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A Study of the Bubble Properties in the Column Flotation System

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Abstract−The bubble properties in the column flotation system are deeply affected by the bubble-generator type, frother dosage, and superficial gas velocity. This study is to determine the bubble-generator type, which effectively produces micro-bubbles to affect the flotation efficiency. Characteristics for two types of bubble generators like the in-line mixer and sparger are examined by bubble properties such as bubble diameter, holdup and bubble velocity. Micro bubbles generated from an in-line mixer result in the increase of the bubble rising velocity and gas holdup. Bubbles produced at the in-line mixer were more effective for operating the flotation system than that of the sparger. It means that the in-line mixer bubble generator is more effective than a sparger in designing or operating the column flotation system.

Key words: Column Flotation, Bubble, Gas Holdup, Bubble Swarm Velocity, Bubble Generator, Frother

INTRODUCTION

In mineral processing, froth flotation is regarded as the best available technique for separating fine particles; however, it becomes inefficient when the particle size was very small (less than approximately 20 μm). Fine particles have a low probability of collision with air bubbles, resulting in low recovery [Yoon, 1993]. Bubble diameter plays a critical role in the froth flotation process. It is believed that the decrease in flotation recovery of fine particles is due to the lower probability of collision between such particles and air bubbles. Experimental work has shown that micro-bubbles can improve the flotation recovery [Cassel et al., 1975; Ahmed and Jameson, 1985]. It has been clearly shown that probability of particle-bubble collision increases with decreasing bubble diameter. Bubble properties including the gas holdup, bubble diameter and bubble swarm velocity play an important role in improving flotation efficiency. Air bubbles are indispensable components in column flotation operations, in which hydrophobic particles are attached to bubbles, and carried upward by the rising bubbles to the froth zone, leaving the hydrophilic particles in the liquid phase to go down with the tailings. This separation process depends on the collision of particles with bubbles in the slurry and their ability to remain in contact long enough. In order to understand the mechanism of particle-bubble collision and attachment, it is necessary to know the behavior of bubbles and the bubble flow characterization.

Previous work carried out by Dobby et al. [1988] and Finch and Dobby [1990] has provided bubble properties under the different conditions of the frother dosage and superficial gas velocity. Several methods were used to determine bubble size and swarm bubble velocity. Zhou et al. [1993] measured bubble swarm velocity in aqueous media in the absence or in the presence of surfactants such as MIBC (Methyl IsoButyl Carbinol), Dowfroth 250 and pine oil experimentally. Patel et al. [1990] studied the bubble diameters as wax types using the photograph technique, and Pal and Masliyah [1989] investigated the flow characteristics of bubbles with different size. Shen and Finch [1996] established experimentally bubble swarm velocity in the column flotation experimentally using a conductivity meter. These works have focused on improving the flotation efficiency though analyzing the bubble properties such as the bubble diameter, gas holdup and bubble swarm velocity. Work on bubble properties as variable frother dosage and superficial gas velocity has been reported in literature and has been reviewed by several authors [Shah et al., 1982; Dobby and Finch, 1988; Zhou et al., 1993; Yoon, 1993]; however, work on bubble properties by different bubble-generator types has not been conducted. And the effect of different bubble-generators has not been considered in most of these investigations.

Bubble properties are determined by factors such as bubble size, gas holdup and bubble swarm velocity in the column flotation, and these three factors are closely correlated with each other. Nicklin [1962] established that the motion of bubbles in two-phase bubble flow arises partly from buoyancy and partly from the superficial velocity caused by the entry of the two phases into the column. In the case of countercurrent column flotation process, at least two main factors contribute to the actual average bubble rise velocity: gas holdup and bubble diameter. Therefore, the average swarm bubble rise velocity, U_s, can be given by:

$$U_S = \frac{J_g}{\varepsilon_g} \tag{1}$$

where, J_g is the superficial gas velocity and ε_g is the fractional gas holdup. Shah [1982] suggested that for gas holdup of less than 30% the drift flux relationship of Richardson and Zaki [1954] was the most suitable expression for relating swarm velocity to terminal rise velocity. The expression is

$$U_S = U_T(1-\varepsilon_g)^{m-1} \tag{2}$$

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‡This paper is dedicated to Dr. Youn Yong Lee on the occasion of his retirement from Korea Institute of Science and Technology.

where the terminal velocity, U_T, is defined as

$$U_T = \frac{gd_b^2(\rho_f - \rho_b)}{18\mu(1+0.15Re_b^{0.687})} \tag{3}$$

$$m = \left(4.45 + 18\frac{d_b}{d_c}\right)Re_b^{-0.1} \qquad 1<Re_b<200 \tag{4}$$

$$m = 4.45Re_b^{-0.01} \qquad 200<Re_b<500 \tag{5}$$

$$Re_b = (d_b U_T \rho_f)/\mu_f \tag{6}$$

where m is a function of bubble Reynolds number, d_b is bubble diameter, d_c is column diameter, and ρ_f is liquid density, and μ_f is liquid viscosity, respectively.

This implies that swarm bubble velocity may decrease as gas holdup increases and bubble diameter decreases. Masliyah [1979] has drawn Eq. (2) with respect to the bubble rise velocity in a flotation system. When the swarm bubble velocity decreases, the effect of the collision between bubbles and particles increases, which can improve flotation efficiency at column flotation. The main disadvantage of the equation is that it cannot directly predict the bubble size and frother effects on the rise velocity, which are important in the analysis of flotation kinetics and process optimization. In order to investigate the bubble motion in a flotation column, it is necessary to establish the relationship between the rise velocities and the sizes of bubbles in a swarm, in the presence of dosages of frother. This paper describes the technique for bubble generators and conforms them by comparison with the properties of the bubble size, gas holdup and swarm bubble velocity under different conditions of superficial gas velocity and frother dosage.

The aim of this paper is to extend the study of different properties of the bubble formed by in-line mixer and sparger bubble-generator. To grasp the bubble properties as a function of two bubble generators, a column flotation (2,400 mm height by 80 mm in diameter) which can set up the in-line mixer and sparger bubble generator at the lower portion of the column is used.

EXPERIMENTAL

1. Countercurrent Column Flotation

The experiments to measure bubble characteristics are carried out in countercurrent column flotation, which is composed of column, bubble generator, photographic measurement, conductivity meter and water manometer. Fig. 1 shows a schematic diagram of the setup. A laboratory flotation column, 80 mm in diameter and 2,400 mm in height is used. The liquid level in the column is kept at 2,300 mm from the static in-line mixer or sparger bubble-generator. The compressed air (98 kPa) is introduced at the bottom of the column and gas flow rate is adjusted by the regulator valve above the venturi and the rotameter (Dwyer, RMA). At that time, gas flow rate (Q_g) supplied to the column through venturi tube is changed from 1 to 8 lpm for generating the bubbles, which is related to the superficial gas velocity (J_g). Table 1 shows the changes of the superficial gas velocity as supplied gas flow rate where ranges from 0.33 to 2.65 cm/s. J_g can be obtained by dividing the air flow rate by the cross section of column. When the cross section of 50.2 cm^2 is considered, the change of air flow rate from 1 to 8 lpm causes that J_g to range from 0.33 to 2.65 cm/s. Fig. 2 shows the general

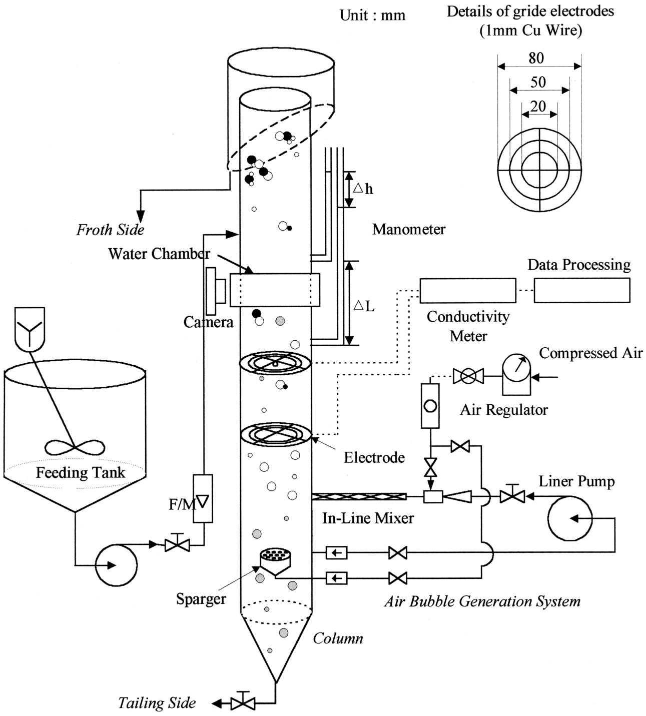


Fig. 1. Schematic diagram of the experimental apparatus for measuring gas holdup, bubble size and bubble swarm velocity.

design of bubble generators. Fig. 2(a) shows the design of the in-line mixer bubble-generator and Fig. 2(b) shows the sparger bubble-generator. The in-line mixer bubble-generator consists of venturi and tube with 300 mm length by 25 mm in diameter. When compressed air (98 kPa) is introduced through the in-line mixer, a pressure drop takes place at the venturi tube in order to generate bubble easily. Inside tube, helical vane swirlers are set up to enhance swirling flow when the air-water mixture flows through the tube. Pressure drop is generated by the effect of the vortex flow, the loss of kinetic momentum and vane angular is generated across swirlers [Dave and Gray, 1989]. In the state of mixing compressed air and water, micro-bubbles are generated by swirling flow in the helical vane swirlers. Helical vane angle is set at 50° and pitch is set at 40 mm. Fig. 2(b) shows that sparger bubble-generator is made from glassy filter with 45 mm diameter and porosity of 45 μm in surface as presented from Fig. 2(c). Compressed air flows in the sparger and is discharged through the surface of the sparger to generate the bubbles. In order to evaluate the characteristics of bubbles generated by this two bubble generator, column flotation variables such as superficial gas velocity (J_g) and froth addition (C_F), bubble size, gas holdup and swarm bubble velocity can be obtained from the bubble-generator. The most commonly used frother in flotation, MIBC (Methyl IsoButyl Carbinol), is tested in this study under different concentration.

2. Gas Holdup Measurement

Gas holdup is one of the most important parameters characterizing the hydrodynamics of a bubble column. It can be defined as the percentage by volume of the gas in the two phase mixture in the column and depends on mainly on the flotation variables such as superficial gas velocity and frother [Shah et al., 1982]. The meth-

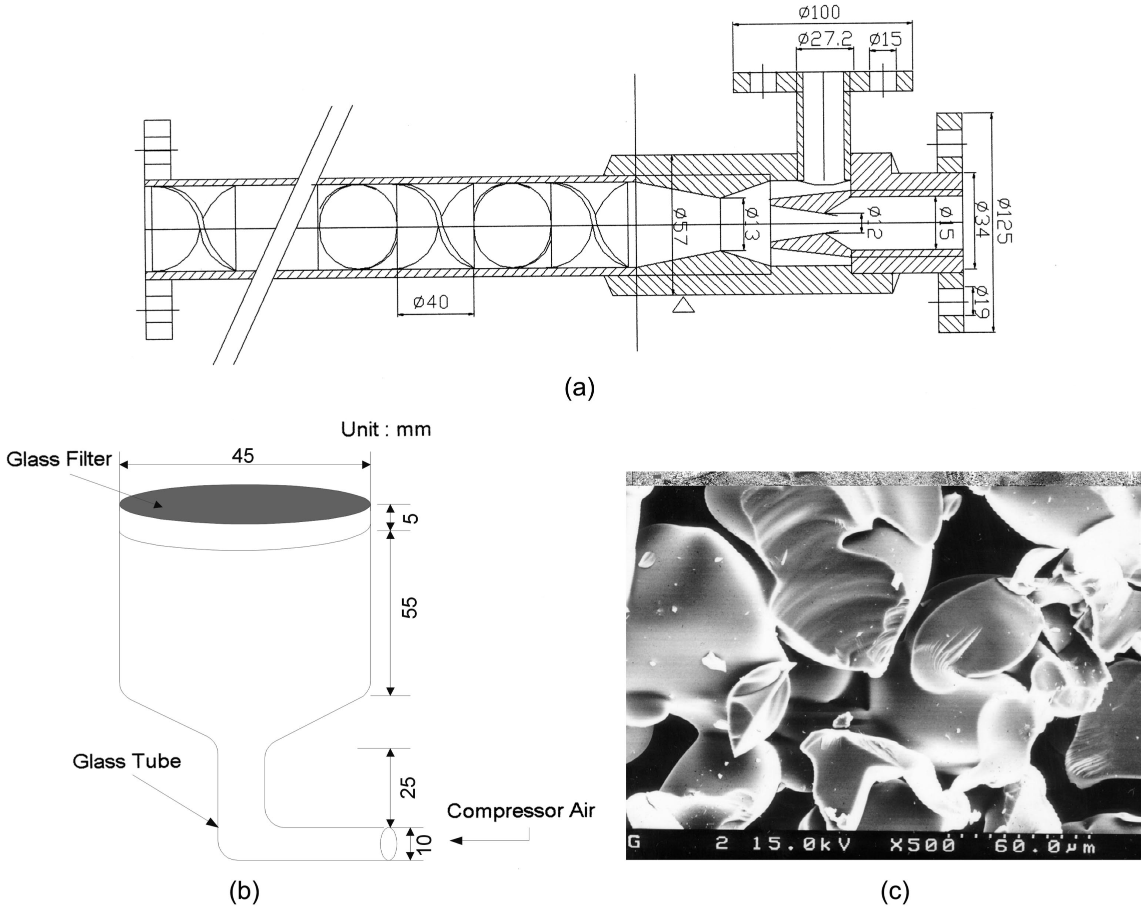


Fig. 2. Schematic diagram of bubble generators in this study.
(a) in-line mixer bubble generator, (b) sparger bubble generator, (c) SEM of sparger surface

od related to measure gas holdup is shown in Fig. 1. It is measured with a water manometer located 58 cm from the top of the column. With this arrangement the gas holdup of the aerated column is calculated by dividing the distance from the column lip to the water level in the manometer by the length of the column of water in the absence water. The average gas holdup ε_g, between the two manometers is given by

$$\varepsilon_g = \frac{\Delta h}{\Delta L} \tag{7}$$

where Δh is the distance between the water level in the two manometers, and ΔL is the distance between the location of the manometers as shown in Fig. 1. This measurement represents the average value of the gas holdup in the section of the column between the pressure tap and column tip. Gas holdup is determined by different operating variables such as the frother dosage and the superficial gas velocity.

3. Bubble Diameter Measurement

Photographic techniques have been the most common approach for measuring the bubble diameter. The bubble diameters are measured by photographic techniques and the average bubble diameters are estimated as the sauter mean diameter [Yianatos et al., 1987]. Photographic technique to measure the bubble diameter is shown in Fig. 1. The bubble diameter is determined by Nikon camera with micro-lens and Tmax-400 film. The light source is provided by two 500-W bulbs. It is found that a good picture can be obtained at a shutter speed of 1/1,000. In order to minimize the optical distortion caused by the curved wall of the column, the focused area is kept as small as possible as the water chamber is being built at the column surface. All the photographs are taken at a fixed height of about 90 mm above from bubble-generator. A calibrated ruler is attached to the column wall and in the field of view of camera so that the bubble diameter can be estimated. The photographs are measured to use a film scanner (Sprint Scan-35, Polaroid) and counted to use an image analyzer. About 200-400 bubbles are measured at every operating condition. The sauter mean diameter, d_{bs}, which considered the most consistent representation of mean bubble diameter, was used as

$$d_{bs} = \frac{\sum n_i d_{bi}^3}{\sum n_i d_{bi}^2} \tag{8}$$

4. Bubble Swarm Velocity Measurement

The bubble swarm velocities at the different frother concentrations and superficial gas velocities are estimated by conductivity meter (3200, YSI), which is shown in Fig. 1. Two grid electrodes covering the cross-sectional area of the column are separated by 360 mm. The grid electrode consists of three constant rings (80, 50

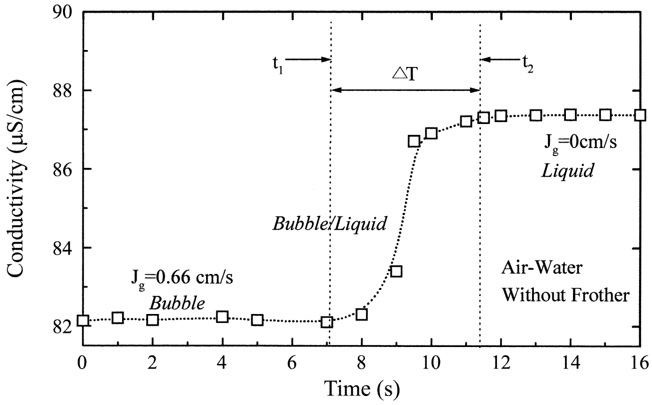


Fig. 3. Relation with conductivity and bubble behavior time in the flotation cell.

and 20 mm) soldered to a cross. Copper wire of 1 mm diameter is used. The grid electrodes are located one above the other along the vertical axis of the column. Fig. 3 illustrates the conductivity vs. time in cell. Before t_1, the rising interface has not reached the lower cell and conductivity is constant. At t_1 the interface contacts lower cell and the conductivity in the cell increases as the interface rises from lower cell to upper cell. After t_2, the interface has left cell and conductivity is again constant. The swarm bubble velocity, U_s, is calculated from

$$U_S = \frac{\Delta Le}{\Delta T} \tag{9}$$

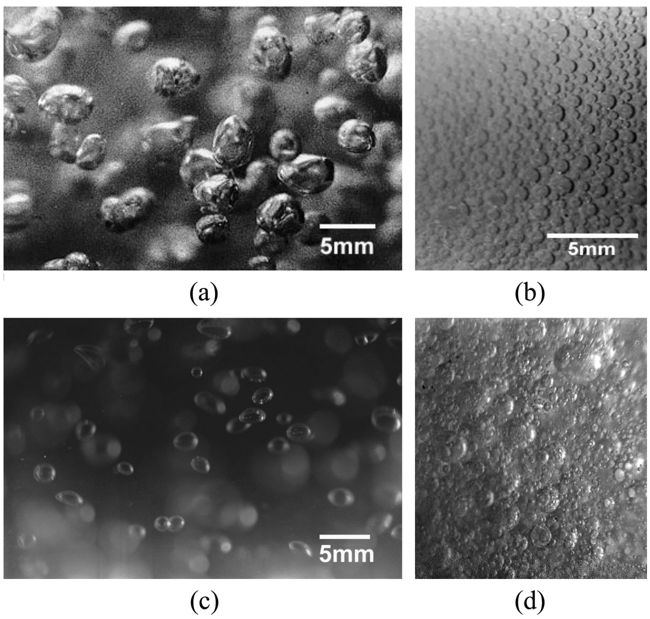


Fig. 4. Photographs of gas bubbles formed by bubble-generators in the column flotation: (a), (b) was formed by in-line mixer bubble-generator under the condition of C_F=0 mg/L, J_g=2.32 cm/s (a), C_F=120 mg/L, J_g=2.32 cm/s (b), and (c), (d) was formed by the sparger bubble-generator under the condition of C_F=0 mg/L, J_g=2.32 cm/s (c), C_F=120 mg/L, J_g=2.32 cm/s (d).

where ΔLe is the distance between cells. The conductivity measurements are performed with a conductivity meter with the analog output fed to a data acquisition system. ΔT is obtained, and bubble swarm velocity, U_s, is calculated by Eq. (9).

RESULTS AND DISCUSSION

1. Photographic Shapes and Diameter of Bubbles

Photographs are taken in order to observe the shape of bubble and determine the bubble size. In any experimental run, photographs are taken at different locations in a bubbly zone as a function of flotation variables. Fig. 4 shows typical photographs of bubbles generated from the in-line mixer bubble generators when superficial gas velocity is 2.32 cm/s for 0 and 120 mg/L of frother dosage, respectively. From the photographs, bubble shapes are irregular something like ellipsoidal and much larger without frother, but they are spherically shaped with frother dosage. The ability to resist the rupture and coalescence of bubbles is stronger in the presence of a frother than in its absence [Zhou et al., 1993]. Under the range of superficial gas velocity in this experiment, bubbles are much larger in the absence of a frother, due to an effect of coalescence of bubbles. Therefore, the experiments to compare with bubbles formed by two types of bubble-generator are completed in the presence of a frother.

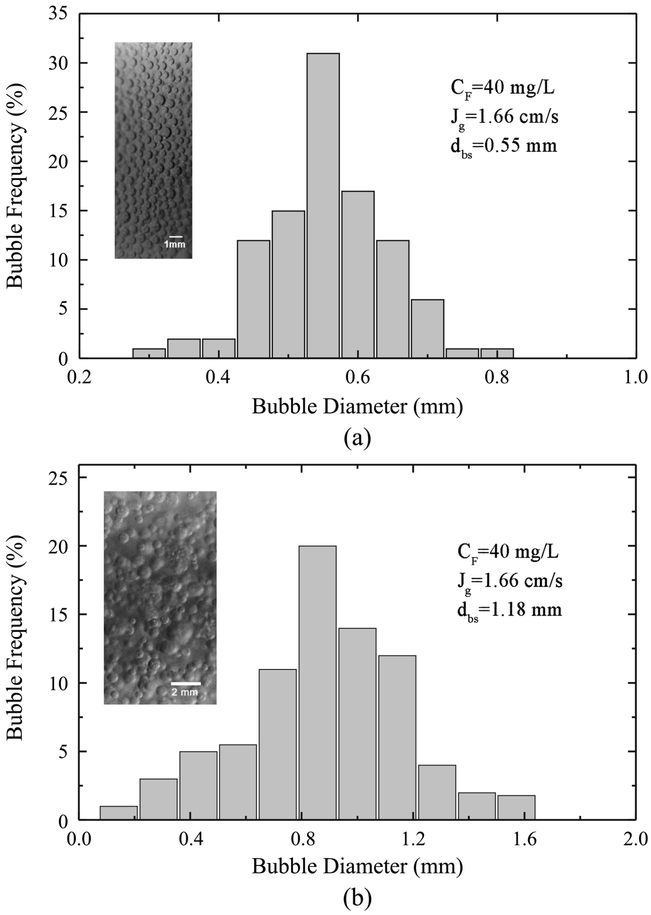


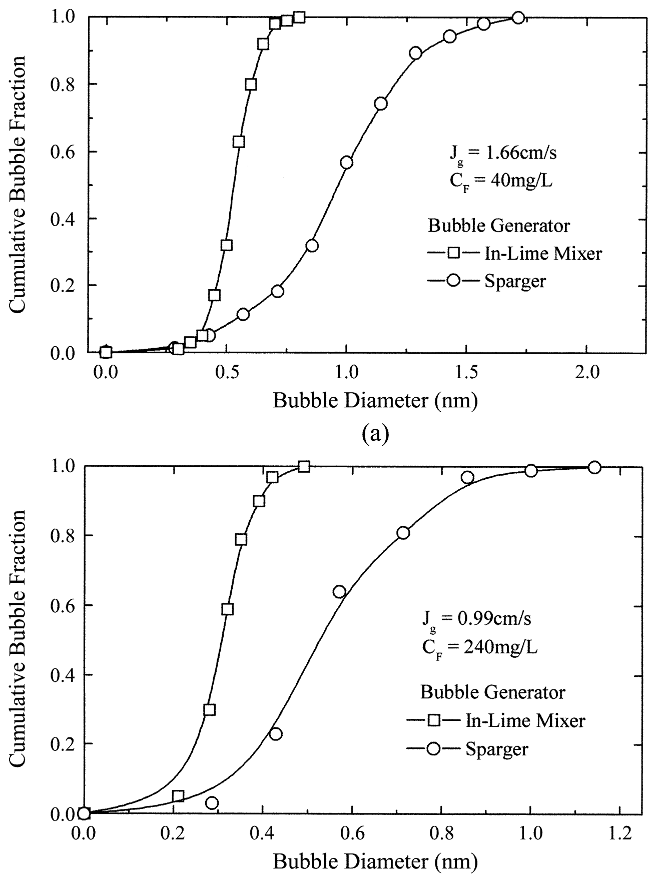
Fig. 5. Size distribution formed by bubble-generators under the condition of C_F=40 mg/L and J_g=1.66 cm/s: (a) formed by in-line mixer generator, (b) formed by sparger.

Fig. 5 presents the size distribution of bubbles generated from the in-line mixer and the sparger bubble-generator in the column flotation under the conditions of the frother dosage of 40 mg/L and superficial gas velocity of 1.66 cm/s. Fig. 5(a) shows size distribution of bubbles by in-line mixer bubble-generator; the analysis shows that the distribution type is normal distribution and Sauter mean diameter is 0.55 mm. Yianatos et al. [1987] suggested that the distribution type was normal distribution and Sauter mean diameter was 1.13 mm on the basis of the results of bubble diameter measured by the sparger bubble generator. Bubble size distribution is a normal distribution like one of Yianatos' works, but bubble diameter from these results is believed to be smaller than the Yianatos results. Fig. 5(b) represents the form of the bubble size distribution and sauter mean diameter by sparger bubble-generator. It can be seen that bubble size distribution is a normal distribution and Sauter mean diameter is 1.18 mm, so this is similar to the results by the above researchers. It is revealed that the bubble diameter formed by in line mixer is much smaller than that formed by a sparger. This result is similar to that determined by photographic techniques as shown in Fig. 4.

Fig. 6 shows the bubble diameter with cumulative bubble frequency, where Fig. 6(a) presents the bubble size distribution of two types of bubble-generator under the conditions of the superficial gas velocity (J_g=1.66 cm/s) and the frother dosage (C_F=40 mg/L), and Fig. 6(b) indicates the results in the case of conditions of the superficial gas velocity (J_g=0.99 cm/s) and the frother dosage (C_F= 240 mg/L). Through both conditions, the difference of the bubble size distribution would be understood. In both cases, the bubble size formed by the in-line mixer is much smaller than that by the sparger. Results of cumulative bubble frequency as bubble diameter are similar to that by Pal and Masliyah [1989] and Patel et al. [1990]. Therefore, it was observed that bubbles produced at the in-line mixer bubble generator are significantly smaller than those by sparger under the condition of superficial gas velocity and frother dosage.

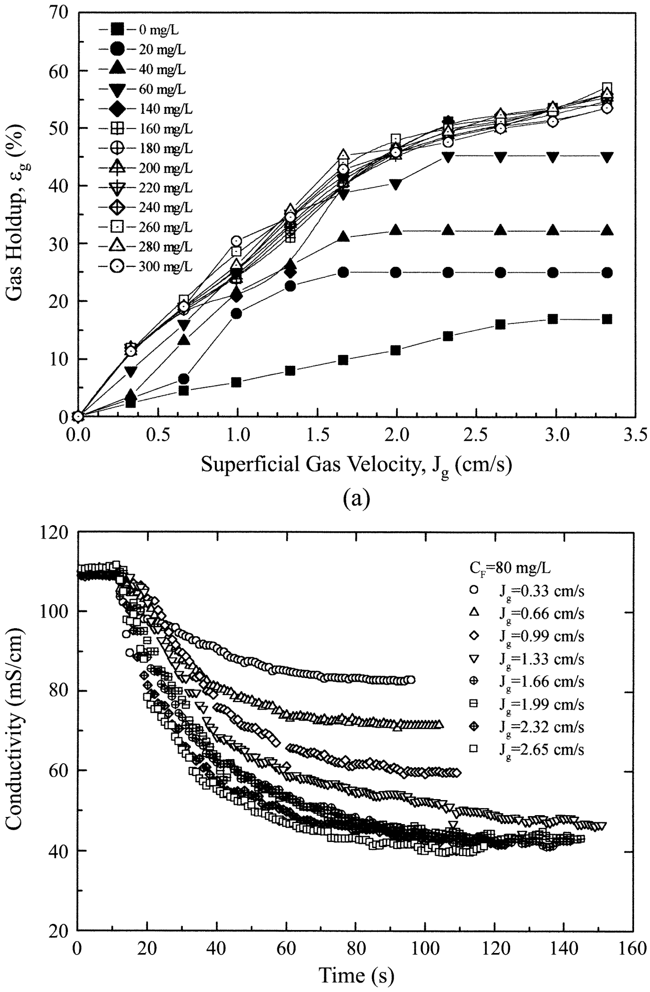
2. Gas Holdup Effect

Nicklin [1962], Maliyah [1979], and Dobby et al. [1988] have defined that gas holdup increased as bubble size decreased. Gas holdup itself is one of the most important parameters characterizing the hydrodynamics of bubble columns. It can be defined as the percentage by volume of the gas in two or three phase mixture in the column. The rate of particle collection is a function of gas flow rate and bubble diameter, both of which affect gas holdup. At this point, gas holdup depends mainly on the bubble diameter deter-



(b)

Fig. 6. Comparison with cumulative fraction of bubble formed by in-line mixer and sparger bubble-generator under the different operating condition: (a) C_F=40 mg/L and J_g=1.66 cm/s, (b) C_F=240 mg/L and J_g=0.99 cm/s.



(b)

Fig. 7. Gas holdup of bubble formed by in-line mixer bubble-generator: (a) the measurement by the water manometer, (b) by the conductivity meter.

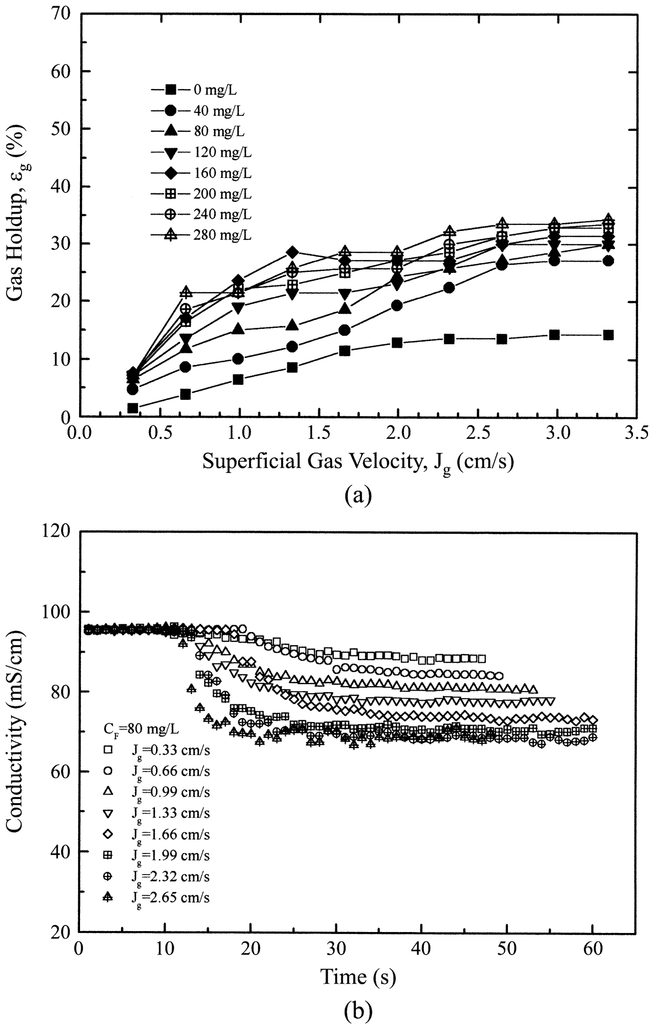


Fig. 8. Gas holdup measurement of bubble formed by sparger bubble-generator: (a) by the water manometer, (b) by the conductivity meter.

mined by superficial gas velocity and frother dosage. Studies conducted by Yoon [1993] show that collision effects depended on the bubble size, in which probability of collision increased from 0.003 to 0.04 with the particle diameter reducing from 0.5 mm to 0.1 mm. Thus, this implies that the increase of gas holdup as the decrease of the bubble size is attributed mainly to the increase of flotation efficiency.

Fig. 7 and Fig. 8 represent the gas holdup of the bubbles formed by in-line mixer and sparger bubble generator, respectively. Fig. 7(a) shows the gas holdup obtained by measuring the pressure difference of the water manometer in the column flotation. Gas holdup shows a tendency to increase with the increase of superficial gas velocity and frother dosage, which is in good agreement with that obtained by Finch and Dobby [1990]. The difference of gas holdup did not happen when superficial gas velocity was up to 2 cm/s as well as when over 16 mg/L frother is added. Finch and Dobby [1990], Dobby et al. [1988] and Zhou et al. [1993] also have reported that gas holdup within a definite range of variables such as the frother dosage and the superficial gas velocity is not changed, because bubbles become saturated within the column. In the case of bubbles formed by the in-line mixer, gas holdup is 50% when a frother is added to 160 mg/L, and then gas holdup does not increase with increasing frother dosage. Hence, it is speculated that the bubbles reach the saturated state based on the fact that gas holdup has been changeless in the column. The gas holdup obtained by means of measuring the conductivity difference of liquid and gas with conductivity meter is shown in Fig. 7(b). This is performed with a cell formed by two "grid" electrodes, which provide conditions for the free movement of the phases and for the establishment of near-to-uniform electrical fields. The results obtained in this work suggest that cell conductivity decreases depending on the increase of superficial gas velocity. Conductivity depends on the gas fraction in the column, which means that the gas holdup increases with increasing amount of bubble (gas fraction). From Fig. 7(b), it is seen that the value of conductivity decreases with increase of gas flow rate. According to the conductivity results measured under the frother dosage of 8 mg/L and superficial gas velocity of 0.33-2.65 cm/s, conductivity is reduced from 100 μS/cm to 50 μS/cm with increase of superficial gas velocity. So it is found that the amount of bubbles in the column reduced almost to 50%. This result indicates that Fig. 7(a) was remarkably valid.

Fig. 8 shows the gas holdup of bubbles formed by the sparger. Gas holdup was about 30% when frother dosage was 160 mg/L and superficial gas velocity was 2.5 cm/s. This result is consistent with

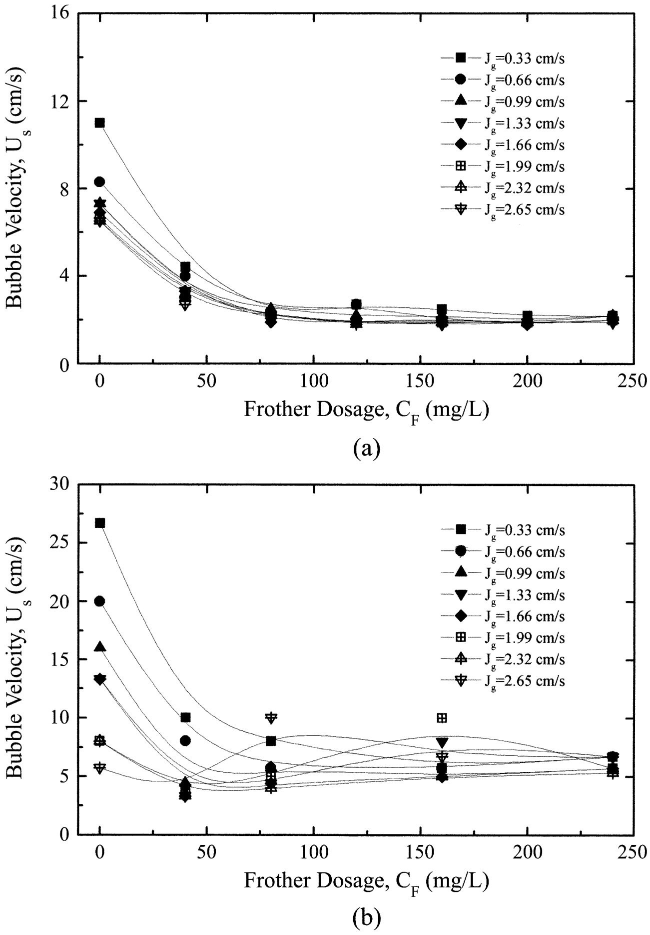


Fig. 9. Bubble swarm velocity as a function of a superficial gas velocity (J_g) and frother dosage (C_F): (a) in-line mixer, (b) sparger.

that of Finch and Dobby [1990]. Comparison of the results from Fig. 7 and Fig. 8 reveals that gas holdup of bubbles formed by the in-line mixer is 20% larger than that by a sparger. This corresponds well with the results from analyzing bubble size distribution. Gas holdup should be larger for the in-line mixer than for the sparger, because bubble diameter formed by the in-line mixer is much smaller than that by the sparger. When bubble size is small and gas holdup is large should mean that the collisions between bubbles and particles increased, and hence flotation efficiency may improve remarkably. These results are related to the bubble swarm velocity. From Eq. (3), the variables to determine the bubble swarm velocity in the flotation column are dependent on bubble diameter and gas holdup in the condition of given frother dosage and superficial gas velocity.

3. Bubble Swarm Velocity Effect

Fig. 9 shows the results of swarm bubble velocity (U_s), using a conductivity meter, as shown in Fig. 3. Fig. 9(a) presents the swarm velocity of bubble formed by in-line mixer bubble generator, in which U_s would indicate higher velocity (6-10 cm/s) as the difference of superficial gas velocity without frother. However, bubble velocity (U_s) decreases markedly to 3 cm/s as soon as frother dosage is added. At this point, swarm bubble velocity is slightly influenced by superficial gas velocity. And the distribution of swarm bubble velocity is also considerably uniform. In the case of bubbles formed by the sparger, like that depicted in Fig. 9(b), without frother, swarm bubble velocity was 5-28 cm/s and it is widely distributed. Furthermore, this case is largely affected by superficial gas velocity. Although frother dosage is allowed to increase, bubble swarm velocity is observed not only to be as large as 5-10 cm/s but also not to be constant at all.

According to the results of analysis the bubble size, the gas holdup, and swarm bubble velocity, respectively, it is found that bubbles produced at the in-line mixer are more effective than at the sparger to improve the floating performances. This is because bubble size is smaller, gas holdup is larger, and hence bubble swarm velocity is lower in in-line mixer than in sparger. So the probability of collisions with bubbles and particles in the column should be increased. Table 1 shows the values of the bubble diameter, the gas holdup and the bubble swarm velocity that produced at the in-line mixer and the sparger, under superficial gas velocity of 0.99, 1.66, and 2.32 cm/s, with frother dosage of 80 and 240 mg/L. For example, when frother 80 mg/L is added and superficial gas velocity is 1.66 cm/s, it can be seen from Tables 1 and 2 that sauter mean diameter is 0.47 mm, gas holdup is 32.19%, and bubble swarm velocity is 2.2 cm/s for bubble formed by in-line mixer, while, for sparger, sauter mean diameter is 1.43 mm, gas holdup is 18.57%, and bubble swarm velocity is 4.4 cm/s. A similar trend is also resented under different conditions. Therefore, the flotation system is strongly influenced by frother dosage and bubble-generator type. This study indicates that the in-line mixer bubble generator is more effective than the sparger to develop flotation efficiency.

Table 1. Bubble diameter, bubble gas holdup and bubble swarm velocity from the in-line mixer bubble generator as a function of frother dosage

Frother dosage (mg/L)	Superficial gas velocity (cm/s)	Bubble diameter (mm)	Gas holdup (%)	Bubble swarm velocity (cm/s)
80	0.99	0.4	20.83	2.6
	1.66	0.47	32.19	2.2
	2.32	0.56	52.38	1.9
240	0.99	0.35	25	2.2
	1.66	0.38	37.67	2.0
	2.32	0.43	48.81	1.8

Table 2. Bubble diameter, bubble gas holdup and bubble swarm velocity from the sparger bubble generator as a function of frother dosage

Frother dosage (mg/L)	Superficial gas velocity (cm/s)	Bubble diameter (mm)	Gas holdup (%)	Bubble swarm velocity (cm/s)
80	0.99	1.09	15	5.7
	1.66	1.43	18.57	4.4
	2.32	1.76	25.71	4.0
240	0.99	0.47	21.43	5.7
	1.66	0.7	25.71	5.6
	2.32	1.37	30	5.3

CONCLUSION

The factors that affect the bubble properties are frother dosage, superficial gas velocity determined by air flow rate and the type of bubble generator. It can be concluded that factors to determine bubble properties should be affected more by bubble-generator types than by operating factors. Based on the results of bubble size, gas holdup, and swarm bubble velocity formed by in-line mixer and sparger bubble-generator, the following conclusions can be drawn from this study.

1. Three relations on bubble properties can be recognized in this study: (a) bubble size decreases typically with increase of frother dosage and superficial gas velocity. (b) gas holdup increases with decrease of bubble size, (c) swarm bubble velocity decreases with increase of gas holdup.
2. According to the comparison between bubbles formed by in-line mixer and sparger bubble-generator, the in-line mixer bubble-generator produced micro-bubbles whose sauter mean diameter was less than 1 mm.
3. Gas holdup of the micro-bubble produced by in-line mixer bubble-generator results in up to 50%, so that it can generate a stable froth layer in the column.
4. Swarm bubble velocity of bubble formed by in-line mixer bubble-generator is lower than by sparger bubble-generator, so it is more suitable to produce micro-bubbles and a more stable frother layer can be formed.

Consequently, the in-line mixer bubble-generator is determined to be useful for improving flotation efficiency in the column flotation system.

NOMENCLATURE

C_F : frother dosage

d_b : bubble diameter
d_{bs} : Sauter mean diameter
d_c : column diameter
Δh : the distance between the water level in the two manometers
J_g : superficial gas velocity
ΔL : the distance between the locations of the manometers
ΔLe : the distance between cells
m : function of bubble Reynolds number
Re_b : bubble Reynolds number
U_s : bubble rise velocity
U_T : terminal gas velocity
ε_g : gas holdup
ρ_f : liquid density
ρ_b : bubble density
μ_f : liquid viscosity

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