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AMPLITUDE MODULATION OF SUBMILLIMETER WAVE GYROTRON OUTPUT

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Abstract

Amplitude modulation of gyrotron by a small modulation of the anode voltage is calculated using an energy transfer formula. Experimental measurements using a submillimeter wave gyrotron are in good agreement. One hundred percent modulation of the output at frequencies up to several hundred kilohertz has been achieved with anode modulation levels of only a few percent. Numerical calculations lend further support to the experimental results.

Key words : Gyrotron, Amplitude modulation, submillimeter wave.

1. Introduction

Amplitude modulation of the output of a gyrotron is important for the extension of gyrotron applications into new areas. For example, plasma wave excitation by a ponderomotive force, coupling of millimeter-wave energy into a plasma by an electron cyclotron resonance antenna at its surface, phase sensitive detection of the scattered signal from a plasma, studies of relaxation phenomena in plasmas and other materials, and so on. While the importance is well-recognized, few experiments have been carried out.[1] Some theoretical work has been reported,[2] but no comparison with experiment was made.

In this paper, we consider amplitude modulation of gyrotron output by applying a small modulating signal to the anode voltage. First, a simple calculation based on an energy transfer formula is carried out. Next, we describe experimental amplitude modulation results for a medium power, submillimeter wave gyrotron. One hundred percent modulation of the gyrotron output up to 5 kHz can be achieved with modulation levels of only a few percent in the anode voltage. Smaller amounts of modulation at frequencies up to 400 kHz have been obtained. Finally, a full numerical calculation including the spread of electron velocities and the interaction between the electron beam and the high-frequency field is carried out. Agreement with the experimental results is good.

2. Calculation of amplitude modulation using an energy transfer formula

The magnetron injection gun that produces the electron beam consists of a cathode with an emitting ring, an anode and the main body of the gyrotron or resonator which is grounded. When the anode voltage V_a of such a gun is modulated, the velocity distribution of the electrons in the beam is modulated. Modulation of the energy transfer between the electrons and the high frequency field follows which in turn leads to modulation of the gyrotron output. A first analysis of this amplitude modulation of the gyrotron output can be carried out using an equation for energy transfer. An expression for the real part of the rate of energy transfer P_{ss} from electrons to electromagnetic wave fields in a TE$_{mln}$ mode cavity which has been given by Brand,[3]

$$P_{SS} = \frac{2e^2|E_0|^2\nu}{k^2 p_{//}} G\left(\frac{\beta_\perp^2\omega}{2k v_{//}} \frac{1}{2G} \frac{dG}{dX} P - Q\right) \tag{1}$$

where

$$P = \frac{\left[J_{N+m}^2(\xi_{ml} r_0/R) + J_{N-m}^2(\xi_{ml} r_0/R)\right](N\beta_\perp)^{2N-2}}{2^{2N}\left[(N-1)!\right]^2} \tag{2}$$

$$Q = NP \tag{3}$$

for a cold electron beam having a small gyroradius, and

$$G = -\frac{(-1)^n \cos(n\pi X) - 1}{2(X^2-1)^2} \tag{4}$$

and

$$X = -\frac{\omega - N\omega_c}{kv_{//}} \tag{5}$$

E_0 is the amplitude of the electromagnetic wave in cavity, ν the electron charge density per unit length in z-direction, k the wave number of the wave, $p_{//}$ and $v_{//}$ the axial momentum and velocity of the electrons, $\beta_\perp = v_\perp / c$, $v_\perp$ the perpendicular velocity, r_0 the injection point of the electron beam in the cavity, ω_c the electron cyclotron frequency, J_m Bessel functions of the first kind, R the cavity radius, ξ_{ml} the lth zero of $dJ_m(x)/dx$ and $N = \omega / \omega_c$ the cyclotron harmonic number.

The output power P_{out} from a cavity is calculated as follows,

$$P_{out} = P_{ss} - P_{ohm} \quad , \tag{6}$$

where $P_{ohm} = \omega\, \varepsilon / Q_{ohm}$ is the power lost to the walls of the cavity by ohmic heating, ε is electromagnetic wave energy stored in the cavity and Q_{ohm} the ohmic Q factor of the cavity. The energy transfer function P_{ss} for the second harmonic operation (N=2) has a maximum value near $X = -1$. Under these conditions, equation (1) becomes

$$P_{ss} = \frac{\pi^2 e^2 |E_0|^2 \nu}{16\, k^2 p_{//}}\, P\left[\frac{\beta_\perp^2 \omega}{2kv_{//}} - 4\right] \quad . \tag{7}$$

Note that from Eq.(2), the factor P is proportional to $\beta_\perp^2$ when N=2. When the anode voltage V_a is modulated as

$$V_a = V_{a0} + \Delta V_a \sin\omega_m t \quad , \tag{8}$$

where ω_m is the modulation frequency. The quantities $\beta_\perp$, $v_{//}$, $p_{//}$ and ν are modulated as follows,

$$\beta_\perp = \beta_{\perp 0}\left\{1 + \left(\Delta V_a / V_{a0}\right) \sin\omega_m t\right\} \quad , \tag{9}$$

$$v_{//}\,,\, p_{//} = v_{//0}, p_{//0}\left\{1 - \alpha^2 \left(\Delta V_a / V_{a0}\right)\sin\omega_m t\right\} \quad , \tag{10}$$

$$\nu = \nu_0\left\{1 + \alpha^2 \left(\Delta V_a / V_{a0}\right)\sin\omega_m t\right\} \quad , \tag{11}$$

where $\alpha = v_\perp / v_{//}$ is the velocity ratio at the cavity. Higher order terms of small modulation ratio $\Delta V_a / V_{a0}$ have been neglected. Substituting Eqs.

(8)~(11) into Eq. (7), and making some simplifications based on experimentally obtained values for Q_{ohm}, $\beta_{\perp}$, $v_{//}$, $p_{//}$, ω, r_0 and ν , we can write an expression for the amplitude modulation of the output $\Delta P_{out} / P^0_{out}$ as follows,

$$\frac{\Delta P_{out}}{P^0_{out}} = \frac{1}{1 - P_{ohm} / P_{ss}} \left(3\,\alpha^2 + 4 \right) \frac{\Delta V_a}{V_{a0}} \quad . \tag{12}$$

Substituting Eq.(6) into this equation and reawanging gives an expression for what may be called the efficiency for amplitude modulation. $(\Delta P_{out} / P^0_{out}) / (\Delta V_a / V_{a0})$,

$$\frac{\Delta P_{out} / P^0_{out}}{\Delta V_a / V_{a0}} = \left[1 + \frac{P^0_{ohm}}{P^0_{out}} \right] \left(3\alpha^2 + 4 \right) \quad . \tag{13}$$

One conclusion from this equation is that amplitude modulation can be obtained more easily when the velocity ratio α is large and the output power P^0_{out} is small. Under such conditions, P^0_{out} is very sensitive to any modulation of P_{ss}. Eq. (13) will be compared with the experimental results in a latter section.

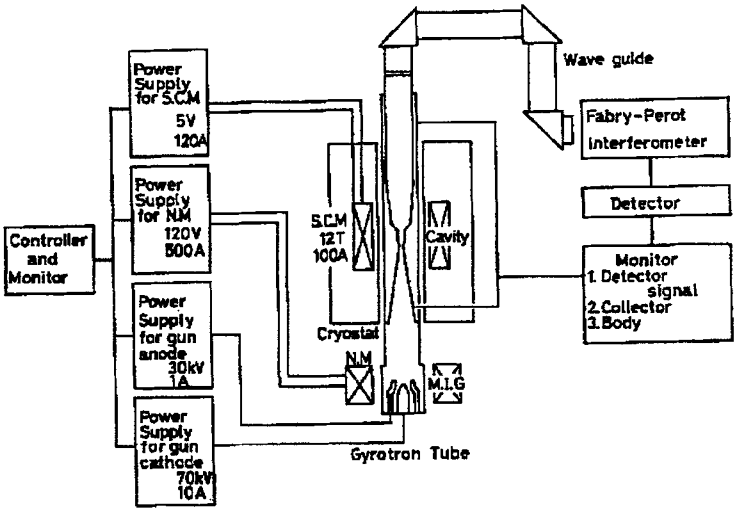

Fig.1 The complete experimental arrangement

Table 1 Typical experimental conditions for the amplitude modulation studies.

Frequency of output	f	444 GHz
Harmonic number	$N = \omega / \omega c$	2
Cavity mode		TE_{161}
Output power	P^0_{out}	300 W
Cathode voltage	V_k	-30 kV
Anode voltage, with respect to resonator*	V_{ar}	-20 kV
Modulation frequency of V_a	f_m	< 1 MHz
Modulation rate of V_a	$\Delta V_a / V_{a0}$	<0.25 (f_m < 5 kHz) <2×10^{13}(f_m < 1 MHz)
Modulation mode		sinusoidal or square wave

3. Experimental arrangement

The experimental apparatus is the same as the one described in an earlier paper.[4] The gyrotron tube (Gyrotron FU III) is mounted in a superconducting magnet whose field intensity can be varied up to 12T. The gyrotron is step tunable and emits submillimeter waves at frequencies up to 316 GHz, when operating at the fundamental of the electron cyclotron frequency and up to 636 GHz under single mode operation at the second harmonic. Typical output powers are several kilowatts at the fundamental and several hundred watts at the second harmonic.

Fig.1 shows the whole experimental arrangement including the power supplies and the detector system. In the modulation experiments in the frequency range from 0.5 to 5 kHz, we used a pulsed high-voltage supply with a special facility to allow sinusoidal or square wave modulation of the anode voltage up to levels of 25 percent. In the higher frequency experiments, the output of a high power radio frequency oscillator was superimposed on the DC high voltage pulse to the anode. In this case only sinusoidal modulation was possible. Typical values of the operating parameters are shown in Table 1.

The output power of the gyrotron is transmitted by a circular waveguide system into the experimental area and analyzed as follows:

1. The power is fed to a Fabry-Perot interferometer and the output of the interferometer is detected by a pyroelectric detector. This gives the wavelength of the gyrotron output and the frequency can be calculated.

2. The power is attenuated and fed to a pyroelectric detector directly. This enables the modulation of the output power to be monitored.
3. The power is Fourier-analyzed to give the frequency spectrum of the modulated gyrotron output.
4. The absolute output power of gyrotron is measured by placing a water load at the top of the gyrotron tube and calibrating the pyroelectric detector. The pyroelectric detector is used in the range where its sensitivity to input power is linear.

4. Experimental results

4.1 *Low frequency amplitude modulation*

The amplitude modulation experiment is carried out by using the arrangement shown in Fig.1. The power supply for the anode gives a pulse of approximately 1 millisecond on the anode. The anode voltage is modulated with a sinusoidal or square wave at a frequency up to 5 kHz and up to a level of 25%. The power supply for the cathode is adjusted so that its pulse width is a little shorter than that on anode.

Fig.2 shows some results of modulating the submillimeter wave gyrotron when it is operating at the second harmonic of the cyclotron frequency. The cavity mode is TE_{161}, the frequency is 444 GHz and the output power is about 300 watts The modulating frequency f_m was 5 kHz. The upper traces show the high-voltage pulse applied to the anode. The small sinusoidal modulating signal is just visible in the traces. The lower traces show the output of the gyrotron. The modulation $\Delta P_{out} / P^0_{out}$ of the gyrotron output increases with $\Delta V_a / V_{a0}$. One hundred percent modulation ($\Delta P_{out} / P^0_{out} = 1$) is attained when $\Delta V_a / V_{a0}$ is only several percent ($\Delta V_a / V_{a0} \sim 0.055$).

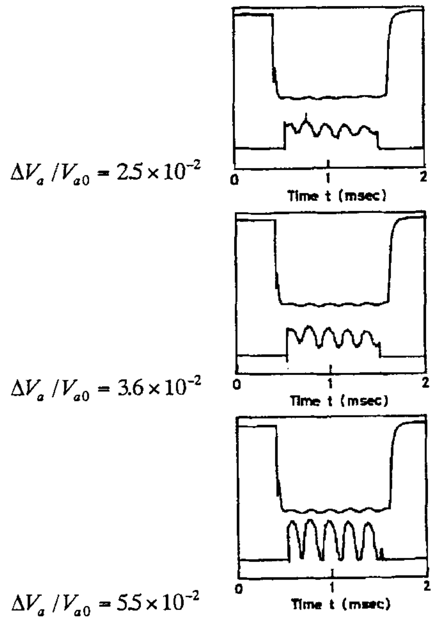

Fig.2 (upper trace) Voltage pulse applied to anode of submillimeter wave gyrotron and (lower trace) modulated output for three modulation levels $\Delta V_a / V_{a0}$. Gyrotron operation conditions: V_k=-30kV, V_{a0}=8.6kV, and I_b=420mA. Gyrotron frequency f =444 GHz at the second harmonic. Modulating signal: f_m=300 kHz, sinusoidal.

4.2 *High frequency amplitude modulation*

Amplitude modulation of gyrotron output at higher frequencies up to 1 MHz has been attempted. In the modulation frequency range from 5 kHz to 1 MHz, the experimental arrangement is slightly different. Only one power supply is used to provide the high voltage pulse to both cathode and anode. A voltage divider is used for the anode. The pulse width is 1 millisecond. To modulate the anode voltage, the output of a high power oscillator is connected to the anode. The output power from this oscillator and its frequency can be changed up to 30 W and 1 MHz.

Fig.3 shows a typical result of the high frequency amplitude modulation experiment. The upper traces show the modulation voltage applied to the anode and the lower traces the output power of gyrotron. In this case,modulation frequency is 300 kHz and modulation level $\Delta V_a / V_{a0}$ of anode voltage is very small, only 1.7×10^{-3}. Small amounts of modulation of gyrotron output are observed. The modulation level $\Delta P_{out} / P^0_{out}$ is about 7×10^{-2}.

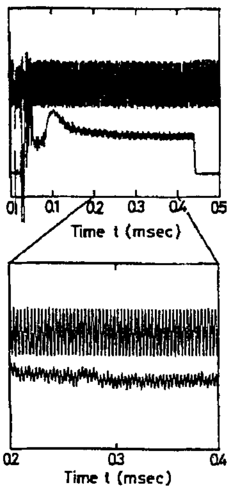

Fig.3 (upper trace) Modulation voltage applied to anode and (lower trace) modulated output of gyrotron. Gyrotron operating conditions: V_k=-21.4 kV, V_{a0}= 8.6kV and I_b=420 mA. Gyrotron frequency f=312 GHz at the second harmonic (TE_{431} mode). Modulating signal: f_m=300 kHz, sinusoidal.

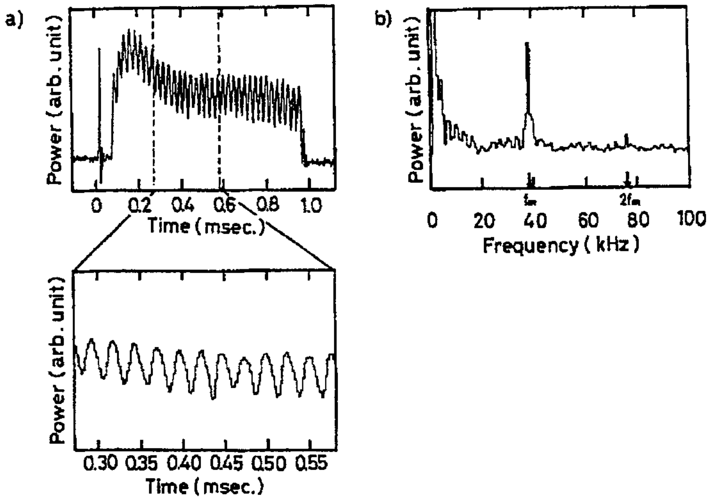

Fig.4 (a) High frequency modulated output of the gyrotron.
(b) Frequency spectrum of the output. Gyrotron operating conditions: V_k=-30kV, V_{a0}=8.6kV and I_b=420 mA. Gyrotron frequency f=444 GHz at the second harmonic. Modulating signal: f_m=38.5 kHz, sinusoidal. $\Delta V_a / V_{a0} = 9.0 \times 10^{-3}$.

The modulation efficiency $(\Delta P_{out} / P^0_{out})/(\Delta V_a / V_{a0})$ is about 41. High frequency modulation up to 600 kHz has been attained. At 400 kHz, modulation efficiency of about 50 was achieved, under a low modulation level of anode voltage $\Delta V_a / V_{a0} = 1.1 \times 10^{-3}$.

4.3 *Frequency spectrum of modulated output power*

The output power of the gyrotron is Fourier-analyzed using a signal analyzer (SM-2100, Iwatsu Co., Ltd) to give a frequency spectrum of the modulation. This provides a check on how closely the modulation of the gyrotron output is sinusoidal.

Fig.4(a) shows the gyrotron output and Fig.4(b) shows the corresponding frequency spectrum. The modulation frequency observed in the spectrum is 38.5 kHz as expected and the second harmonic component is -17 dB lower than the fundamental one. This means the amplitude modulation is almost sinusoidal and suggests the linearity between the modulation level at the anode and the modulated output is quite good.

4.4 *Modulation efficiency versus the output power*

Fig.5 shows the modulation of the gyrotron output $\Delta P_{out} / P^0_{out}$ as a function of the modulation level $\Delta V_a / V_{a0}$ for several values of beam current I_b, for the second harmonic operation in the TE161 cavity mode. $\Delta P_{out} / P^0_{out}$ is proportional to $\Delta V_a / V_{a0}$, when the modulation level of the anode voltage is low $(\Delta V_a / \Delta V_{a0} \leq 2 \times 10^{-2})$.This supports the frequency spectrum result shown in Fig.4.

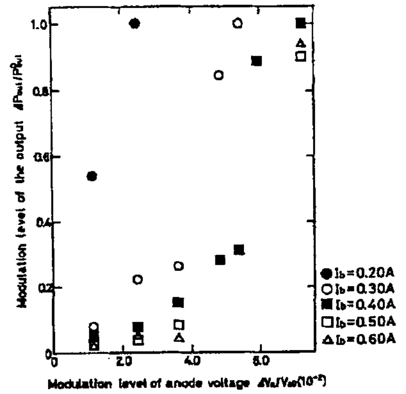

Fig.5 Modulation level of gyrotron output $\Delta P_{out} / P^0_{out}$ as functions of the modulation level of anode voltage $\Delta V_a / V_{a0}$, with the beam current I_b as a parameter. Gyrotron operating conditions: V_k=-30kV. Gyrotron frequency f =444 GHz at the second harmonic (corresponding cavity mode is TE161 mode). Modulating signal: f_m=5 kHz, sinusoidal wave.

Fig.6 shows the efficiency for amplitude modulation $(\Delta P_{out} / P^0_{out})/(\Delta V_a / V_{a0})$ as a function of a beam current I_b. When I_b is small, the output power P^0_{out} is low and the efficiency is fairly large. With the beam current I_b increased, P^0_{out} becomes higher and the efficiency goes down and tends towards its minimum value. The behavior can be expected from Eq.(14). The expected minimum value is equal to $3\alpha^2 + 4$. This corresponds to the situation where the term $\Delta P^0_{out} / P^0_{out}$ can be neglected. This value is indicated by the broken line in Fig.6. Fig.7 and Fig.8 are similar to Figs. 5 and 6 but show results for the fundamental operation in the TE431 cavity mode (frequency is 444 GHz). Amplitude modulation is perhaps best in the low beam current region, because the output power is so low that a small modulation level has a large effect on $\Delta P_{out} / P^0_{out}$.

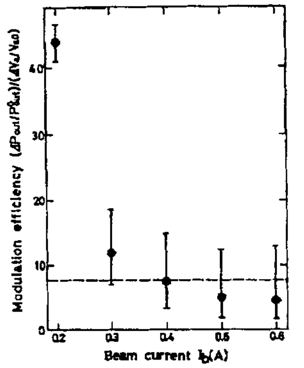

Fig.6 Modulation efficiency $(\Delta P_{out} / P^0_{out})/(\Delta V_a / V_{a0})$ as a function of beam current I_b All operation conditions are the same as those in Fig.7. Gyrotron frequency f =444 GHz at the second harmonic (corresponding cavity mode is TE161 mode). Modulating signal: f_m=5 kHz, sinusoidal wave.

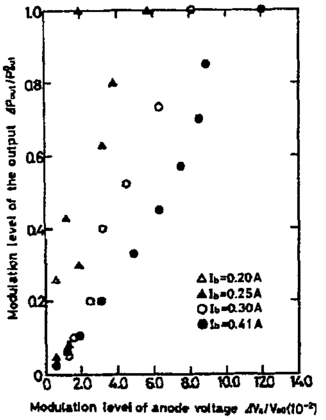

Fig.7 Modulation level of gyrotron output $\Delta P_{out} / P^0_{out}$ as functions of the modulation level of anode voltage $\Delta V_a / V_{a0}$, for different beam current I_b. Gyrotron operating conditions: V_k=-30kV. Gyrotron frequency f =314 GHz at the fundamental (corresponding cavity mode is TE431 mode). Modulating signal: f_m=5 kHz, sinusoidal wave.

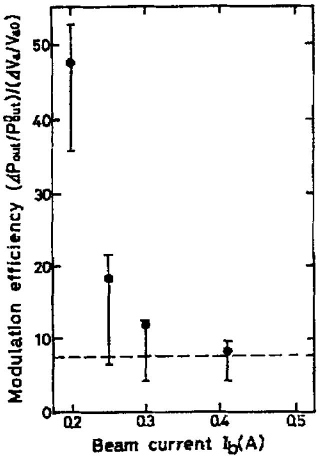

Fig.8 Modulation efficiency $(\Delta P_{out} / P^{u}_{out})/(\Delta V_a / V_{a0})$ as a function of beam current I_b. All operation conditions are the same as those in Fig.5.

4.5 *High frequency switch on and off of gyrotron output*

Figs. 5 and 7 show that, in many cases, we can attain one hundred percent amplitude modulation with a modulation level of the anode voltage of only 12%. This suggests that the gyrotron output may be switch on and off at the square wave modulation of the anode voltage.

Modulation of the gyrotron output like this would be useful for studies of relaxation phenomena in plasma and other materials and studies of the propagation of electron cyclotron waves in tokamak plasmas. Fig.9 shows a typical result of high-frequency switching by a 5 kHz square wave modulation of anode voltage. The upper trace shows the high voltage pulse applied to the anode and the lower trace shows the output power of the gyrotron. The modulation level $\Delta V_a / V_{a0}$ of anode voltage in this case is 9.1 $\times 10^{-3}$. Under these conditions the output power of the gyrotron is switched on and off completely. Switching experiments in the higher frequency range are now under way.

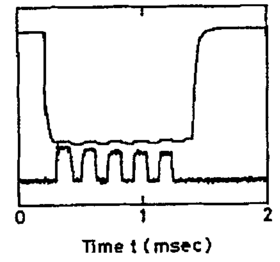

Fig.9 (upper trace) Voltage pulse applied of anode of gyrotron and (lower trace) modulated output of gyrotron. Gyrotron operation conditions: V_k=-30kV, V_{a0}=7.7kV and I_b=165 mA. Gyrotron frequency f =444 GHz at the second harmonic (corresponding cavity mode is TE_{161} mode). Modulating signal: f_m =5 kHz, sinusoidal wave.

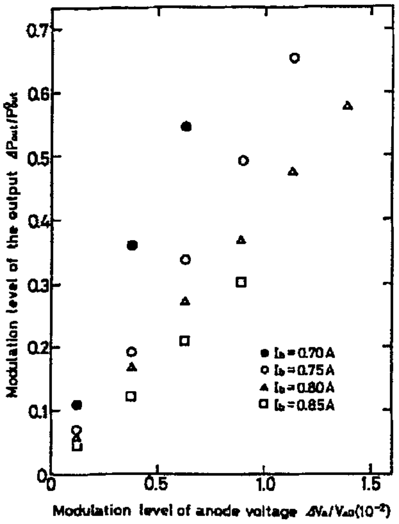

Fig.10 Results of numerical calculation of the modulation level of the gyrotron output $\Delta P_{out} / P^0_{out}$ as a function of the modulation level of anode voltage $\Delta V_a / V_{a0}$, for different beam currents I_b. Gyrotron operation condition: V_k=-30kV. Gyrotron frequency f=444 GHz at the second harmonic. Corresponding cavity mode is TE161 mode. Injection point of electron beam R_{inj}=0.50 mm.

5. Numerical calculation

In order to obtain a more complete analysis of the amplitude modulation of gyrotron output we have included the spread of electron velocities and the interaction between the electron beam and the high-frequency field. Since the time of flight of electrons from the gun region to the cavity region is much shorter than one period of the anode voltage modulation, we can treat the problem as a steady-state one. The calculation have been carried out for the same conditions as Fig.5. The results are shown in Fig.10. There is excellent qualitative agreement between the computed results in Fig.10 and the experimental results in Fig.5. The beam current used in the simulation 3 or 4 times higher than that in the experiment. This is because a less ideal injection point R_{inj} of the electron beam and magnetic field intensity B were chosen.

6. Conclusion

Amplitude modulation of the gyrotron output due to the modulation of the anode voltage has been studied experimentally and the results are found to be in good agreement with a simple linear theory and a numerical calculation. The main features of the results are as follows:

1. $\Delta P_{out} / P^0_{out}$ is proportional to $\Delta V_a / V_{a0}$ for low modulation levels ($\Delta V_a / \Delta V_{a0} \leq 2 \times 10^{-2}$).

2. Almost sinusoidal modulation is possible for low modulation levels ($\Delta V_a / \Delta V_{a0} \leq 2 \times 10^{-2}$).
3. One hundred percent modulation of gyrotron output or switching has been obtained with less than 10 percent modulation of the anode voltage.
4. High frequency modulation up to 600 kHz has been achieved.
5. The modulation efficiency defined as $(\Delta P_{out} / P^0_{out})/(\Delta V_a / V_{a0})$ decreased with increased output power P^0_{out}.

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