Korean J. Chem. Eng., 17(6), 619-624 (2000)

Sliding Mode Control of a Bioreactor

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(*Received 24 April 1999 • accepted 23 October 2000*)

Abstract–In this paper, sliding mode control (SMC) of a bioreactor is considered and is compared with PID control. The magnitude of the error in SMC is found to be lower than that in PID control. Moreover, the magnitudes of cells and nutrients were very close to the selected reference values in SMC, whereas they were quite different in PID control. Overall, SMC was more robust against disturbances and had better performance than PID control.

Key words: Bioreactor System, Sliding Mode Control, PID Control

INTRODUCTION

Chemical processes are often highly nonlinear and difficult to control; however, it may be easy to make approximate models for them. The problem of controlling them by using conventional controllers has been widely studied. In spite of the extensive work on self-tuning controllers and model-reference control, there are many problems in the chemical processing industries for which current techniques are inadequate. A study of a bioreactor benchmark for adaptive network-based process control, compared chemical and robotic process control and suggested a problem in the control of bioreactors which gives a sequence of problems of increasing difficulty [Ungar, 1991; Morari and Zafiriou, 1989; Agrawal et al., 1982].

Chemical systems may have few variables, but are often very difficult to control due to strong nonlinearities that are difficult to model accurately. Extensive theoretical and experimental studies have been made on both batch and continuous stirred tank reactors (CSTRs). Although such reactors can be (approximately) described by simple equations, they can exhibit complex behaviours such as multiple steady states and periodic and chaotic behaviour [Agrawal et al., 1982; Zhao, 1997]. One example of such reactors that presents special problems is the bioreactor. It is difficult to model and difficult to control because of the complexity of the living organisms in it and variances between different batches. They can have markedly different operating regimes, depending on whether the buds (bacteria or yeast) are rapidly growing or producing product.

The simplest version of the bioreactor problem is a continuous flow stirred tank reactor (CFSTR) in which cell growth depends only on the nutrient being fed to the system. The target value to be controlled is the cell mass yield. This system is difficult to control for several reasons: the uncontrolled equations are highly nonlinear and exhibit limit cycles. Optimal behaviour occurs within or around an unstable region. The problem exhibits multiplicity: two different values of control parameter (flow rate) can lead to the same set point in cell mass yield. This problem has proved challenging for conventional controllers [Agrawal et al., 1982; Agrawal and Lim, 1984].

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In this study, we show that Variable Structure System (VSS) with sliding mode is a robust nonlinear control technique for a bioreactor process. The SMC has good control performance for nonlinear systems, applicability to MIMO systems, design criteria for discrete time systems, etc. The best property of the SMC is its robustness. Loosely speaking, a system with an SMC is insensitive to parameter changes or external disturbances [Hung et al., 1993]. Some of the application areas of SMC can be listed as robots, aircraft, motors, power converters, and chemical process control [Utkin, 1992; Slotine and Li, 1991; Wang et al., 1997].

This study reports the control of a bioreactor system by using the SMC and PID control techniques. The results of PID control and SMC techniques for a nonlinear bioreactor system are compared. However, the PID control results were found to be unsuccessful since it is not easy to tune the parameters of the PID controller when the process has relatively large time delay [Sung and Lee, 1998]. On the other hand, the SMC had many advantages and was successful for various disturbance changes and set points of feed flow rates.

PLANT MODEL

The bioreactor considered in this paper is a tank containing water, nutrients, and biological cells as shown in Fig. 1. Nutrients and cells are introduced into the tank where the cells are mixed with the nutrients. The state of this process is characterized by the number of cells and the amount of nutrients. The liquid volume in the tank is maintained at a constant level by removing tank contents at

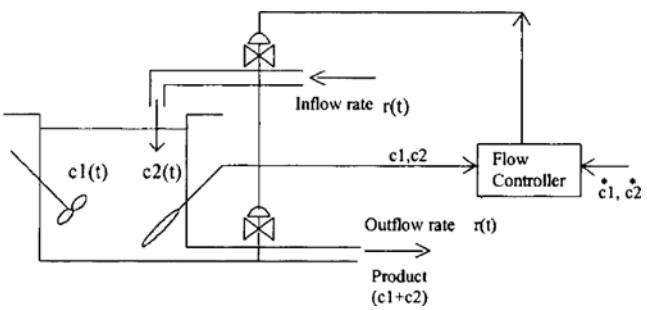

Fig. 1. Schematic diagram of the bioreactor plant.

a rate equal to the incoming rate. This rate is called the flow rate and is the only variable by which the bioreactor is controlled. The main problem in controlling the bioreactor system is maintaining the amount of cells at a desired level during the continuous operation of the plant.

The plant dynamics are given as follows [Agrawal et al., 1982]:

Scaled equations of motion:

$$\alpha_1(t) = -c_1(t)r(t) + c_1(t)(1-c_2(t))e^{\frac{c_2(t)}{\gamma}} \tag{1}$$

$$\alpha_2(t) = -c_2(t)r(t) + c_1(t)(1-c_2(t))e^{\frac{c_2(t)}{\gamma}}\frac{1+\beta}{1+\beta-c_2(t)} \tag{2}$$

Constraints:

$0 \le c_1, c_2 \le 1$ Cell and nutrients amounts are between 0 and 1.

$0 \le r \le 2$ Flow rate is positive and less than or equal to 2.

The volume in the tank is maintained at a constant level by removing tank contents at a rate equal to the incoming rate (inflow rate $-r$ = outflow rate $-r$).

Initial conditions:

$c_1[0]$, $c_2[0]$: are random variables from uniform distribution on the intervals (c_1^*, c_2^*).

Control: r (flow rate)

Parameters: β: 0.02 growth rate parameter

γ: 0.48 nutrient inhibition parameter

Δ: 0.01 sampling interval

Control input and output: input: $c_1[t]$ and $c_2[t]$

output: r[t]

The objective is to achieve and maintain a desired cell amount, c_1^*, by altering the flow rate. The bioreactor is a challenging problem for nonlinear control techniques for several reasons. Although the task involves few variables and is easily simulated, its nonlinearity makes it difficult to control because small changes in parameter values can cause the bioreactor to become unstable. The issues of delay, nonlinearity, and instability can be studied with bioreactor control problems.

DESIGN OF VARIABLE STRUCTURE SYSTEM WITH SLIDING MODE

The SMC is a technique derived from the Variable Structure System (VSS). In VSS, the control can modify its structure. The first step in SMC is to define a sliding surface S, and the goal is to reach the sliding surface and to keep on it [Slotine and Li, 1991]. One of the main features of this approach is the fact that all we need is to derive the error to a switching surface on which the system will not be affected by any modeling uncertainties and disturbances [Utkin, 1977]. If the system is defined as,

$$\dot{x} = f(x) + Bu(t) \tag{3}$$

where rank(B)=m, $x \in R^n$, $u \in R^m$. In VSS control, the goal is to keep the system motion on manifold S, which is defined as

$$S = \{x : \sigma(x, t) = 0\} \tag{4}$$

The solution to achieve this goal would be calculated from the requirement that $\sigma(x, t)=0$ is stable. The control should be chosen such that the candidate Lyapunov function satisfies the Lyapunov stability criteria. The aim is to force the system states to the sliding surface. The sliding surface equation for the control of the system can be selected as follows:

$$\sigma(x, t) = G.(x_{ref} - x) = G.e \tag{5}$$

In this equation x_{ref} represents the state vector of the reference, and the constant G matrix represents the slope of the sliding surface. First, a candidate positive Lyapunov function is selected. This function must bring the system on a sliding surface and avoid chattering [Ertugrul et al., 1994].

$$v = \frac{\sigma^T\sigma}{2} > 0 \quad \text{and} \quad \dot{v} = \sigma^T\dot{\sigma} < 0 \tag{6}$$

The aim is to define the derivative of the Lyapunov function as negative definite. This can be assured if it can somehow made sure that

$$\dot{v} = -\sigma^T D\sigma < 0 \tag{7}$$

D is always positive definite. Therefore Eqs. (6) and (7) satisfy the Lyapunov conditions. From Eqs. (6) and (7)

$$\dot{\sigma} = -D\sigma \tag{8}$$

can be written. If Eq. (8) is equated to zero, then equivalent control is obtained. In other words, the control that makes the derivative of the sliding function equal to zero is called "equivalent control"; i.e., if $\sigma(x, t)=0$, u is equal to u_{eq}. Equivalent control holds the system on a sliding surface but does not bring the system to a sliding surface [Slotine and Lee, 1991]. Derivative of Eq. (5) is

$$G\dot{x}_{ref} - G(f(x,t) + Bu_{eq}) = 0 \tag{9}$$

As a result, the equivalent control can be written in the following form:

$$u_{eq} = -(GB)^{-1}G(f(x,t) - \dot{x}_{ref}) \tag{10}$$

From the derivative of Eq. (5) and using Eq. (10):

$$\frac{d\sigma}{dt} = (GB)(u_{eq} - u) \tag{11}$$

Then, from Eq. (11), another equation for equivalent control can be written as follows:

$$u_{eq}(t) = u(t) + (GB)^{-1}\frac{d\sigma}{dt} \tag{12}$$

From Eqs. (3), (5) and (8)

$$G(\dot{x}_{ref} - \dot{x}) = G(\dot{x}_{ref} - f(x,t) - Bu) = -D\sigma \tag{13}$$

$$u = (GB)^{-1}(G(\dot{x}_{ref} - f(x,t)) + D\sigma) \tag{14}$$

Using the Eq. (4) for the equivalent control then

$$u(t) = u_{eq}(t) + (GB)^{-1}D\sigma \tag{15}$$

By looking at Eq. (12) an estimation for u_{eq} can be made by using the property that u(t) is continuous and cannot change too much in a short time as

$$u_{eq}(t) = u(t - \delta t) + (GB)^{-1}\frac{d\sigma(t)}{dt} \tag{16}$$

where δt is a short delay time. This estimation is also consistent with the logic that u_{eq} is selected as the average of u. By substituting the Eq. (16) into the Eq. (15), the last form of the equation for controller is

$$u(t)=u(t-\delta t)+(GB)^{-1}\left(D\sigma(t)+\frac{d\sigma(t)}{dt}\right) \tag{17}$$

By using Euler interpolation algorithm,

$$u(t)=u(t-\delta t)+\frac{(GB)^{-1}}{\delta t}[(D\delta t+1)\sigma(t)-\sigma(t-\delta t)] \tag{18}$$

In discrete time applications, δt is called short delay time or it has to be chosen as the sampling time. As seen from controller Eq. (17), there is no need to know the plant parameters exactly. Only the knowledge about the control input matrix B, is sufficient and also its range of change is adequate to design a stable system.

CONTROL OF BIOREACTOR SYSTEM

The block diagram of the closed loop system is given in Fig. 2. For a controller, first a velocity type PID controller [Clenant and Chidambaram, 1997; Yamamoto et al., 1997] is used for comparison purposes. Second, a chattering-free sliding mode structure is proposed for the control of the bioreactor.

The structure of SMC is given as follows:

Sliding surface function:

$$\sigma(t)=[G]\cdot[e(t)]^T \tag{19}$$

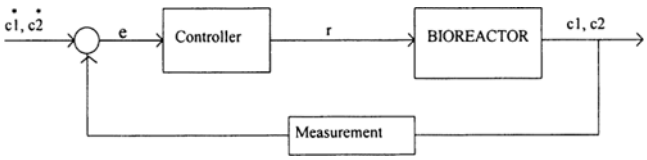

Fig. 2. The block diagram of the nonlinear bioreactor system.

$$[G]=[G1\ G2] \tag{20}$$

$$[e(t)]=[e1(t)\ e2(t)] \tag{21}$$

where

$$e1(t)=c_1^*-c1(t)$$
$$e2(t)=c_2^*-c2(t)$$

From Eq. (18), inflow and outflow rate [r(t)] is

$$r(t)=r(t-\delta t)+\frac{(GB)^{-1}}{\delta t}((D\delta t+1)\sigma(t)-\sigma(t-\delta t)) \tag{22}$$

In Eq. (22), G is the slope of the sliding mode manifold and D is the decay of the Lyapunov function. The transients of the system, the time change in the Lyapunov function (ν), the distance from sliding mode manifold (σ) and control error (e(t)) were examined in the phase plane with a control error on the horizontal axis and its derivative on the vertical axis. Parameters G and D are chosen from the reaching stage in transient and by changing the value of G and D. The range of change of B is adequate to design a stable system. Therefore, we have chosen B as the average value of references c1 and c2. The initialization of the state variables should be very important for a bioreactor. The initial values may cause an unstable result or a stable result. Each controller is tested with different initial and reference values that are given in Table 1.

Table 1. References and initial values for bioreactor control system

Reference values-A (Ref. A)	Reference values-B (Ref. B)
$c_1^*=0.120$	$c_1^*=0.120+0.05*\sin(2.0*3.14*t/50.0)$
$c_2^*=0.880$	$c_2^*=0.880-0.05*\sin(2.0*3.14\,t/50.0)$
Initial values (A, B, C)	
1st states (Init. A) : $r_o=0.675$, $c_{1o}=0.109$, $c_{2o}=0.792$	
2nd states (Init. B) : $r_o=0.009$, $c_{1o}=0.009$, $c_{2o}=0.009$	
3rd states (Init. C) : $r_o=1.359$, $c_{1o}=0.135$, $c_{2o}=0.540$	

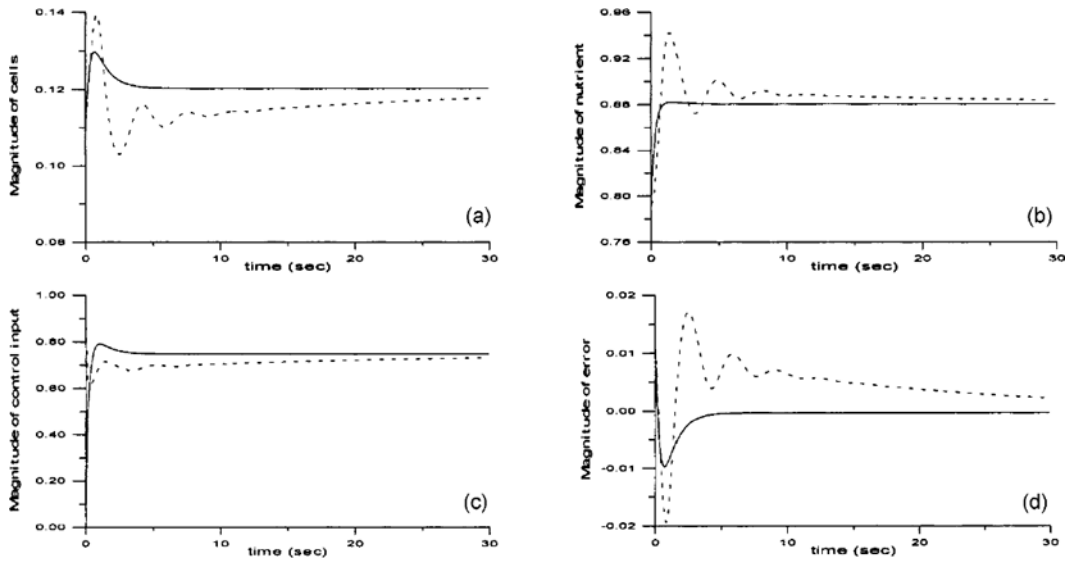

Fig. 3. Response of PID and SMC for Init. A and Ref. A.
(---) indicates response of PID and (—) indicates response of SMC. (a) Variation of cell amount, (b) Variation of nutrient, (c) Controllers outputs, (d) Variation of error.

RESULTS AND DISCUSSION

The simulation results for three initial conditions and two different reference values are given in Figs. 3 to 6. In Figs. 3 and 4, the system quickly reaches steady state conditions (less than 5 seconds) without oscillations in case of SMC control. In case of PID control, however, the system slowly reaches steady state conditions (more than 10 seconds) with a few oscillations. With the initial conditions A, B and reference value A in Figs. 3 and 4, the magnitude of errors in SMC control became 0 in a very short time interval (less than 4 seconds), and the magnitude of cells and nutrient responses fit well to that reference value with no more than one oscillation. But with the same initial conditions and reference value, the magnitude of errors in PID control is higher with a few oscillations in Fig. 3 and the magnitude of the error in Fig. 4 has not become 0 in a short time interval (in 10 seconds), and it also has a few oscillations. The magnitude of cells and nutrient responses with a few oscillations in PID control does not sufficiently fit that reference value.

As seen in Figs. 3 and 4 and according to the initial conditions, the bioreactor system quickly reaches steady state conditions in the case of sliding mode control, but it slowly reaches steady state operational conditions with a PID controller. However, the initial con-

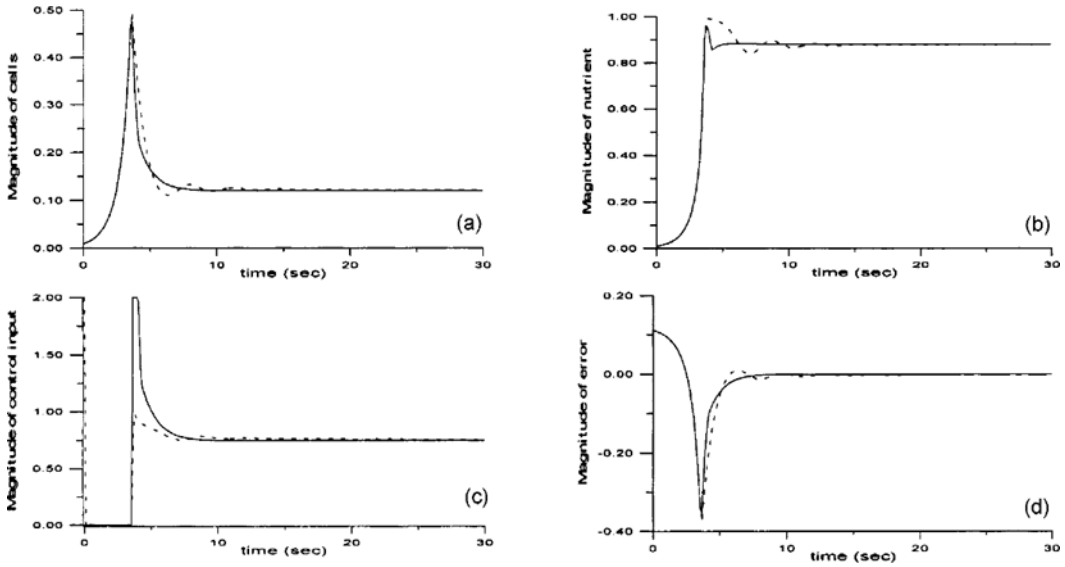

Fig. 4. Response of PID and SMC for Init. B and Ref. A.
(---) indicates response of PID and (−) indicates response of SMC. (a) Variation of cell amount, (b) Variation of nutrient, (c) Controllers outputs, (d) Variation of error.

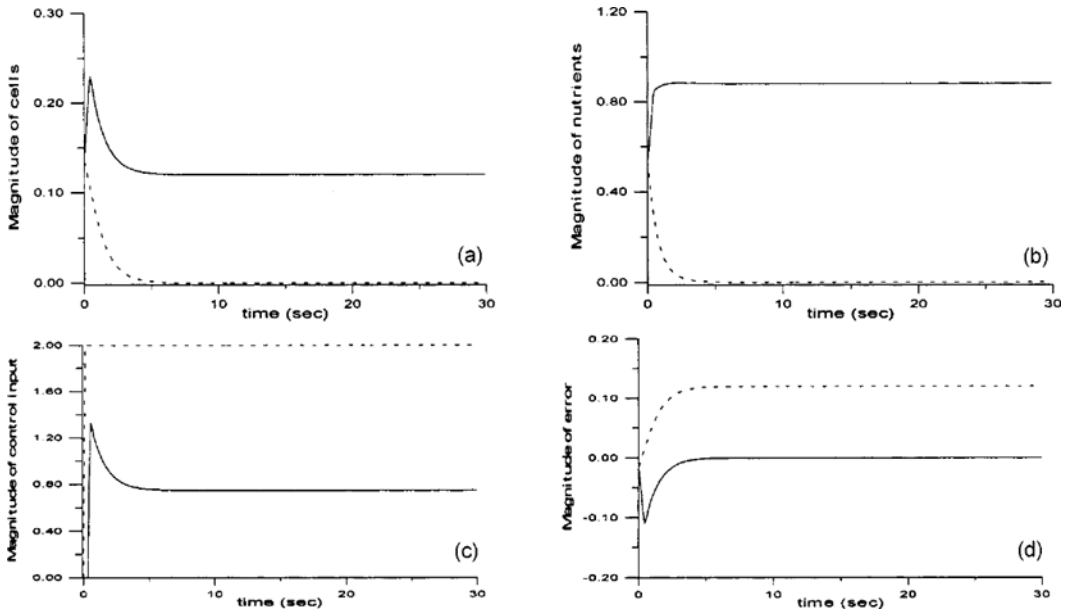

Fig. 5. Response of PID and SMC for Init. C and Ref. A.
(---) indicates response of PID and (−) indicates response of SMC. (a) Variation of cell amount, (b) Variation of nutrient, (c) Controllers outputs, (d) Variation of error.

ditions used in Fig. 3 are the values chosen to bring the system to steady state conditions without a controller. Thus, the controller should quickly bring the system to steady state. Also, in Fig. 3, the PID control brings the system to steady state, but in this case, a time delay occurs. On the other hand, the SMC control quickly brings the bioreactor system to the steady state operational conditions without any large changes of the system responses. In addition, it is observed that in Fig. 4 the control is not applied for about the first 4 seconds, which can be understood when Fig. 4a, is examined, i.e., the magnitude of the cells is increasing rapidly during this time period. In Fig. 4, although the bioreactor system quickly bring the steady state conditions in the case of both controller techniques, the response of SMC control is better than PID control.

In Fig. 5, the PID and SMC have been tested with different initial conditions and same reference value, and it has been observed that the PID control did not overcome that nonlinear problem to manage the system. However, the SMC responded quickly and confidently to manage the system. The magnitude of the error in SMC control became 0 in a short time interval, but never became 0 in PID control. The magnitude of cells and nutrient responses was sufficiently close to the reference value in SMC control, but the magnitude of cells and nutrients responses did not reach that selected reference value.

In Fig. 6, a variable reference trajectory is given for desired magnitude of cells and nutrients. The bioreactor system successfully follows both trajectories with SMC, since the magnitude of cells and nutrients has good agreement with the reference value and also the magnitude of the error is quite low with a lower range of amplitude ratios. But with PID control it is not responding efficiently because the magnitudes of cells and nutrients have some differences. They follow the references with phase lag, and also the magnitude of the error is quite high with higher range of amplitude ratios.

Finally, the above results show the success and robustness of SMC. Overall, SMC was more robust against disturbances and had better performance than PID control.

CONCLUSIONS

So far in this study, the VSS with SMC algorithm for a nonlinear bioreactor process have been developed and the results presented in Figs. 3 to 6. According to the results, the comparisons of SMC and PID control techniques were made; in some cases the failure and low accuracies of the PID control technique have been observed and the good capability, higher accuracies and successful applications of the SMC control technique have clearly been presented here for a nonlinear bioreactor plant. The main consideration for proposing this control technique is the robustness and ability of these types of controllers for nonlinear bioreactor processes. Since the process dynamics changes very often with load and disturbances, this method should gain more importance for practical applications.

NOMENCLATURE

VSS	: variable structure system
c_1	: amount of cells
c_2	: amount of nutrient
D	: sliding mode control parameters
r	: flow rate (inflow rate or outflow rate)
δt	: short delay time (or sampling time)
B	: the control input matrix
G	: (mxn) slope matrix of the sliding surface
x(t)	: state
x_{ref}	: represents the state vector of the reference
e(t)	: error
$\sigma(x, t)$	: sliding surface function
$\sigma(t-\delta t)$	: previous value of $\sigma(t)$
T	: number of time steps in a trial
u(t)	: control action
β	: growth rate parameter
γ	: nutrient inhibition parameter
Δ	: sampling interval

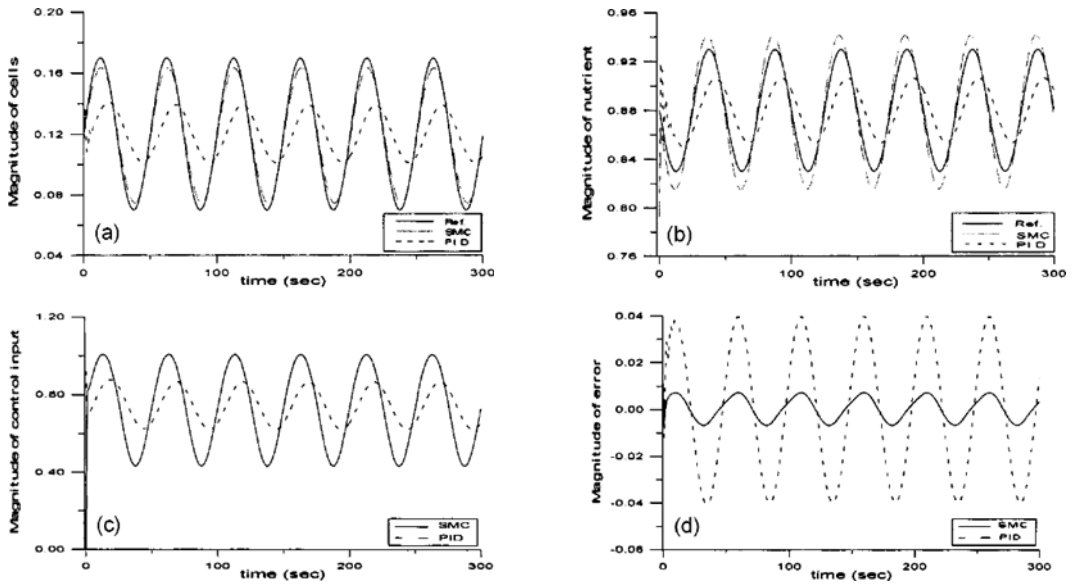

Fig. 6. Response of PID and SMC for Init. A and Ref. B.
(a) Variation of desired and actual cell amount, (b) Variation of desired and actual nutrient, (c) Controllers outputs, (d) Variation of error.

ν : represents selected Lyapunov function

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