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THEORY AND TECHNOLOGY OF THE PROCESS OF ARTICLES FORMING

VARIANT OF THE PLASTICITY CONDITION FOR POWDERED SOLIDS

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A plasticity criterion is proposed for powdered materials to account for the local character of slip and plastic deformation of the particles. The degree of localization of the contact interaction between the particles is quantitatively evaluated by using the relative fraction of the active volume, after M. Yu. Balshin. Good agreement is obtained between the theoretical and experimental data in describing of the process of the isostatic pressing of spherical powders of lead and tin. The effect of the density of the material and pressure on the form of the yield surface and the character of the volume deformation is examined. A method is presented for determination of the empirical parameters of the plasticity criterion. These parameters are evaluated for sand.

Keywords: *porous bodies, plastic deformation, isostatic compaction.*

The following plasticity condition has been substantiated and come into wide use for porous materials having an ideally plastic matrix which is bonded into a total framework [1, 2]:

$$\frac{\sigma^2}{2\psi} + \frac{T^2}{\varphi} = \rho\tau_s^2 . \tag{1}$$

Here, σ is the average stress; T is the intensity of the shear stresses; ρ is relative density; τ_s is the yield strength in shear; ψ and φ are functions of relative density and are equal to

$$\psi = \frac{2}{3}\frac{\rho^3}{1-\rho}, \ \varphi = \rho^2.$$

Shear stress intensity T in (1) is determined through the components of the stress deviator s_{ij} by means of the relation $T = (0.5 s_{ij} s_{ij})^{1/2}$ and differs from the shear stress intensity $\tau = (s_{ij}s_{ij})^{1/2}$ used in [2] and other investigations. This accounts for the presence of the multiplier 2 ahead of ψ in the denominator.

Yield condition (1) describes the mechanical behavior of porous bodies composed of an incompressible isotropic substance whose yield strength τ_s is independent of the mean stress σ. A powder is a collection of particles which are capable of being plastically deformed both by plastic shear of the particles and by slip of the contacting particles. Therefore, two yield strengths should be examined for powdered solids: the yield strength in sliding friction τ_{sd} and the yield strength in shear τ_s. The yield strength in sliding friction τ_{sd} depends on the mean stress σ, and we will express this dependence in the form of Coulomb law of friction

$$\tau_{sd} = K_0 - f\sigma, \tag{2}$$

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where K_0 is the constant associated with shear friction; f is the coefficient of internal friction. As the compressive stress σ increases, the value of τ_{sd} approaches a certain limiting value τ_0. That value is equal to the yield strength τ_s for ductile particles or to the ultimate strength in shear τ_b for brittle particles. The von Mises plasticity condition for a hypothetical nonporous Coulomb body is a piecewise-smooth condition and is written in the form

$$T = K_0 - f\sigma \text{ at } \sigma \le \sigma*,\ T = \tau_0 \text{ at } \sigma > \sigma*. \tag{3}$$

Here, σ^* is the average value at which plastic deformation or fracture of the particles begins: $\sigma^* = (K_0 - \tau_0)/f$. The plasticity condition for a porous Coulomb body, examined in [3], will also be piecewise-smooth:

$$\frac{\sigma^2}{2\psi} + \frac{T^2}{\varphi} = \rho(K_0 - f\sigma)^2 \text{ at } \sigma \le \sigma*;$$

$$\frac{\sigma^2}{2\psi} + \frac{T^2}{\varphi} = \rho\tau_0^2 \text{ at } \sigma > \sigma*. \tag{4}$$

Plasticity criteria (1) and (4) follow from the postulate on the identity of the limiting total elastic energy of a porous body and its material [1]. It is assumed here that the entire volume of the material is in the limiting state relative density ρ in the right side of Eqs. (1), (4) is considered to be the quantitative measure of that state. At the same time, it is known that plastic deformation is localized around pores in porous materials or in the regions of contact between the particles in powders [4-7]. In this investigation, we examine a variant of the plasticity condition that accounts for the local character of the plastic deformation of the particles of a powdered medium.

The contact interaction that occurs inside particles is accompanied by the formation of two zones [5]: the active volume, which is adjacent to the contact surfaces; the inert volume, which is not in contact with neighboring particles. The stresses are concentrated in the active volume, and it is here that slip or plastic deformation of the particles begins. The inert volume of the substance is in the elastic state, and we will regard it as a perfectly rigid inclusion. Then only the active volume will be in the limiting state, and the plasticity condition for a dispersed medium is written in the form

$$\frac{\sigma^2}{2\psi} + \frac{T^2}{\varphi} = \alpha(K_0 - f\sigma)^2 \text{ at } \sigma \le \sigma*;$$

$$\frac{\sigma^2}{2\psi} + \frac{T^2}{\varphi} = \alpha\tau_0^2 \text{ at } \sigma > \sigma*, \tag{5}$$

where α is the relative fraction of the active volume. The fraction of the active volume α is connected with the relative density of the powder ρ by the relation [5]

$$\alpha = \rho^2\left(\frac{\rho - \rho_0}{1 - \rho_0}\right)^b, \tag{6}$$

where ρ_0 is the bulk density of the powder; b is a constant.

Relation (6) was obtained in [5] from a model that gives a probabilistic description of the formation of the contact bonds between the particles of a powder. A similar expression with the exponent $b = 1$ was obtained in [8] from a model describing the consolidation which takes place due to the increase in the radii of spherical ductile particles around fixed centers. The spheres subsequently overlap one another and contact bridges are formed.

The main difference between plasticity condition (5) and condition (4) is in the evaluation of the amount of the substance that is in the limiting elastic state. According to (4), plastic flow of a powdered material begins when the limiting elastic energy is accumulated throughout the volume of the solid phase. It follows from (4) that when a powder characterized by its bulk density undergoes pure shear, the shear stress is nontrivial: $T = K_0\sqrt{\rho_0\varphi}$. At the same time, the experiment shows

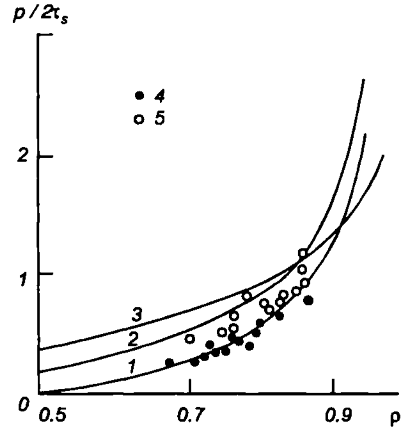

Fig. 1. Dependence of relative pressure during isostatic pressing on the relative density of the powders. 1, 2, 3) Calculation based on (5), (4), and (7), respectively; 4, 5) experimental points (lead and tin, respectively).

that a powder in the bulk state has no resistance to shear. According to condition (5), only the active volume of the solid phase is in the limiting state. In the bulk state at $\rho = \rho_0$, the fraction of the active volume $\alpha = 0$, and according to (5) the shear stress in the powder is also equal to zero. Plasticity condition (5) also agrees well with the experimental data for a stress state with a substantial mean compressive stress σ. The experimental data presented in Fig. 1 characterizes consolidation during the isostatic pressing of spherical powders of lead and tin [9]. The figure also shows the relationship between the relative density ρ of a powder and the relative pressure $p/2\tau_s$ calculated on the basis of plasticity condition (4), plasticity condition (5), and the equation proposed in [8, 10] to describe isostatic pressing:

$$p = -2\sqrt{3}\tau_s \ln(1 - \rho). \tag{7}$$

In calculating the fraction of the active volume α, bulk density was taken equal to $\rho_0 = 0.5$ (this value was obtained by extrapolating empirical data for lead), and the exponent $b = 1$. It is apparent from Fig. 1 that the best agreement between the calculated and experimental data throughout the range of densities is obtained with plasticity criterion (5). The difference between the experimental and calculated data for tin powder is related to the fact that this material has a bulk density lower than $\rho_0 = 0.5$. The pressure at which the porous medium undergoes consolidation is proportional to the volume fraction of the plastically deformed substance. The physical model followed by Eqs. (4) and (7) does not account for localization of plastic flow during the contact interaction of the particles and implies plastic deformation throughout the volume of the substance. Therefore, a calculation based on Eqs. (4), (7) shows a higher relative pressure compared to the experimental data.

The deformation of powdered materials can cause both their consolidation and decrease in density. A predictive estimate of the character of the change in density can be obtained on the basis of the form of the trace of the yield surface in the plane $p - T$, where $p = -\sigma$ is hydrostatic pressure. The form of the yield surface corresponding to plasticity criterion (4) was analyzed in [3]. We will perform a similar analysis for plasticity condition (5). We write the first equation in (5) as follows:

$$\Phi p^2 - 2\alpha f K_0 p + (T^2/\varphi) = \alpha K_0^2, \ p \le p^*, \tag{8}$$

where Φ is a function of density and internal friction and is equal to

$$\Phi = \frac{1}{2\psi} - \alpha f^2. \tag{9}$$

The form of the second-order curve (8) depends on the sign of the function Φ and, thus, on the density of the powdered material. If ρ_k is the solution of the equation $\Phi = 0$, then the function $\Phi > 0$ at $0 < \rho < \rho_k$, and Eq. (8) describes an ellipse

$$\frac{(p-\Delta)^2}{a^2} + \frac{T^2}{c^2} = 1 \tag{10}$$

with the semi-axes

$$a = \frac{K_0}{\Phi}\sqrt{\frac{\alpha}{2\psi}}\,;\; c = K_0\sqrt{\frac{\alpha\varphi}{2\psi\Phi}}\,, \tag{11}$$

which are shifted along the p axis by the amount $\Delta = \alpha K_0 f \Phi^{-1}$. The strain-rate vector is directed along a normal to the yield curve, and its projections in the directions of the σ and T axes are respectively equal to the rate of change in volume e and shear-strain intensity H. The rate of volume deformation $e = 0$ on the equator of ellipse (10) at $p = \Delta$; $e > 0$ at $p < \Delta$, and deformation is accompanied by a reduction in density; $e < 0$ at $p > \Delta$, and the powder undergoes consolidation. When $\rho = \rho_k$, the function $\Phi = 0$, and Eq. (8) is a parabola:

$$p = \frac{T^2}{2\alpha\varphi f K_0} - \frac{K_0}{2f}\,. \tag{12}$$

The rate of volume deformation $e > 0$ and the density of the powder decrease for any pressure $p < p^*$ on the curve (12). Thus, the quantity ρ_k determines the density of the powder above which consolidation cannot take place as a result of slip. When $\rho_k < \rho < 1$, the function $\Phi < 0$, and Eq. (8) is the equation of a hyperbola

$$\frac{(p+\Delta_1)^2}{a_1^2} - \frac{T^2}{c_1^2} = 1 \tag{13}$$

with the parameters

$$a_1 = \frac{K_0}{\Phi}\sqrt{\frac{\alpha}{2\psi}}\,;\; c_1 = K_0\sqrt{-\frac{\alpha\varphi}{2\psi\Phi}} \tag{14}$$

and a center of symmetry that is shifted to the left along the p axis by the amount $\Delta_1 = -\alpha f K_0 \Phi^{-1}$. As in the case of parabola (12), loading on the hyperbolic yield surface leads to a decrease in the density of the material. In the nonporous state at $\rho = 1$, yield condition (8) takes the form of linear relation (3): $T = K_0 - f\sigma$. The material becomes looser when it is subjected to plastic deformation on this yield surface.

At hydrostatic pressure $p > p^*$, the plastic flow of the powder is described by the second equation of plasticity condition (5). This equation describes an ellipse in the plane $\sigma - T$. The equator of the ellipse, at which $e = 0$, has the abscissa $p = 0$. Since $p^* > 0$, only consolidation of the powder occurs on the yield surface. As consolidation proceeds and $\rho \to 1$, the generatrix of the ellipse degenerates into the straight line $T = \tau_0$. This line corresponds to the von Mises yield condition for an incompressible material.

Plasticity condition (5) contains several empirical parameters. Those parameters are bulk density ρ_0, pressure p^*, the adhesion constant K_0, the coefficient of internal friction f, the limiting stress τ_0, and the exponent b. Bulk density ρ_0 is the simplest parameter to determine. The other parameters are found in tests that involve measuring components of the stress state and density ρ. To determine the exponent b and the strength characteristics τ_0, we write plasticity condition (5) in the form:

$$\ln\frac{\sqrt{u}}{\rho} = \frac{b}{2}\ln\bar{\alpha} + \ln\left(K_0 - f\sigma\right);\; \ln\frac{\sqrt{u}}{\rho} = \frac{b}{2}\ln\bar{\alpha} + \ln\tau_0\,. \tag{15}$$

Here, u represents the left side of Eqs. (5) and $\bar{\alpha}$ represents the expression $\bar{\alpha} = (\rho - \rho_0)/(1 - \rho_0)$. Then the point of inflection on the curve of Eq. (15) reflects a change in the mechanism of plastic flow — slip ends and plastic deformation (or fracture)

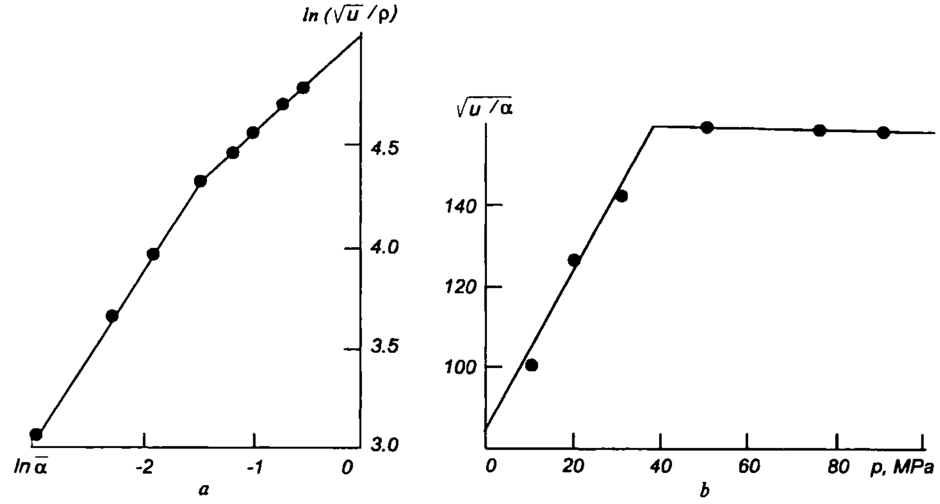

Fig. 2. Linearization of plasticity condition (5) in determining the rheological properties of sand.

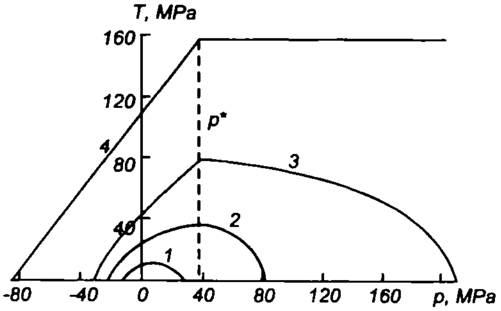

Fig. 3. Trace of the yield surfaces of sand in the plane $p - T$. $\rho = 0.65$ (1), 0.724 (2), 0.83 (3), and 1 (4).

of the particles of the powder begins. We use the parameters of the linear section of Eq. (15) to find the exponent b and the strength characteristic τ_0. Having known values of b and the active volume α, we can find the characteristics of internal friction f, K_0, and pressure p^*. To do this, we reduce Eqs. (5) to the form

$$\sqrt{\frac{u}{\alpha}} = K_0 - f\sigma; \quad \sqrt{\frac{u}{\alpha}} = \tau_0 \qquad (16)$$

and we study their dependence on pressure p. If we put *a priori* $b = 1$ — which is acceptable for spherical powders — then the procedure of finding the rheological characteristics is simplified and is limited to the analysis of Eqs. (16).

Figure 2 shows results of analysis of experimental data from the pressing of sand in a closed die [11]. The values $b =$ $= 1.06$ and $\tau_b = 160$ MPa were determined when we approximated the linear section of the graph (Fig. 2a) of Eqs. (15). Then we used the graphs of Eqs. (16) in Fig. 2b to obtain the coefficient of internal friction $f = 1.97$, the adhesion constant $K_0 = 85$ MPa, and the pressure $p^* = 38$ MPa. The solution of the equation $\Phi = 0$ was used to determine the density $\rho_k = 0.724$, at which consolidation can take place only as a result of crushing of the particles of sand. We note that the value of ρ_k here differs little from the relative density of the most solid packing of monodispersed spheres, which is $\rho = 0.74$. Figure 3 shows the yield curves of the sand for different values of ρ. These curves clearly show the dependence of the plasticity properties of the powders on their structural state and the magnitude and sign of the mean stress. The last factor causes the anisotropy of the mechanical properties of dispersed materials.

The proposed plasticity condition, which accounts for the local character of the slip and plastic deformation of particles of a dispersed medium, shows good agreement between theoretical and experimental data for different deformation schemes and can be used to solve problems in the applied plasticity of powdered materials.

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