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Seismic reliability analysis of large electric power systems

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Abstract: Based on the De.Morgan laws and Boolean simplification, a recursive decomposition method is introduced in this paper to identify the main exclusive safe paths and failed paths of a network. The reliability or the reliability bound of a network can be conveniently expressed as the summation of the joint probabilities of these paths. Under the multivariate normal distribution assumption, a conditioned reliability index method is developed to evaluate joint probabilities of various exclusive safe paths and failed paths, and, finally, the seismic reliability or the reliability bound of an electric power system. Examples given in the paper show that the method is very simple and provides accurate results in the seismic reliability analysis.

Keywords: seismic reliability; electric power system; multivariate normal distribution; conditioned fractile index; correlation coefficient; joint probability

1 Introduction

Electric power systems can be represented by network graphs with a weight assigned to each edge or node. In seismic reliability analysis, the weight denotes the seismic reliability of transmission lines (network edges) or power stations and substations (network nodes). In general, the failure of transmission lines can be neglected in the seismic reliability analysis of electric power systems, thus electric power systems can be basically regarded as a node weighted network, i.e. only nodes are assigned weights.

In the seismic reliability analysis of a large electric power system, we should generate all exclusive safe paths, and then sum up all the joint probabilities of the exclusive safe paths to get the seismic reliability. Thus, there are two steps in the seismic reliability analysis of a large electric power system, i.e., (1) generate the exclusive safe paths, and (2) calculate the joint probabilities of these paths.

In early algorithmic approaches to generating exclusive safe paths, the first step was to enumerate all min-paths, and then to create exclusive safe paths, as described by Aggarwal (1978), and Arunkumar and Lee (1979). It is well-known that the safe paths must be non-polynomial in the size of the system, thus those approaches are too complex for practical applications. In order to reduce the computational complexity, various approaches have been proposed to cope with the problem. In 1988, Yoo and Deo (1988) compared four current approaches and concluded that the min-paths disjoint idea may be the best in terms of redcuing this computational complexity. In 2002, Li and He (2002) proposed a recursive decomposition method to directly identify the main exclusive safe paths and failed paths of a network. By using the recursive decomposition method, the reliability or the reliability bound of a network can be formulated.

Having generated the exclusive safe paths, the next step in seismic reliability analysis of an electric power system is to calculate the joint probabilities of these exclusive safe paths. This step often involves the integration of multivariate normal distribution. An approximate method for the integration of multivariate normal distribution proposed by Terada and Takahashi (1988) and Pandey (1998) can be used to evaluate the joint probabilities of series systems. However, the conditional fractiles defined in the above studies must be developed to evaluate the joint probabilities of exclusive safe paths in the seismic reliability analysis of an electric power system, which will be described in next section.

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2 The recursive decomposition method

For a network with m components, its structure function, $\Psi(S)$, is defined as

$$\Psi(S)=\bigcup_{\zeta=1}^{k} L_{\zeta} \tag{1}$$

where L_ζ is the ζ-th min-path of the network, and K is the number of min-paths of the network.

According to absorption laws and the inclusion-exclusion principle, the structure function can be transformed into

$$\Psi(S) = L_1 \bigcup (\bigcup_{\zeta=1}^{K} L_\zeta) = L_1 + \bar{L}_1 \cdot \Psi(S) \tag{2}$$

where $L_1 = e_1 e_2 ... e_i$, $i \le m$ is any one min-path of the network searched by the BFS (Broad First Search) method, e_j, $j = 1, \cdots, i$ is the event that component j is operating (for a electric power system, component j denotes one power station or one substation), and $\bar{L}_1$ is the event that causes L_1 to fail.

From absorption laws and De.Morgan's law $\bar{L}_1$ can be expressed as

$$\bar{L}_1 = \bar{e}_1 + e_1 \bar{e}_2 + \cdots + e_1 e_2 \cdots \bar{e}_i \tag{3}$$

where $\bar{e}_j$, $j = 1, \cdots, i$ is the event in which j-th component fails.

After substituting Eq. (3) into Eq. (2) and applying Boolean simplification, Eq. (2) becomes

$$\Psi(S) = L_1 + \bar{e}_1 \cdot \Psi(S_{-e_1}) + e_1 \bar{e}_2 \cdot \Psi(S_{-e_2}) + \cdots + e_1 e_2 \cdots \bar{e}_i \cdot \Psi(S_{-e_i}) \tag{4}$$

where S_{-e_j}, $j = 1, \cdots, i$ is a subsystem excluding the component e_j from the original network and $\Psi(S_{-e_j})$ is the structure function of a subsystem S_{-e_j}, which is a union of all min-paths in the subsystem.

The next step is to iteratively decompose Eq.(4) following the above process, and apply Boolean simplification to the obtained structure function expression until no connected subsystem remains in the system. The structure function of the network can be given by

$$\Psi(S) = L_1 + \sum_{u=2}^{N} S_u = \sum_{u=1}^{N} S_u \tag{5}$$

where S_u is u-th exclusive safe path of the network, N is the number of exclusive safe paths of the network, and $S_1 = L_1$.

During the decomposition process, we can also obtain exclusive failed paths leading to network failure. Summing up these exclusive failed paths, the complement of $\Psi(S)$ (system failure status) can be written as

$$1 - \Psi(S) = \sum_{v=1}^{M} F_v \tag{6}$$

where 1 is the space of system status, F_v is the v-th exclusive failed path of the network, and M is the number of exclusive failed paths of the network.

Therefore, the reliability and failure probability of the network can be expressed as, respectively

$$p_r\{S\} = p_r\{\Psi(S)\} = \sum_{u=1}^{N} p_r\{S_u\} \tag{7}$$

$$p_f\{S\} = p_r\{1 - \Psi(S)\} = \sum_{v=1}^{M} p_r\{F_v\} \tag{8}$$

where $p_r\{\cdot\}$ and $p_f\{\cdot\}$ denote the reliability and the failure probability of a stochastic event, respectively.

For a large system, we cannot accumulate all exclusive safe paths or failed paths. Therefore, the reliability bound can be expressed as

$$\sum_{u=1}^{N'} p_r\{S_u\} \le p_r\{S\} \le 1 - \sum_{v=1}^{M'} p_r\{F_v\} \tag{9}$$

where N' and M' are the numbers of accumulated exclusive safe paths and the failed paths, respectively, and in general, $N' < N$, $M' < M$.

The diagram expression of the recursive decomposition method can be illustrated as follows. Figure 1 is a node weight network system with the source 1 and the terminal 4, and Fig. 2 shows the reliability analysis based on the algorithm.

According to Fig. 2, there are two exclusive safe paths and four failed paths, thus Eqs. (7) and (8) can be expressed as, respectively

$$p_r\{S\} = \sum_{u=1}^{2} p_r\{S_u\} = p_r\{124\} + p_r\{1\bar{2}34\} \tag{10}$$

$$p_f\{S\} = \sum_{v=1}^{4} p_r\{F_v\} = p_r\{\bar{1}\} + p_r\{12\bar{4}\} + p_r\{1\bar{2}\bar{3}\} + p_r\{1\bar{2}3\bar{4}\} \tag{11}$$

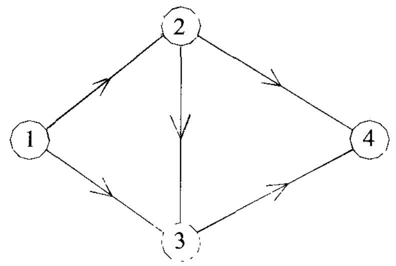

Fig. 1 A five components network

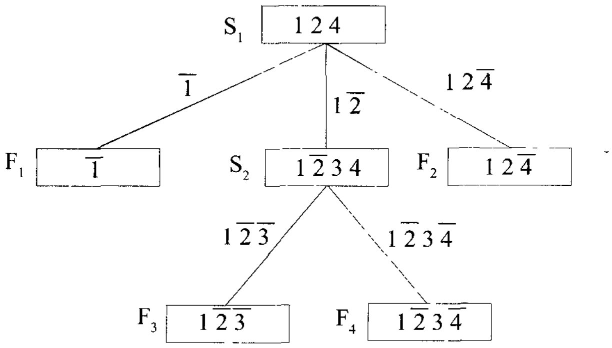

Fig. 2 Diagram scheme of the reliability analysis

3 Joint probabilities of exclusive safe paths or failed paths

From Fig. 2, we can see that there are three kinds of exclusive safe paths and failed paths, i.e., 1) a group in which all components are operating, 2) a group in which all components failed, and 3) a group in which some components are operating and other components failed.

If the failure function of a component can be described using a standard normal variable X_k (if the variable is not a standard normal variable, then some transformations can be used to obtain the standard normal variable as described by Nowak and Collins (2000)), then a general form for various exclusive safe paths and failed paths can be written as $\bigcap_{k=1}^{n}(X_k < c_k)$, where n is the number of components of an exclusive safe or failed path, and c_k is a fractile of the standard normal variable X_k. In order to make $\bigcap_{k=1}^{n}(X_k < c_k)$ to express various exclusive safe and failed paths, the fractile c_k is redefined as:

$$\begin{cases} c_k = \beta_k, & \text{if component } k \text{ is operating} \\ c_k = -\beta_k, & \text{if component } k \text{ is failed} \end{cases} \tag{12}$$

where β_k is the reliability index of the k-th component.

Thus, probability of various exclusive safe paths and failed paths can be generally written as

$$\begin{aligned} & p_r\{\bigcap_{k=1}^{n}(X_k < c_k)\} \\ & = p_r\{(X_n < c_n)\left|\bigcap_{t=1}^{n-1}(X_t < c_t)\right.\}\cdot p_r\{(X_{n-1} < c_{n-1})\left|\bigcap_{s=1}^{n-2}(X_s < c_s)\right.\}\cdots p_r\{X_1 < c_1\} \end{aligned} \tag{13}$$

where $p_r\{(X_n < c_n)\left|\bigcap_{t=1}^{n-1}(X_t < c_t)\right.\}$ denotes the probability of $X_n < c_n$ given $\bigcap_{t=1}^{n-1} X_t < c_t$, and c_t can be defined by Eq. (12).

According to Terada and Takahashi (1988), Eq. (13) can be approximated to

$$p_r\{\bigcap_{k=1}^{n}(X_k < c_k)\} \approx \Phi(c_1)\cdot\prod_{q=2}^{n}\Phi(c_{q|q-1}) \tag{14}$$

where $\Phi(.)$ is the cumulative distribution function of a standard normal variable, $c_{q|q-1}$ denotes the conditioned fractile of the standard normal variable X_q when the event $\bigcap_{p=1}^{q-1}(X_p < c_p)$ has occurred, which can be generally recursively derived as follows (Terada and Takahashi, 1988; Pandey, 1998):

$$c_{q|h} = \frac{c_{q|h-1} + \rho_{q,h|h-1}A_{h|h-1}}{\sqrt{1-\rho_{q,h|h-1}B_{h|h-1}}} \tag{15}$$

in which

$$A_{h|h-1} = \phi(c_{h|h-1})[\Phi(c_{h|h-1})]^{-1} \tag{16}$$

$$B_{h|h-1} = A_{h|h-1}(c_{h|h-1} + A_{h|h-1}) \tag{17}$$

where $\phi(\cdot)$ is the probability density function of a standard normal variable.

In Eq. (15) $\rho_{q,h|h-1}$ is the conditioned correlation coefficient between the event $X_q<c_q$ and $X_h<c_h$ when event $\bigcap_{w=1}^{h-1}(X_w < c_w)$ has occurred, which can be recursively derived from the following general formula

$$\rho_{q,h|o} = \frac{\rho_{q,h|o-1} - \rho_{q,o|o-1}\rho_{h,o|q-1}B_{o|o-1}}{\sqrt{1-\rho_{q,o|o-1}^2 B_{o|o-1}}\sqrt{1-\rho_{h,o|o-1}^2 B_{o|o-1}}} \tag{18}$$

in which $o<h$ and $o<q$.

Similarly, in order to make Eq. (18) suitable to various exclusive safe paths and failed paths, we define the initial value (in this case, o equals 1)

$$\begin{cases} \rho_{q,h} = \gamma_{q,h}, & \text{if component } q \text{ and component } h \text{ are all operating or failed} \\ \rho_{q,h} = -\gamma_{q,h}, & \text{if component } q \text{ is operating and component } h \text{ is failed, or reverse} \end{cases} \tag{19}$$

where $\gamma_{q,h}$ is the correlation coefficient between X_q and X_h.

Substituting Eqs. (15) and (18) into Eq. (14), the joint probabilities of responding exclusive safe paths and failed paths can be evaluated, and finally, the seismic reliability of an electric power system can be evaluated. For a large system, Eq. (9) can be used to obtain the seismic reliability bound of the system.

4 Examples

An Intel Pentium II personal computer with storage memory of 64MB was used to execute the algorithm in the following examples.

4.1 Example 1

The network shown in Fig. 1 is used in this example. The reliability indexes of all nodes are 2.8 and the correlation coefficients between any two performance function variables of all responding nodes are 0.8.

Because the system is so simple, in that there are only two recursive safe paths and four recursive failed paths, respectively, the accurate system reliability and failure probability can be computed as $p_r(S)$=0.9942439 and $p_f(S)$=0.0057561.

4.2 Example 2

The second example considers a series system with 100 components, which has been analyzed by Pandey (1998). The components have equal reliability indices of β=2.883 and failure correlation coefficients of $\gamma_{q,h}$=0.6909, $q \neq h$.

Table 1 System failure probability calculated by various methods

Methods	Failure probability
Monte Carlo Simulation	3.242×10^{-2}
Ditlevsen's Narrow Bound	$(1.969\times10^{-3},\ 6.217\times10^{-2})$
Rackwitz's Method	1.818×10^{-3}
FORMS	2.572×10^{-2}
Present paper	3.156×10^{-2}

The system failure probability calculated using existing methods (Pandey, 1998) and the method presented in this paper are listed in Table 1. Table 1 reveals that the results calculated using the method presented in this paper is in closer agreement with the numerical results estimated by Monte Carlo simulation than Rackwitz's Method and FORMS. It should be also

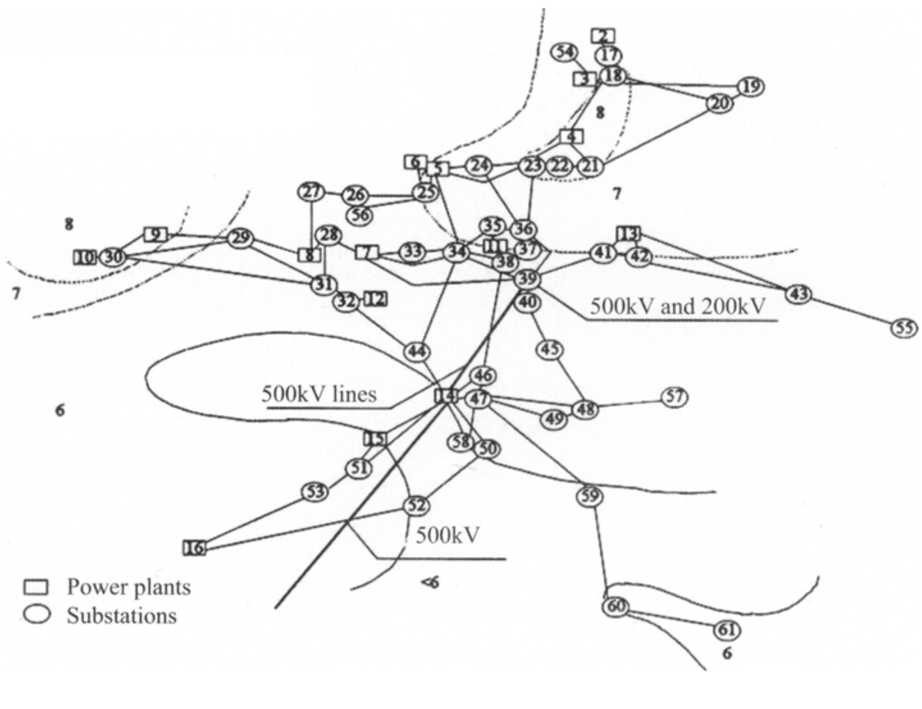

Fig. 3 Network graph of an electric power system

noted that the results obtained by the method presented in this paper are within Ditlevsen's narrow bounds.

4.3 Example 3

Figure 3 shows an actual large electric power system with 60 nodes (power stations and substations) and 128 edges (transmission lines). Among these nodes, all nodes from node 2 to node 16 represent 15 power stations and other nodes denote substations. Through seismic reliability analysis of power stations and substations under a given earthquake excitation, the seismic reliabilities of all nodes approximate to 2.75 and the correlation coefficients between any two performance function variables of all responding nodes approximate to 0.67.

Because the electric power system under consideration is so large, there are too many exclusive safe and failed paths to evaluate individually. Therefore, the reliability bound expressed by Eq. (9) should be used to analyze the seismic reliability of this electric power system. For example, seismic reliability bound from node 4 (the most important power station of the electric power system) to node 47 (a central substation of the region) can be evaluated with the computer result [0.9850227, 0.9853205]. This means that as a network, the seismic reliability of the electric power system (from the power station 4 to the substation 47) is approximately 0.9851716 with an upper bound of 0.9850227 and a lower bound of 0.9853205.

5 Conclusion

This paper first introduces the recursive decomposition method for identifying exclusive safe and failed paths of a network. Secondly, through redefining the conditional fractiles, the approximate method for joint probability of series systems proposed by Terada and Takahashi (1988) and Pandey (1998) is developed to calculate the joint probabilities of exclusive safe and failed paths. Finally, the seismic reliability or the seismic reliability bound of an electric power system is obtained. It can be seen that the method is exact and very simple for seismic reliability analysis, as shown in the examples.

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