature profile in the heating zone depends exponentially on the coordinate. The exponent is mainly determined by the effective value of the temperature diffusivity of the layered system, i.e., mass transfer does not make a noticeable contribution to heat transfer to the heating zone. The main amount of heat is transferred upward by thermal conduction over the substrate, and the fuel-film thickness in region 1 is assumed to be constant. Therefore, the simplest variant of the model takes into account only the heat transfer by thermal conduction over the substrate. It is assumed that the fuel layer completely evaporates over the length δ in region 2.

Within the framework of these assumptions, we derive quasi-one-dimensional equations for the longitudinal heat distribution in the "substrate–fuel" system. In the frame of reference fitted to the moving flame, the gas and the substrate with a fuel layer are incoming onto the flame with a constant velocity u along the z axis. The equations of heat propagation over the substrate and in the liquid fuel layer have the form

$$\begin{aligned} c_{p,\mathrm{s}}\rho_{\mathrm{s}} u \frac{\partial T_{\mathrm{s}}}{\partial z} - \lambda_{\mathrm{s}} \frac{\partial^2 T_{\mathrm{s}}}{\partial z^2} - \lambda_{\mathrm{s}} \frac{\partial^2 T_{\mathrm{s}}}{\partial x^2} &= 0, \\ c_{p,\mathrm{f}}\rho_{\mathrm{f}} u \frac{\partial T_{\mathrm{f}}}{\partial z} - \lambda_{\mathrm{f}} \frac{\partial^2 T_{\mathrm{f}}}{\partial z^2} - \lambda_{\mathrm{f}} \frac{\partial^2 T_{\mathrm{f}}}{\partial x^2} &= 0, \end{aligned} \tag{1}$$

where c_p is the specific heat capacity at constant pressure. The x coordinate is counted along the axis normal to the substrate surface. The fuel-film thickness is h_{f}, and the substrate thickness is h_{s}. The surface separating the substrate and the cold gas is defined as $x = 0$, and the boundary between the fuel and the substrate is described by the equation $x = h_{\mathrm{s}}$. The boundary between the fuel and the gas is defined as $x = h_{\mathrm{s}} + h_{\mathrm{f}}$. For the case of convective heat transfer, we can write the following conditions for heat fluxes at these boundaries:

$$\lambda_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial x} = \alpha(T_{\mathrm{s}} - T_0) \tag{2}$$

for $x = 0$,

$$\lambda_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial x} = \lambda_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial x}, \quad T_{\mathrm{s}} = T_{\mathrm{f}}, \tag{3}$$

for $x = h_{\mathrm{s}}$ and $z < \delta$,

$$\lambda_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial x} = -\alpha(T_{\mathrm{f}} - T_0) \tag{4}$$

for $x = h_{\mathrm{s}} + h_{\mathrm{f}}$ and $z < 0$, and

$$\lambda_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial x} = -\alpha(T_{\mathrm{f}} - T_b) + q\rho_{\mathrm{f}} u \frac{dh_{\mathrm{f}}}{dz}. \tag{5}$$

for $0 < z < \delta$.
Here q is the specific heat of fuel evaporation. For $x = h_{\mathrm{s}}$ and $\delta < z < L$, we have

$$\lambda_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial x} = -\alpha(T_{\mathrm{s}} - T_b), \tag{6}$$

and for $x = h_{\mathrm{s}}$ and $z > L$, we have

$$\lambda_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial x} = -\alpha(T_{\mathrm{s}} - T_0). \tag{7}$$

In writing the boundary conditions, it was assumed that the fuel is located only in the region $z < \delta$ and is completely absent in the combustion region $z > \delta$. Assuming the "fuel–substrate" system to be thermally thin, we integrate system (1) with respect to the transverse coordinate x from 0 to h_{s} for the substrate and in the interval from h_{s} to $h_{\mathrm{s}} + h_{\mathrm{f}}$ for the fuel layer. We also assume that the temperatures of the substrate $T_{\mathrm{s}}(z)$ and fuel $T_{\mathrm{f}}(z)$ are identical and equal to their mean values along the x axis:

$$\langle T_{\mathrm{s}}(z)\rangle = \langle T_{\mathrm{f}}(z)\rangle = T(z).$$

Here

$$\langle T_{\mathrm{s}}(z)\rangle = \frac{1}{h_{\mathrm{s}}} \int_0^{h_{\mathrm{s}}} T_{\mathrm{s}}(z)\, dx,$$

$$\langle T_{\mathrm{f}}(z)\rangle = \frac{1}{h_{\mathrm{f}}} \int_{h_{\mathrm{s}}}^{h_{\mathrm{s}}+h_{\mathrm{f}}} T_{\mathrm{f}}(z)\, dx.$$

After integration, system (1) acquires the following form:

$$\begin{aligned} & c_{p,\mathrm{s}}\rho_{\mathrm{s}} u h_{\mathrm{s}} \frac{\partial T_{\mathrm{s}}}{\partial z} - \lambda_{\mathrm{s}} h_{\mathrm{s}} \frac{\partial^2 T_{\mathrm{s}}}{\partial z^2} \\ & \qquad - \lambda_{\mathrm{s}} \Big(\frac{\partial T_{\mathrm{s}}}{\partial x}\Big|_{h_{\mathrm{s}}} - \frac{\partial T_{\mathrm{s}}}{\partial x}\Big|_{0} \Big) = 0, \\ & c_{p,\mathrm{f}}\rho_{\mathrm{f}} u h_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial z} - \lambda_{\mathrm{f}} h_{\mathrm{f}} \frac{\partial^2 T_{\mathrm{f}}}{\partial z^2} \\ & \qquad - \lambda_{\mathrm{f}} \Big(\frac{\partial T_{\mathrm{f}}}{\partial x}\Big|_{h_{\mathrm{s}}+h_{\mathrm{f}}} - \frac{\partial T_{\mathrm{f}}}{\partial x}\Big|_{h_{\mathrm{s}}} \Big) = 0. \end{aligned} \tag{8}$$

Adding the equations in system (8) and taking into account the boundary conditions (2)–(7), we obtain quasi-one-dimensional equations for the temperature $T(z)$:

$$u \frac{dT_1}{dz} - \varkappa \frac{d^2 T_1}{dz^2} + 2\omega(T_1 - T_0) = 0 \tag{9}$$

for $-\infty < z < 0$,

$$u \frac{dT_2}{dz} - \varkappa_{\mathrm{s}} \frac{d^2 T_2}{dz^2} + \omega_{\mathrm{s}}(2T_2 - T_b - T_0) = 0 \tag{10}$$

for $\delta < z < L$, and

$$u \frac{dT_3}{dz} - \varkappa_{\mathrm{s}} \frac{d^2 T_3}{dz^2} + 2\omega_{\mathrm{s}}(T_3 - T_0) = 0. \tag{11}$$

for $L < z < \infty$.
Here

$$\omega = \frac{\alpha}{c_{p,\mathrm{s}}\rho_{\mathrm{s}} h_{\mathrm{s}} + c_{p,\mathrm{f}}\rho_{\mathrm{f}} h_{\mathrm{f}}}, \qquad \omega_{\mathrm{s}} = \frac{\alpha}{c_{p,\mathrm{s}}\rho_{\mathrm{s}} h_{\mathrm{s}}}.$$

The subscripts 1, 2, and 3 refer to the regions $-\infty < z < 0$, $\delta < z < L$, and $L < z < \infty$, respectively.

We consider the conditions that relate the derivatives and the values of the function $T(z)$ at the points $z = 0$ and $z = L$. The temperature and its derivative are continuous at the boundary of regions 2 and 3:

$$T_2\Big|_{z=L} = T_3\Big|_{z=L}, \quad \frac{dT_2}{dz}\Big|_{z=L} = \frac{dT_3}{dz}\Big|_{z=L}. \qquad (12)$$

To obtain the boundary condition that relates the temperature and heat fluxes at the boundary of regions 1 and 2, we use the assumption that the length of the fuel-evaporation zone is much smaller than the characteristic length of heating of the substrate by combustion products [$\delta \ll (\varkappa_s\omega_s)^{1/2} \approx 3$ mm] and much smaller than the length of the combustion zone ($\delta \ll L \approx 3$ cm).

We consider two variants of conditions that relate heat fluxes at the boundaries of the region $0 < z < \delta$. The first variant is obtained in the limiting case $\delta \to 0$; the second variant implies that $h_f \ll \delta \ll L$. We write the equation for the substrate and fuel temperature in the region $0 < z < \delta$ in the quasi-one-dimensional approximation:

$$u\frac{dT}{dz} - \varkappa(z)\frac{d^2T}{dz^2} + \omega(z)(2T - T_b - T_0) - \frac{q\rho_f u}{c_{p,s}\rho_s h_s + c_{p,f}\rho_f h_f(z)}\frac{dh_f}{dz} = 0. \qquad (13)$$

This equation is derived by the same method as (10) taking into account the boundary condition (5).

Relation (13) takes into account the heat consumption necessary for fuel evaporation. To obtain the relation between the heat fluxes at the boundaries of the region δ, we integrate Eq. (13) over the z coordinate from 0 to δ:

$$u\left(T\big|_{z=\delta} - T\big|_{z=0}\right) - \left(\varkappa_s\frac{dT}{dz}\Big|_{z=\delta} - \varkappa\frac{dT}{dz}\Big|_{z=0}\right) + \frac{1}{2}(\omega_s + \omega)\delta\left(2T\big|_{z=0} - T_b - T_0\right) + \frac{qu}{c_{p,f}}\ln\left(1 + \frac{c_{p,f}\rho_f h_f}{c_{p,s}\rho_s h_s}\right) = 0. \qquad (14)$$

In integrating (13), the integrals of terms containing the variable thickness of the fuel film h_f are calculated approximately using the formulas

$$\int_0^\delta \varkappa(z)\frac{d^2T}{dz^2}\,dz \simeq \varkappa_s\frac{dT_2}{dz}\Big|_{z=\delta} - \varkappa\frac{dT_1}{dz}\Big|_{z=0},$$

$$\int_0^\delta \omega(z)(2T - T_b - T_0)\,dz \simeq (\omega_s + \omega)\delta\left(2T\big|_{z=0} - T_b - T_0\right)/2.$$

The boundary condition for the heat fluxes is obtained from Eq. (14):

$$\varkappa_s\frac{dT_2}{dz}\Big|_{z=\delta} = (\varkappa + u\delta)\frac{dT_1}{dz}\Big|_{z=0} + \frac{qu}{c_{p,f}}\ln\left(1 + \frac{c_{p,f}\rho_f h_f}{c_{p,s}\rho_s h_s}\right) + \delta(\omega_s + \omega)(2T_v - T_b - T_0)/2. \qquad (15)$$

The temperatures at the boundaries of the region δ are related by the equation

$$T_2(z)\big|_{z=\delta} = T_v + \delta\frac{dT}{dz}\Big|_{z=0}, \qquad (16)$$

where $T_v = T\big|_{z=0}$. It is assumed in the model that the temperature at the boundary $z = 0$, where the diffuse plume in the gas phase appears, is equal to the temperature T_v for which the concentration of fuel vapors on the liquid surface reaches the stoichiometric value under equilibrium conditions.

VELOCITY OF COMBUSTION-WAVE PROPAGATION

We consider solutions that describe diffusion combustion of a liquid-fuel film on a metal substrate. The solution of the linear system (9)–(11) with the boundary conditions (12), (15), and (16) does not involve severe mathematical difficulties. The temperature distribution in the condensed phase along the z axis obtained by solving this system has the form

$$T_1 = T_0 + (T_v - T_0)\exp(\lambda_0 z), \qquad (17)$$

$$T_2 = \frac{T_b + T_0}{2} + A\exp(\lambda_+ z) + B\exp(\lambda_- z), \qquad (18)$$

$$T_3 = T_0 + C\exp(\lambda_- z). \qquad (19)$$

Here

$$\lambda_0 = \frac{u}{2\varkappa} + \sqrt{\left(\frac{u}{2\varkappa}\right)^2 + \frac{2\omega}{\varkappa}},$$

$$\lambda_\pm = \frac{u}{2\varkappa_s} \pm \sqrt{\left(\frac{u}{2\varkappa_s}\right)^2 + \frac{2\omega_s}{\varkappa_s}},$$

$$A = \frac{\lambda_-(T_b - T_0)\exp(-\lambda_+ L)}{2(\lambda_+ - \lambda_-)},$$

$$B = -A\exp((\lambda_+ - \lambda_-)\delta) - (T_b + T_0 - 2T_v) \times \exp(\lambda_-\delta)/2 + (T_v - T_0)\lambda_0\delta\exp(-\lambda_-\delta),$$

$$C = B + \lambda_+ A\exp((\lambda_+ - \lambda_-)L)/\lambda_-.$$

Substituting the solutions of (17)–(19) into the boundary condition (15), we obtain the transcendental equation for the velocity of combustion-wave propagation

$$\lambda_-(\Theta - (\Theta + 2)\exp(-\lambda_+(L - \delta))/2 + \lambda_0\Big(\frac{\varkappa}{\varkappa_{\rm s}} - \delta\Big(\lambda_- - \frac{u}{\varkappa_{\rm s}}\Big)\Big) + \frac{qu}{\varkappa_{\rm s} c_{p,{\rm f}}(T_v - T_0)} \ln\Big(1 + \frac{c_{p,{\rm f}}\rho_{\rm f} h_{\rm f}}{c_{p,{\rm s}}\rho_{\rm s} h_{\rm s}}\Big) - \frac{\omega_{\rm s} + \omega}{2\varkappa_{\rm s}}\Theta\delta = 0, \qquad (20)$$

where $\Theta = (T_b + T_0 - 2T_v)/(T_v - T_0)$.

If the conditions $u^2 \gg 8\omega\varkappa$, $L \gg \varkappa_{\rm s}/u$, and $\delta \ll \varkappa/u$ are satisfied, the equation for the combustion-wave velocity can be written in the simple form:

$$u^2 = \frac{\alpha\varkappa_{\rm s}}{c_{p,{\rm s}}\rho_{\rm s} h_{\rm s}} \times \Big[1 + \frac{q}{c_{p,{\rm f}}(T_v - T_0)} \ln\Big(1 + \frac{c_{p,{\rm f}}\rho_{\rm f} h_{\rm f}}{c_{p,{\rm s}}\rho_{\rm s} h_{\rm s}}\Big)\Big]^{-1} \times \frac{T_b + T_0 - 2T_v}{T_v - T_0}. \qquad (21)$$

DISCUSSION

We compare the numerical and experimental dependences of the flame-propagation velocity on the substrate thickness and initial temperature. We also compare the experimental temperature profile of the system with that calculated using Eqs. (17)–(19).

The calculation requires knowledge of the corresponding parameters of the problem. The temperature in the region of combustion products T_b was assumed to be equal to 2000 K. The temperature for which the pressure of saturated vapors of the fuel under equilibrium conditions corresponds to a stoichiometric composition in the mixture with air was calculated using the known dependences of the pressure of saturated vapors on temperature [11]: $T_v = 344$ K for *n*-undecane and $T_v = 319$ K for *n*-butanol. The data on evaporation heat, thermal conductivity, heat capacity, and density for fuels and substrates are borrowed from [11, 12]. The substrate thickness was measured by a micrometer and calculated using the mass of a foil of known area. The measurement accuracy of the foil thickness was 10%. The film thickness determined by the weighting method was 7 μm for *n*-undecane and 8 μm for *n*-butanol (the accuracy of determining the fuel-film thickness was ≈15%). However, when the flame propagates from top to bottom on a vertical substrate, it is seen that a fuel roller moves over the substrate ahead of the flame edge at a distance of ≈1 cm from it, and the excess fuel flows out from this roller in the form of separate "fingers" [10]. Obviously, the film at the flame edge is thinner than that measured before the experiment. Its actual thickness was evaluated as follows. The liquid was applied to the surface of a horizontally located foil strip not over its entire length but on a certain section (to avoid the nonuniform outflow of the liquid from the roller under the action of the forces of gravity). After that flame propagation was initiated. It turned out that the flame propagates significantly farther than the limits of the wetted section. The reason is that not all the fuel corresponding to the initial film thickness burns out in the flame front per unit time. Part of the fuel is transported forward by the surface-tension forces. Therefore, after passing the section with the initially applied fuel, the flame continues its motion. The ratio of the distance covered by the flame to the length of the section with the applied film characterizes the ratio of the initial thickness of the fuel film to the film thickness directly under the flame edge. It turned out that the film thickness under the flame edge is two to three times thinner than the initially measured value. In model calculations, the film thickness was assumed to be constant and equal to 3 μm.

The heat-transfer coefficient α was estimated on the basis of notions on free-convective heat transfer between the flame front and the substrate [13]:

$$\mathrm{Nu} = 0.516\mathrm{Ra}^{0.25} = 0.516\Big(\frac{gd^3T_b}{\nu^2T_0}\,\mathrm{Pr}\Big)^{0.25},$$

$$\alpha \simeq 28\ \mathrm{W/(m^2\cdot K)}.$$

Here Ra and Pr = 0.7 are the Rayleigh and Prandtl numbers, respectively, g is the acceleration of gravity, $d = 1$ cm is the characteristic transverse size of the zone of combustion products, and ν is the kinematic viscosity of the gas, which was assumed to be equal to the viscosity of air at the initial temperature. The value of α for all types of fuels remained unchanged in calculations; it was assumed to be equal to 23 W/(K · m^2).

The length of the evaporation zone δ is determined by the conditions of foil wetting by a particular fuel and depends on the angle of wetting and the temperature of the system. This quantity was measured on the photographs of the propagating flame. We obtained $\delta = 2.5$ mm for the "*n*-undecane–foil" system and $\delta = 15$ mm for the "*n*-butanol–foil" system.

The dependence of the combustion-wave velocity u on the substrate thickness $h_{\rm s}$ for the layered system "*n*-undecane on an aluminum foil" calculated by formula (21) is plotted in Fig. 3 (curve 2). The same figure shows the experimental points and the

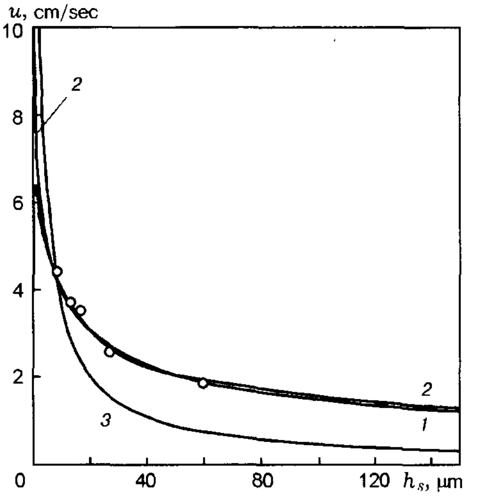

Fig. 3. Velocity of combustion-wave propagation versus the substrate thickness: n-undecane on an aluminum substrate; the points refer to the experiment, curves 1–3 refer to the exact solution, the solution by formula (21), and the solution of [2], respectively.

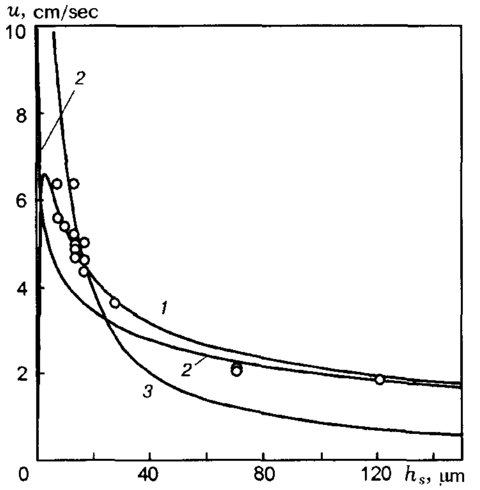

Fig. 4. Velocity of combustion-wave propagation versus the substrate thickness: n-butanol on an aluminum substrate; the points refer to the experiment, curves 1–3 refer to the exact solution, the solution by formula (21), and the solution of [2], respectively.

exact solution of Eq. (20) (curve 1). It was shown by numerical calculations that the plume length L for $L > 1$ cm has almost no effect on the calculated value of the combustion-wave velocity. The plume length in the experiments was within $1.5 < L < 5$ cm. In the calculations, it was assumed that $L = 3$ cm. It is seen from Fig. 3 that, for a foil thickness greater than 5 μm, the approximate formula (21) and the exact solution yield an almost identical result, which is in agreement with experimental data. The coincidence of the results calculated by the approximate (21) and exact (20) formulas is caused by the fact that the condition posed in deriving the approximate formula is satisfied for n-undecane for which the value of δ is small. For the layered system "n-butanol–aluminum foil," the exact solution gives a rather good description of experimental results (curve 1 in Fig. 4). However, the approximate formula (21) yields a noticeable deviation (curve 2). The reason is that the flame-propagation velocity increases as the substrate thickness decreases, and the condition $\delta \ll \varkappa/u$, which is one of the conditions for deriving the approximate formula (21) for velocity, is not satisfied for n-butanol with a high value of δ. The discrepancy between the exact and approximate solutions is observed in the region of very small thicknesses for which the experimental verification is very complicated. The exact solution of Eq. (20) has a maximum for thicknesses of 2–4 μm. For thinner substrates, the combustion-wave velocity decreases with decreasing substrate thickness, since at small thicknesses the substrate does not ensure the necessary heat input for fuel evaporation ahead of the flame edge.

Thus, for both n-butanol and n-undecane whose properties are rather different (boiling point, wetting angle, etc.), the model proposed gives a good description of experimental results. The approximate solution of Eq. (21) also provides good agreement with the experiment but only if the condition of its derivation is satisfied.

The papers [2–4], which deal with models of flame propagation over the surface of thin layers of condensed fuels, give a formula for the flame velocity, which contains many parameters such as the thermal conductivity of the gas, the heat capacity and density of the fuel, the thickness of the fuel layer, the temperature of combustion products, the boiling point of the fuel, and the initial temperature:

$$u = \frac{\sqrt{2}\lambda_g}{c_f\rho_f h_f}\,\frac{T_b - T_v}{T_v - T_0}.$$

Here the product $c_f\rho_f h_f$ is the heat capacity of a unit surface of the fuel layer. To apply these models to our layered system, which is also thermally thin, we should use the sum $c_f\rho_f h_f + c_s\rho_s h_s$ instead of this product. The quantity T_v in [2] is not clearly defined as the vaporization temperature. In our calculations, we use a value close to the temperature for which a stoichiometric mixture is formed above the fuel surface under equilibrium conditions. Substituting the values of thermophysical parameters for particular layered systems, we obtain the values of the flame-propagation velocity, which are significantly lower

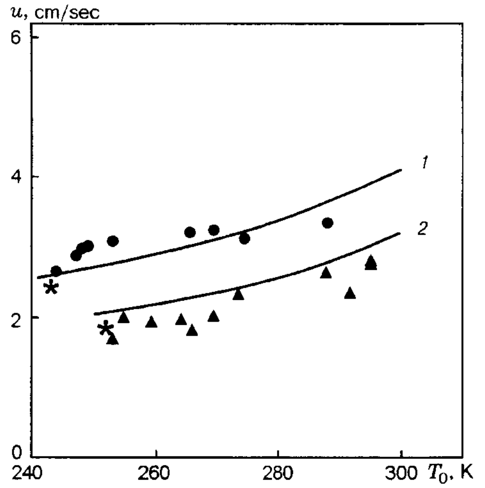

Fig. 5. Velocity of combustion-wave propagation versus the initial temperature: the fuel is *n*-undecane, the substrate is aluminum (1) or copper (2); the points (circles and triangles) refer to the experiment, the curve show the calculation by formula (20), and the asterisks indicate the limiting experimental values.

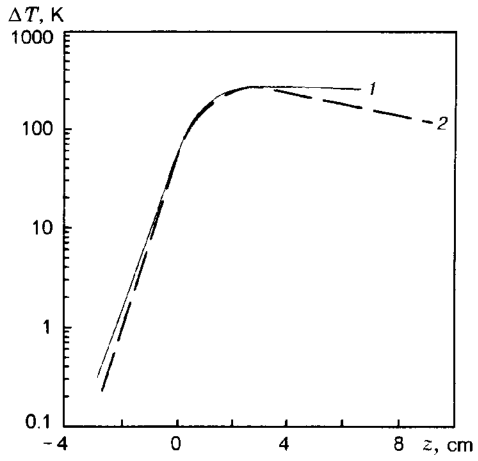

Fig. 6. Temperature profile in the layered system "*n*-undecane on a copper substrate": experiment (1) and calculation (2).

than the experimental values (see curves 3 in Figs. 3 and 4). If the value of T_v is taken equal to the boiling point, the calculated value of velocity [2] is even lower. Only for the thinnest foils (7–15 μm) are the calculated values close to the experimental data. For higher values of h_s, the calculated and experimental results are rather different. Thus, a comparison of the experimental data obtained in the present work with the results calculated by the model of [2] allows us to state that this model does not describe a thermally thin layered system.

It should be noted that it was assumed in the models of diffusion combustion of materials [2] that the temperature on the material surface behind the combustion front is constant and equal to a certain evaporation temperature. For the "liquid fuel–substrate" system, this temperature, as follows from [2], should be close to the boiling point. At the same time, experimental observations and results of mathematical modeling show that the temperature of the "fuel–substrate" system is not constant; under certain conditions, it is even lower than the boiling point of the fuel. In addition, thermocouple measurements of the temperature of the system in the beginning of the diffusion plume confirm the validity of the assumption on the temperature value for which the ratio of fuel vapors above the surface to the gaseous oxidizer is close to stoichiometric.

Figure 5 shows the dependences of the combustion-wave propagation velocity for various fuels on the initial temperature T_0, which were calculated by Eq. (20), and also the corresponding experimental data. It follows from these data that the simple theory proposed correctly reflects the effect of the initial temperature on the combustion-wave velocity in a layered system. However, this theory does not have a mechanism for the limit of the initial temperature, which is observed experimentally (see Fig. 5).

Curve 2 in Fig. 6 shows the temperature distribution in the "*n*-undecane–copper foil" system, which was calculated using formulas (17)–(19) for $u = 2$ cm/sec ($u = 1.9$ cm/sec in the experiment), $h_s = 45$ μm, and $h_f = 3$ μm. This distribution is in qualitative agreement with thermocouple measurements conducted for these parameters (see curve 1 in Fig. 6).

Formula (21) demonstrates a weak dependence of the flame velocity on the thickness of the fuel layer. This result, which is paradoxical at first glance, is confirmed experimentally. The flame velocity does not change with decreasing thickness of the fuel layer by almost two times (during film drying as a result of evaporation before initiation of the combustion wave). This is, apparently, related to the fact that the film thickness under the flame edge is significantly smaller than the initial value. The combustion wave itself forms a certain critical thickness at the point of location of the flame edge. The combustion-wave velocity is independent of the film thickness as long as the initial film thickness is not smaller than this critical value. If the initial film thickness is smaller than the critical value, it is not obvious that the combustion wave is not capable of propagating. A decrease in the plume length and a decrease in the volume of the liquid moving ahead of the flame edge was visually observed. Obviously, because of the weak dependence of the velocity on the plume length (if its size is rather large as compared to the heating zone), the decrease in the plume size due to the decrease in the fuel-layer thickness does not involve the decrease in the flame-propagation velocity. The increase in velocity due to the decrease in losses for evaporation is small.

Thus, despite the rough one-dimensional formulation, which does not take into account the actual temperature distribution in the gas phase, the model proposed gives a good estimate for the flame-propagation velocity for various parameters of the problem.

This work was partly supported by INTAS (Grant No. 96-1173) and the Russian Foundation for Fundamental Research (Grant No. 00-03-32417).

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