THE CHARACTERISTICS OF FLUID FLOW IN BEDS OF SMALL GLASS PARTICLES SPOUTED WITH WATER

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Abstract— The characteristics of beds of small glass particles 0.28, 0.46 and 0.77mm in diameter spouted with water were studied in a half-cylindrical column 51mm in diameter with inlet tube diameter of 3.2mm. The minimum spouting velocity, bed pressure drop at minimum spouting and spout diameter were measured. Assuming Darcy flow, the fluid flow in the annulus is modeled and shown to represent the streamlines quite well. The residence time of the fluid in the annulus is calcuated from the model and compared with experimental data.

INTRODUCTION

Spouting is a technique for bringing a fluid into contact with solid particles in which the fluid is introduced vertically as a jet into a bed of particles. It was invented by Mathur and Gishler[1] as a method for drying wheat. Specific applications of spouted bed technology are discussed by Mathur and Epstein [2]. These includes heating and cooling of solids, drying, coating, granulation and blending. The use of spouted bed reactors for many chemical processes such as combustion and carbonization of coal have also received attention.

The main features of a spouted bed are shown in Fig. 1. The fluid entering the bed produces a central spout containing particles which move upward with the fluid. Although the rate of entrainment of solids is highest close to the fluid inlet, particles enter the spout all along the spout-annular interface. The entrained solids in the spout are carried upward and after reaching the top of the bed fall back on to the top of the annulus. The particles in the annulus move downward and reenter the spout establishing a systematic particle circulation in the bed. Fluid leaks from the spout into the annulus across the spout-annular interface.

There are several parameters for describing the spouting for a given fluid-particle system. These basic parameters are the minimum spouting velocity, the pressure drop at minimum spouting, the maximum spoutable height and spout diameter. These parameters and characteristics for small particles spouting with water are studied and compared with those of coarse particle system [3,4].

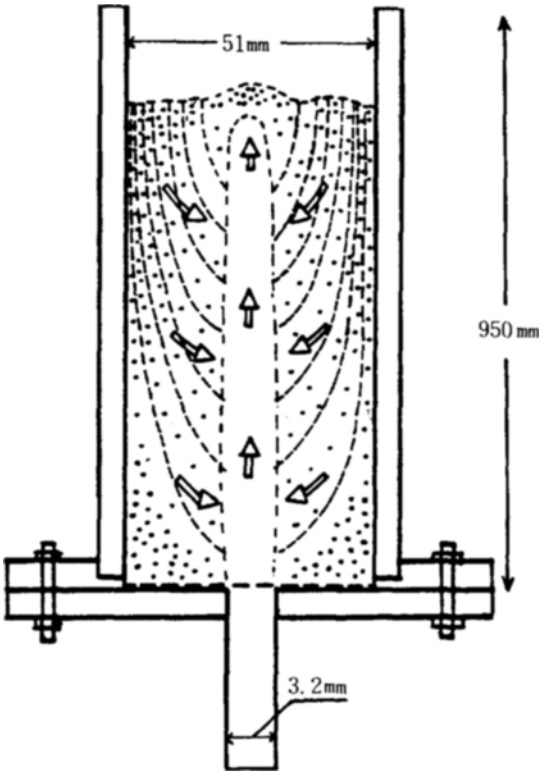

Fig. 1. Schematic Diagram of a Spouted Bed. Arrows Indicate Direction of Solids Movement. Water Passes from Spout to Annulus (Dotted Line).

Spouted beds are commonly thought of as being useful only for processes involving coarse particles, generally of millimeter size, and accordingly many expressions have been reported to predict the basic parameters for coarse particles [5-11]. However, few study has been reported for small particles delineating the basic spouting parameters, the spout-annular interfacial pressure profile and the flow in the annulus of a bed except for the above [3,4]. Small particle systems are of interest because they constitute a new regime of spouting and because the inertial force of the jet at minimum spouting is small relative to the pressure drop in a bed at its maximum spoutable height. For coarse particle-gas system, spout terminates due to fluidization of annular solid [12], however, the termination mechanism changes to the formation of slugs as the particle size is reduced. This instability generates a chain of bubbles which pass upward through it. Because of this, small particle systems with the annular fluidization termination mechanism can only be studied in liquid phase.

For predicting the annual fluid flow in these small particle systems, it is likely that an axisymmetric flow model assuming Darcy flow in the annulus will be applicable. The purpose of this study is to measure the basic parameters for modeling the annular fluid flow and predict the annular pressure and flow fields. These predictions will be compared with experimental measurements for pressure and fluid flow in the annulus.

EXPERIMENT

Experimental Apparatus

The spouted bed used and a schematic layout are given in Fig. 1 and Fig. 2, respectively. The bed is a half cylindrical column 51 mm in diameter and 950mm high. It is made of Plexiglas, has a flat base and designed in such a way that semi-circular orifice plates of different diameters can be fixed into it. The flat and round surfaces of the column have pressure taps. The axial distance between taps is small near the bottom of the column and all the pressure taps are connected to the piezometer tubes.

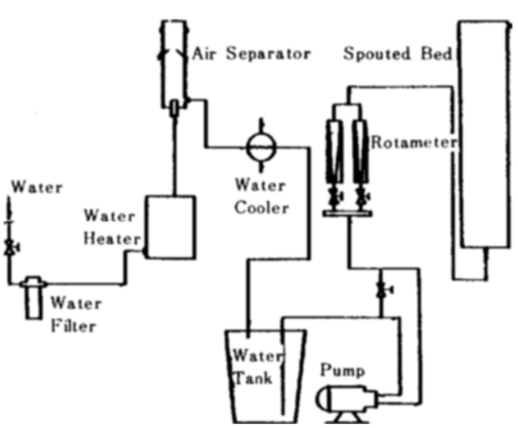

Fig. 2. Schematic Diagram of Experimental Apparatus

Water was de-aerated and filtered with wound-fabric filter. The water heater increases the water temperature from 15-20°C to 55±5°C to reduce the equilibrium solubility of gases dissolved in water. Air bubbles form in the heater and pass out of the system through the small bleed at the top of the main chamber of the air separator. The heated water is cooled in the water cooler and stored in 55 gallon tanks. This air separator prevented the formation of air bubbles in the bed and below the inlet screen. The water in the water storage tank, generally at 23°C, is pumped through a rotameter into a column.

Microscopic photographs were used for determining the size and shape factor of sample particles.

Experimental Procedure

A typical run consists of measuring pressure in the bed, jet penetration and the heights of the annulus and spout fountain as a function of flowrate. u_{mS} is determined as a flowrate at which the jet ceases to penetrate the bed as the flowrate is reduced below minimum spouting. Once u_{mS} is established, the flowrate is set and the pressure readings at various grid points in the spout and annulus are recorded. The spout diameter varies with distance above the spout inlet so that a mean diameter should be calculated. The spouting pressure drop, Δp_S, is determined by measuring the pressure just above the screen covering the spout inlet tube. Δp_{mS} is Δp_S measured in the minimum spouting. p_{Si} is the pressure measured inside the spout close to the spout-annular interface.

For flow visualization experiments, methylene blue solution was injected in the annulus. The streaklines are photographed and residence time was calculated.

FLUID FLOW MODEL

Darcy Flow Model

For fine particles with water as the spouting fluid, Darcy's law is applicable in the flow field in the minimum spouting condition and development of the equation is the same as that of Lefroy and Davidson[13] except for the interfacial boundary conditions.

Thus

$$\nabla\ u=0 \tag{1}$$

$$-\nabla p=f_1\bar{u} \tag{2}$$

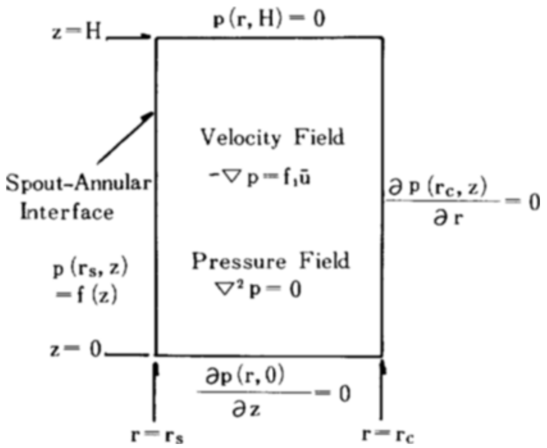

Fig. 3. Darcy Flow Model.

From Equations 1 and 2, pressure field becomes Laplacian.

$$\nabla^2 p = 0 \tag{3}$$

For axisymmetric motion about the z axis of a cylinder, Fig. 3, the boundary conditions are

$$\frac{\partial p(r,0)}{\partial z} = 0 \tag{4}$$

$$p(r, H) = 0 \tag{5}$$

$$\frac{\partial p(r_c, z)}{\partial r} = 0 \tag{6}$$

$$p(r_s, z) = f(z) \tag{7}$$

The experimental spout-annular interfacial boundary condition can be described in terms of polynomial

$$f(z) = p_{si}(z) - p_{si}(H) = \Delta p_{si}(a + b\zeta + c\zeta^2 + d\zeta^3)$$

where a, b c and d are constants.

Pressure Field and Stream Function

Solving Equation 3 using Equations 4-7, the final form for the pressure field in the annulus is

$$p(r, z) = \sum_{n=1,3,5,\cdots}^{\infty} 2\left[k_1 \left(\frac{2}{n\pi}\right)^2 + 2(k_1 + 2k_2 - 3)\left(\frac{2}{n\pi}\right)^3 \cdot \mathrm{SIN}\left(\frac{n\pi}{2}\right) + 6(2 - k_1 - k_2)\left(\frac{2}{n\pi}\right)^4\right] \frac{f_n(r_c, r)}{f_n(r_c, r_s)} \cdot \mathrm{COS}\left(\frac{n\pi}{2}\frac{z}{H}\right)\Delta p_{si} \tag{8}$$

The equation for stream function is

$$\Psi = -\frac{\Delta p_{si}}{f_1} r \left[\frac{K_1(mr_c)I_1(mr) - K_1(mr)I_1(mr_c)}{K_1(mr_c)I_0(mr_s) + K_0(mr_s)I_1(mr_c)}\right] \cdot \sum_{n=1,3,5,\cdots}^{\infty} \frac{2}{n}\left[k_1\left(\frac{2}{n\pi}\right)^2 + 2(k_1 + 2k_2 - 3)\left(\frac{2}{n\pi}\right)^3 \mathrm{SIN}\left(\frac{n\pi}{2}\right) + 6(2 - k_1 - k_2)\left(\frac{2}{n\pi}\right)^4\right] \mathrm{SIN}\left(\frac{n\pi}{2}\frac{z}{H}\right) \tag{9}$$

where

$$\frac{f_n(r_c, r)}{f_n(r_c, r_s)} = \frac{K_1(mr_c)I_0(mr) + K_0(mr)I_1(mr_c)}{K_1(mr_c)I_0(mr_s) + K_0(mr_s)I_1(mr_c)}$$

RESULTS AND DISCUSSION

Flow and Particle Properties

The flow and particle properties of the system studied are given in Table 1. Fluid properties are those of pure water at 23°C. Particles are technical quality glass spheres purchased from Potters industries (Hawthorne, New Jersey, U.S.A.). Particle densities were determined by measuring the volume of water displaced in a burette by adding a known mass of particles. In determining the volume mean diameter and shape factor of the particles, the length of the major and minor axes of each particle were measured from the microscopic photographs. Assuming the particles were spheroid, the surface area was calculated. From the volume mean diameter and surface area, the shape factor of the particle was calculated. For the particles studied, Table 1 shows that the largest particles, 0.77mm, are spherical, but the smallest ones, 0.28mm, are highly non-spherical.

Table 1. Flow and Particles of Water—Glass Particles Studied; D_C = 102mm

d_p mm	ρ_p kg/m³	u_{mF} mm/s	ε_{mF}	ϕ_S	u_T^* m/s
0.28	2.57	0.80	0.365	0.81	0.038
0.46	2.48	2.40	3.390	0.98	0.068
0.77	2.47	5.20	0.393	1	0.115

* Calculated[16]

Spouting Parameters

The basic data for spouting parameters such as u_{mS}, Δp_{ms} and d_s are measured for model calculations. These are listed in Table 2. The spout diameter listed in Table 2 is the mean diameter. The experimental values of

Table 2. Experimentally Measured Spouted Bed Properties in Bed Used; D_C = 51mm, d_i = 3.2mm

d_p mm	H mm	u_{ms} mm/s	d_S mm	ΔP_{mS} mmH₂O	$\frac{\Delta P_{mS}}{\Delta P_{mF}}$
0.28	61.1	0.65	5.1	44.3	0.697
	104.0	0.88	6.8	75.4	0.783
	134.5	0.92	8.2	97.5	0.832
	163.5	1.02	9.5	118.5	0.856
0.46	81.8	2.45	8.5	56.7	0.793
	103.2	2.72	9.8	71.6	0.823
	133.6	2.98	11.5	92.6	0.849
0.77	59.9	5.74	10.1	39.9	0.756
	77.8	6.12	11.5	51.8	0.816
	96.8	6.57	13.4	64.4	0.840

$\Delta p_{ms}/\Delta p_{mF}$ increases with bed height reaching the value of 0.85 which is well above the maxima predicted by Lefroy and Davidson[13] and Mamuro and Hattori[14] (0.64 and 0.75 respectively). The difference in $\Delta p_{ms}/\Delta p_{mF}$ between the coarse and fine particles is dicussed in detail[3] and it will affect the fluid flow in the annulus substantially.

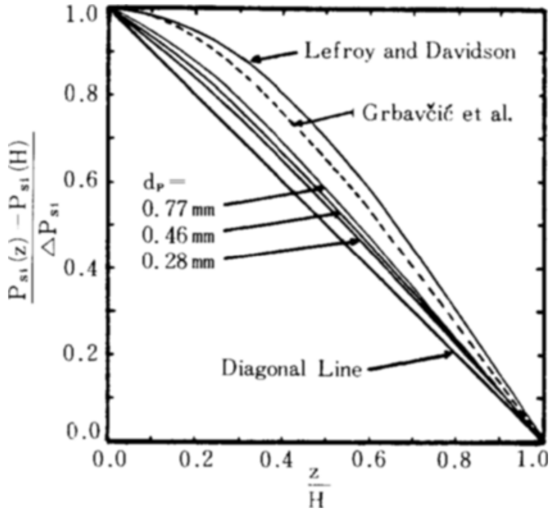

Fig. 4. The Spout Annular Interfacial Pressure Profile

Fig. 4 shows the experimentally measured normalized pressure profiles close to the spout-annular interface in the minimum spouting for bed heights at or slightly below H_m. The predictions of Lefroy and Davidson and Grbavčić et al.[15] give higher values of normalized interfacial pressure for a given bed height, z/H. It is shown that the normalized interfacial pressure profile is practically independent of d_i [3]. However, the normalized interfacial pressure profile changes with d_p. Least square fitting of the data to a third degree polynomial gives

$$\frac{p_{si}(z)-p_{si}(H)}{\Delta p_{si}} = 1.0-0.616\zeta-0.657\zeta^2+0.273\zeta^3$$

for $d_p = 0.28$ mm particles

$$\frac{p_{si}(z)-p_{si}(H)}{\Delta p_{si}} = 1.0-0.530\zeta-0.804\zeta^2+0.334\zeta^3$$

for $d_p = 0.46$ mm particles

$$\frac{p_{si}(z)-p_{si}(H)}{\Delta p_{si}} = 1.0-0.424\zeta-0.985\zeta^2+0.409\zeta^3$$

for $d_p = 0.77$ mm particles.

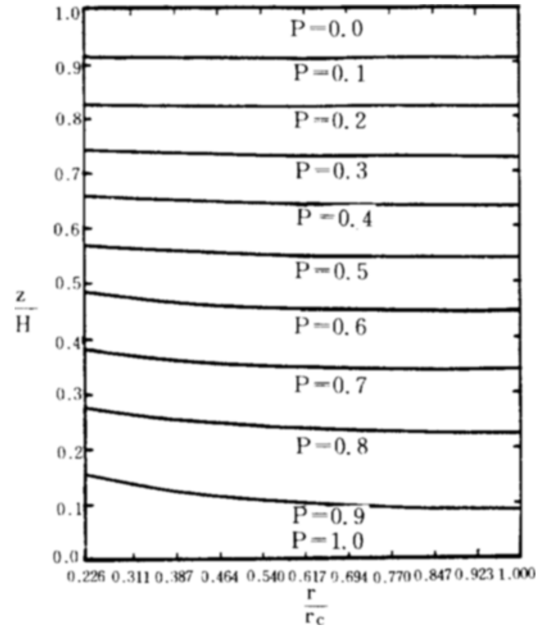

Fig. 5. Annular Pressure Field Calculated by Darcy Flow Model; $dp = 0.46$ mm, $di = 3.2$ mm, $u = 2.93$ mm/s, $H = 133.6$ mm

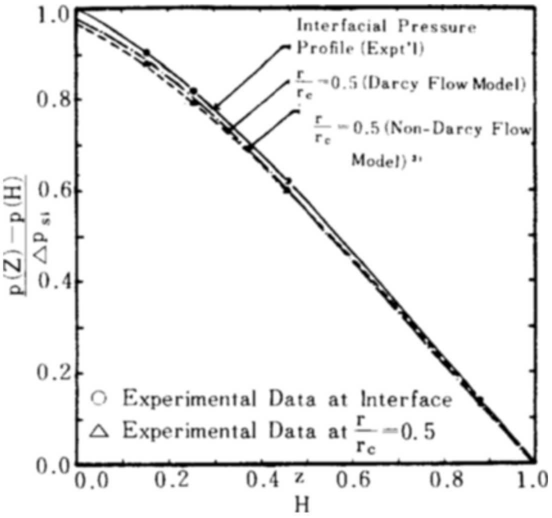

Fig. 6. Comparision of the Calculated Dimensionless Pressure Profiles at the Interface and in the Annulus with Experimental Data; $d_p = 0.46$mm, $d_i = 3.2$mm, $u = 2.98$ mm/s, $H = 133.6$mm

$-k_1$, the slope of the normalized interfacial pressure profile at $\zeta = 0$, decreases as particle size increases and $-k_2$, a slope at $\zeta = 1$, increases with d_p. If the bed is uniformly fluidized, the values of $-k_1$ and $-k_2$ would be 1. Lefroy and Davidson's quarter cosine pressure profile gives $-k_1 = 0$ and $-k_2 = 1.57$ for coarse particles. In this experiment for fine particles, $-k_1$ varies from 0.616 for 0.28mm particles to 0.424 for 0.77mm particles. $-k_1$ decreases substantially while $-k_2$ increases slightly as particle size increases. Comparing the interfacial pressure profiles between the coarse and fine particles, there are substantial differences between these two particles.

Annual Pressure Field

The equation for the pressure field, Equation 8, was solved using the measured interfacial pressure boundary condition.

Fig. 5 presents the pressure fields in the annulus for 0.46mm particles. It was seen that the radial pressure gradient does not vary greatly except close to the spout inlet. As the particle size increases, the radial pressure gradient increases slightly.

Fig. 6 shows the normalized interfacial pressure profile and the normalized pressure profile in the annulus, $r/r_C = 0.5$, as predicted by using Darcy flow model compared with experimental data of 0.46mm particles. The Darcy flow model predicts that the pressure difference between the interface and annulus, $r/r_C = 0.5$, is largest at $z/H = 0$ and decreases gradually as z/H increases becoming zero at $z/H = 1$. Overall, Darcy flow model is in good agreement with experimental data for pressure in the annulus of fine particles.

Stream Lines and Residence Time

The equation for stream function assuming Darcy flow in the annulus, Equation 9, was solved using the measured interfacial pressure boundary condition.

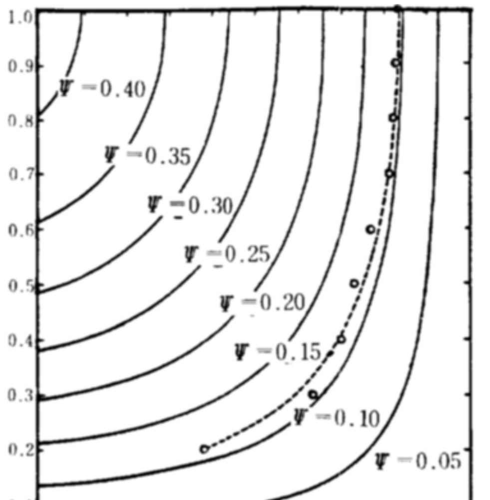

Fig. 7. Streamfunction Field Calculated by Darcy Flow Model; $d_p = 0.28$mm, $d_i = 3.2$mm, $u = 1.00$mm/s, $H = 163.5$mm; ------: Experimentally Measured Streak Lines

Figs. 7, 8 and 9 are the streamline contour maps for 0.28, 0.46 and 0.77mm particles with 0.32mm spout inlet tube. Each bed is at its maximum spoutable height and in the condition of minimum spouting. The shape of the streaklines measured experimentally are plotted with predicted streamlines. The Darcy flow model predicts the streamlines for small particles very well (Figs. 7 & 8). However, as the particle size increases, the experimental streamline shows small but clear deviation from that of the Darcy flow model in the radial direction (Fig. 9).

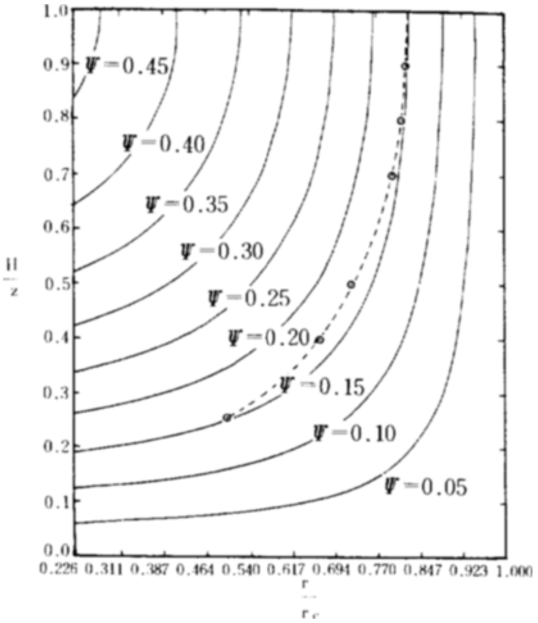

Fig. 8. Streamfunction Field Calculated by Darcy Flow Model; $d_P = 0.46$mm, $d_i = 3.2$mm, $u = 2.93$mm/s, $H = 133.6$mm, ------; Experimentally Measured Streaklines

Residence time of the fluid in the annulus was calculated using Equation 9 along a streamline and compared with experimental data in Table 3. In calculating residence time for each bed height, the properties given in Table 2 were used. The average value of t_C/t_M for the data in Table 3 is 0.66 which shows that the calculated values are substantially smaller than measured values. The voidage is critical in calculating residence time of fluid. Even slight change in voidage will affect much on f_1 which varies fluid velocity. Systematic movement of particles against the direction of fluid flow and expansion of spout diameter as the distance from inlet increases will also affect the residence time of fluid in the annulus. In calculating residence time, however, these effects were neglected.

For the better prediction of fluid residence time in the annulus, the solid circulation and non-Darcy flow effect will be studied later.

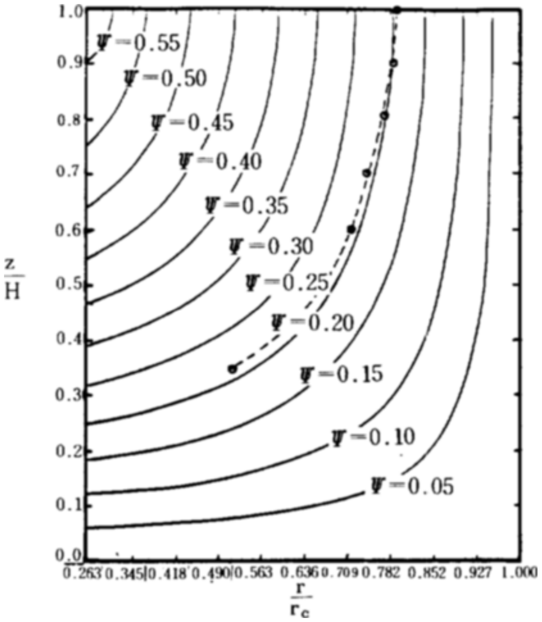

Fig. 9. Streamfunction Field Calculated by Darcy Flow Model; dp = 0.77 mm, di = 3.2 mm, u = 7.01 mm/s, H = 96.8 mm; ------ Experimentally Measured Streaklines.

Table 3. Experimental and Calculated Values of Residence Time: D_C = 51mm, d_i = 3.2mm

d_p mm	H mm	t_M sec	t_C sec	t_C/t_M
0.28	61.1	17.0	10.21	0.601
	104.0	39.7	26.50	0.668
	134.5	60.0	40.06	0.668
	163.5	83.0	53.62	0.646
0.46	81.8	7.2	5.19	0.721
	103.2	12.2	7.42	0.603
	133.6	18.0	11.02	0.612
0.77	59.9	1.5	1.04	0.693
	77.8	2.2	1.56	0.709
	96.8	3.8	3.59	0.682

CONCLUSIONS

1. The basic spouting parameters for modeling fluid flow in the annulus do not correlate with coarse particle data.
2. The model assuming Darcy flow in the annulus represents the pressure and streamlines in the annulus quite well.
3. The measured residence time was always greater than calculated ones with average deviation of 34% (based on measured value).

Acknowledgement

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NOMENCLATURE

d_i: Spout Inlet Tube Diameter
D_C: Column Diameter
d_p: Particle Diameter
d_S: Spout Diameter
f_1 : $[150\,(1-\varepsilon)^2/\varepsilon^3]\,[\mu/d_p^2]$
H: Bed Height
H_m: Maximum Spoutable Height
I_0: Modified Bessel Function (1st Kind, Order Zero)
I_1: Modified Bessel Function (1st Kind, Order One)
K_0: Modified Bessel Function (2nd Kind, Order Zero)
K_1: Modified Bessel Function (2nd Kind, Order One)
k_1,k_2: Slopes of Dimensionless Interfacial Pressure Profile at z/H = 0 and 1, respectively
p: Pressure in the Bed
P: $p/\Delta P_{Si}$
ΔP_{mF}: Minimum Fluidization Pressure Drop in the Bed of Height, H
ΔP_{mS}: ΔP_S at Minimum Spouting
P_{Si}: Pressure at the Spout-Annular Interface
ΔP_S: Spouting Pressure Drop, $P_S(0) - P_S(H)$
r: Distance from the Axis of the Symmetry of the Column
r_C: Column Radius
r_S: Spout Radius
t_C: Residence Time, Calculated
t_M: Residence Time, Experimental
u: Fluid Velocity in the Bed, Superficial
$\bar{u}$: Velocity Vector of Fluid
u_{mF}: Minimum Fluidization Velocity
u_T: Terminal Velocity of the Particle
z: Vertical Height in the Column from the Spout Inlet
ε : Bed Voidage
ε_{mF}: Void at Minimum Fluidization Condition
ζ : z/H
μ : Fluid Viscosity
ρ_f: Fluid Density
ρ_p: Particle Density
ϕ_s: Particle Shape Factor
ψ : Stream Function
Ψ : $\psi/u_{mF}r$

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