A Note on Some Methods Suitable for Verifying and Correcting the Prediction of Climatic Anomaly

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ABSTRACT

The weighted correlation coefficient of the predicted and observed anomalies and the ratio of the weighted norm of predicted anomaly to the observed one, both together, are suggested to be suitable for the estimating of the correctness of climate prediction. The former shows the similarity of the two patterns, and the later indicates the correctness of the predicted intensity of the anomaly. The weighting function can be different for different emphasis, for example, a constant weight means that the correlation coefficient is the conventional one, but some non–uniform weight leads to the ratio of correct sign of the anomaly, the stepwise weight leads to the formulation of correctness of prediction represented by grades of the anomaly, and so on.

Three methods for making correction to the prediction are given in this paper. After subtracting the mean error of the prediction, one method is developed for maximizing the similarity between the predicted and observed patterns, based on the transformation of the spatial coordinates. Another method is to minimize the mean difference between the two fields. This method can also be simplified as similar to the "optimum interpolation" in the objective analysis of weather chart. The third method is based on the expansion of the anomaly into series of EOF, where the coefficients are the predicted but the EOFs are taken as the "observed" calculated from historical samples.

Key words: Weighted correlation, Anomaly, Prediction, Rainfall

I. GENERAL STATISTICAL CONSIDERATIONS

In the verification of the correctness of climate prediction, one has two charts, the predicted and the observed. Similar to the weather prediction, the conventional method is to calculate the correlation coefficient of these two fields, the standard deviation, and maybe some other statistical characteristics. However, in the climatic prediction, not only the pattern but also the intensity of the anomaly are both important. The "intensity" should be somehow defined.

Let $A(x)$ and $B(x)$ be two functions of coordinate set x, and $x \in S$ (a domain in one–, two– or three–dimensional space), as well–known the correlation coefficient r and "normalized deviation" σ are defined as follows:

$$r = \frac{(A,B)}{\|A\| \cdot \|B\|} \quad , \tag{1}$$

$$\sigma = \frac{\|A - B\|}{\|B\|} \quad , \tag{2}$$

where (A,B) is the inner production, $\|A\|$ is the norm of A and so on $\|A\|^2 \equiv (A,A)$; and $\|B\|$ is taken as the factor of normalization in σ. For simplicity, hereafter we take a two–dimensional case, hence

$$(A,B) \equiv \iint_S A(x)B(x)dS \ , \tag{3}$$

where $x \equiv (x_1, x_2)$. We have the following formula

$$\sigma^2 = 1 - 2rs + s^2 \ , \tag{4}$$

where s is the "normalized intensity", given by

$$s \equiv \|A\| / \|B\| \ . \tag{5}$$

For Eq.(4), one can choose a pair of (r,σ) or (r,s) to verify the correctness of the represention of $B(x)$ by $A(x)$. The correctness is perfect, that is to say A and B are identical if and only if ($r = 1$ and $\sigma = 0$) or ($r = 1$ and $s = 1$). Note, $-1 \leqslant r \leqslant 1$, the larger r means the better similarity; but the larger σ or $|1 - s|$ indicates the worse representation of intensity and some other characteristics.

In numerical weather prediction the pair (r,σ) is commonly used, (where the factor of normalization may not be the $\|B\|$, but the climatological mean variability). However, in the prediction of climatic anomaly, it is better to adopt the pair (r,s), where r indicates the similarity of these two patterns, and s is the ratio of predicted mean squared intensity to the observed one. s is a very clear index: $s > 1$ means that A is more intensive, and $s < 1$ means that A is less intensive than the observed. This is just what we are interested in. Therefore it is suggested to apply the pair (r,s) to the verification of the prediction of climatic anomaly. It is worth noting that the pair (r,s) is also superior to (r,σ) in the numerical weather prediction and diagnostic analysis of climate, and that its application to the research on seasonal variations and monsoons has been already done in our previous paper (Zeng and Zhang, 1992).

II. CONVENTIONAL AND WEIGHTED VERIFICATIONS

Let a and b be the predicted and observed anomalies respectively. We have the conventional verification if $A \equiv a$ and $B \equiv b$.

In the verification of climate prediction, due to its difficulty and the practical task of the users, other criteria for the verification are also commonly and even more preferably adopted, such as the skill of correctness of the sign in the prediction of anomalies, the skill of making emphasis on the intensive anomalies, positive and negative, which result in disastrous climatic conditions and cause large damage in the economy. These skills can be precisely formulated by the weighted verifications based on Eqs.(1)–(5).

For example, taking

$$A = aw(a) \ , \qquad B = bw(b) \ , \tag{6}$$

where $w(\bullet)$ is a weighting function, and taking

$$w(c) = \frac{|c|}{c^2} \ , \tag{7}$$

(see Fig. 1), we have the skill of correctness of the sign in the predicted anomalies , i. e.

$$r = \iint_S \mathrm{sign}(a) \bullet \mathrm{sign}(b)dS \Big/ \iint_S dS = \frac{S^+ - S^-}{S^+ + S^-} \ , \tag{8}$$

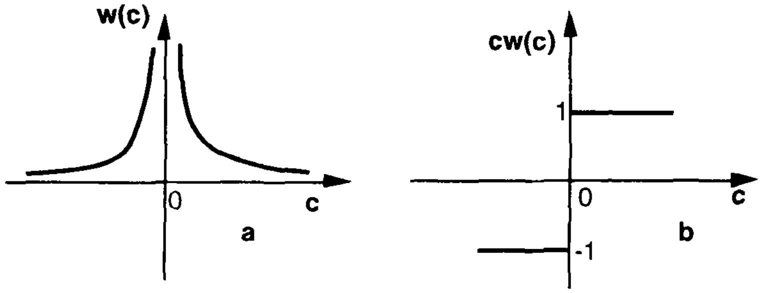

Fig. 1. The weighting function $w(c)=|c|/c^2$, (a); and the associated function of sign $cw=c|c|/c^2$, (b).

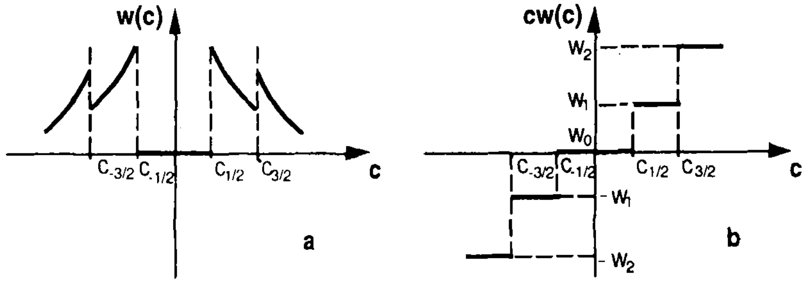

Fig. 2. The weighting function $w(c)=w_j|c|/c^2$, $c_{j-1/2}<c<c_{j+1/2}$, (a); and the associated function of grades $cw=w_j\,\text{sign}(c)$, where $w_{-j}=w_j>0$ and $w_0=0$, (b).

where S^+ and S^- are the areas of correct and incorrect predictions respectively. Another measure of correctness of predicted sign is taken as

$$\rho=\frac{S^+}{S^+ + S^-} \quad . \tag{8$'$}$$

It is clear that $\rho=(1+r)/2$, where r is given by (8).

While taking the function

$$w(c)=w_j\frac{|c|}{c^2} \quad , \quad \text{as} \quad c_{j-1/2}<c<c_{j+1/2} \quad , \tag{9}$$

where $w_j, c_{j\pm1/2}$ are constants (see Fig. 2, where $w(c)$ is a symmetric stepwise function), we have the skill of grade prediction. Especially, we make the emphasis on only the grade c_j and neglect other grades if $w_j=1$ and $w_{j'}=0$ as $j'\neq j$, and it results in the following formula

$$r=r_j=\frac{S_j^+ - S_j^-}{S_j^+ + S_j^-} \quad , \tag{10}$$

where S_j is the area with grade j.

Special emphasis can also be made on the lowest grade with the neglect of the correctness of its predicted sign, because the sign is not important as the absolute value of anomaly is small, (see Fig. 3).

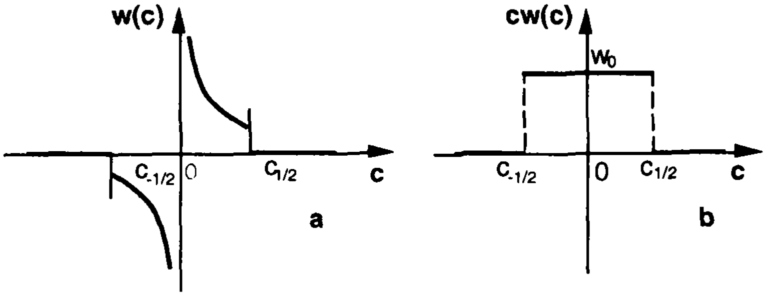

Fig. 3. The same as Fig. 2 but with $w_j = 0$ if $j \neq 0$, and $w(c) = w_0 c^{-1}$ if $c_{-1/2} < c < c_{1/2}$, where $w_0 > 0$ in order to make emphasis on the lowest grade while neglecting its sign.

In general we have the weighted verification given by

$$r = \iint_S aw(a)bw(b)dS \Big/ \iint_S w(a)w(b)dS \ , \tag{11}$$

and

$$s = \iint_S a^2 w^2(a)dS \Big/ \iint_S b^2 w^2(b)dS \ , \tag{12}$$

if (6) is taken, and $w(c)$ can be continuous or discontinuous function of c. $w(c)$ is usually an increasing function of $|c|$ due to the fact that the large anomalies (both positive and negative) are more important. Note that $r = 1$ if $aw(a)$ is proportional to $bw(b)$, and $r = s = 1$ only if $aw(a) = bw(b)$.

III. CORRECTION OF PREDICTION—MAXIMUM SIMILARITY

By using proper method for projection one can make two different curves to be similar or even identical. For example, the linear transformation

$$\begin{cases} \xi = \xi_0 + a_{11}x + a_{12}y \\ \eta = \eta_0 + a_{21}x + a_{22}y \end{cases} \tag{13}$$

can transfer a bilinear form into a quadratic one, hence change an ellipse into a circle (Fig. 4).

This idea is useful for introducing suitable correction into the prediction output of climatic anomaly, and improving the prediction because predictions, say, made by GCM, always possess some systematic errors which are characterized by either non–similarity between the predicted and observed patterns or difference in the locations of the predicted and observed centers of anomalies due to the model errors and so on.

Suppose that we have a pair of sample sets $\{a\}$ and $\{b\}$ with ensemble average $\langle a \rangle$ and $\langle b \rangle$ respectiviely, and the mean systematic error of prediction $\langle a \rangle - \langle b \rangle$ as well. Subtracting this mean error, let

$$\begin{cases} a' = a - (\langle a \rangle - \langle b \rangle) \ , \\ b' = b - \langle b \rangle \ , \end{cases} \tag{14}$$

we still have other systematic errors in a' due to the non–similarity of patterns a' and b' and so on. Let

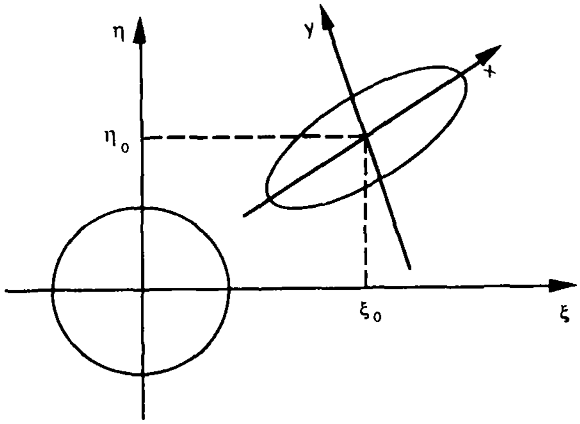

Fig. 4. The sketch-map of the linear transformation ($\xi = \xi_0 + a_{11}x + a_{12}y$ and $\eta = \eta_0 + a_{21}x + a_{22}y$).

$$A \equiv a'w(a') \ , \quad B \equiv b'w(b') \ , \tag{15}$$

and (ξ,η) is given by (13), where (ξ_0,η_0) and the coefficients $\{a_{ij}\}$ are under determination. We require that the transformation leads to a maximum similarity statistically, i.e.

$$\left\langle \frac{\iint_S A(\xi,\eta)B(x,y)dS}{\left(\iint_S A^2(\xi,\eta)dS \cdot \iint_S B^2(x,y)dS\right)^{1/2}} \right\rangle = \max \ . \tag{16}$$

For convenience of computation (16) can be replaced by a weaker one, i.e.,

$$\frac{\left\langle \left(\iint_S A(\xi,\eta)B(x,y)dS\right)^2 \right\rangle}{\left\langle \left(\iint_S A^2(\xi,\eta)dS \cdot \iint_S B^2(x,y)dS\right)^{1/2} \right\rangle} = \max \ . \tag{17}$$

Providing the correction is not large, we can take a truncated Taylor series

$$A(\xi,\eta) \approx A_1 \equiv A(x,y) + \frac{\partial A}{\partial x}[\xi_0 + (a_{11} - 1)x + a_{12}y] + + \frac{\partial A}{\partial y}[\eta_0 + a_{21}x + (a_{22} - 1)y] \ . \tag{18}$$

Substituting (18) into (17), the problem becomes a typical and classical transformation of bilinear forms into quadratics. A successive iterative method must be used if it is not restricted by the linearization (18).

If (16) is adopted, one always has to apply the iteration even though (18) is also taken into account.

After (ξ_0,η_0) and $\{a_{ij}\}$ are obtained, the corrected prediction $a'(\xi,\eta)$ is adopted to replace the old one, $a'(x,y)$. However, this method cannot improve the predicted intensity. For that we need additional correction either by taking the corrected anomaly $a^* = \mu a'(\xi,\eta)$, where

$$\mu^{-2} = \langle \|a'(\xi,\eta)w(a')\|^2 \rangle \; / \; \langle \|b'(x,y)w(b')\|^2 \rangle \;, \tag{19}$$

or by the method of optimum correction (see Section IV).

IV. CORRECTION OF PREDICTION—MINIMUM DIFFERENCE

Similar to the maximum similarity, the method of minimum difference is also suitable for the correction of prediction.

Taking (15), but assuming the corrected prediction is given by

$$a^* = a'(x,y) + a''(x,y,\{c\}) \;, \tag{20}$$

where $\{c\}$ is a set of parameters under determination, we require

$$\langle \|A^* - B\|^2 \rangle = \min \;, \tag{21}$$

where $A^* \equiv a^* w(a^*)$. Therefore, the problem becomes a least square one, either linear or nonlinear.

A general form of a'' is given by

$$a''(x,y,\{c\}) = \iint_S a'(x',y')K(x,y;x',y',\{c\})dx'dy' \;, \tag{22}$$

where $K(x,y;x',y',\{c\})$ is the kernel function. For convenience of computation K is usually localized. For example, in the case of a grid function $a'(x_i,y_j) \equiv a'_{ij}$, a^*_{ij}, is given by some linear combination of values at the surrounding grids, i.e.

$$a^*_{ij} = \sum_{-N \leqslant i',j' \leqslant N} c_{i+i',j+j'} a'_{i+i',j+j'} \;, \tag{23}$$

where N is equal to 1 or 2 in the practices. Moreover one can also simplify (24) by taking

$$c_{i+i',j+j'} = c_{i',j'} \;. \tag{24}$$

Substituting (23) or (24) into (21), the coefficients $c_{i+i',j+j'}$ are easily determined. Especially, (24) is similar to that adopted in objective analysis of weather map, and the associated method is known as the optimum interpolation.

Note that the correction will be more effective if a' in (23) is calculated according to $a'(\xi,\eta)$ given in Section III.

V. CORRECTION OF PREDICTION—APPLICATION OF EOFs

Suppose that there are two sets of normalized EOFs obtained from the sample sets $\{a'\}$ and $\{b'\}$ and denoted as $\{X_a(x,y)\}$ and $\{X_b(x,y)\}$, respectively. Expanding a' into a series of EOFs, we have

$$a'(x,y) = \sum_k c_k X_{ak}(x,y) \;. \tag{25}$$

The error of prediction, $a' - b'$, consists of two parts, the error of predicted coefficients c_k and the error of EOFs, i.e. $X_{ak} - X_{bk}$.

Suppose that the difference $X_{ak} - X_{bk}$ is not large, it is beneficial to replace X_{ak} by X_{bk} in order to avoid the differences between the predicted and observed patterns. Therefore, the corrected prediction can be taken as

$$a^*(x,y)=\sum_k c_k X_{bk}(x,y) \ . \tag{26}$$

As a measure of the error of corrected predictions, we have

$$\langle \|a^* - b'\|^2 \rangle = \left\langle \sum_k (c_k - e_k)^2 \right\rangle \ , \tag{27}$$

where e_k is the expansion coefficient of b', i.e.,

$$b'(x,y)=\sum_k e_k X_{bk}(x,y) \ . \tag{27$'$}$$

Taking $A = a^*$, $B = b'$, for the sample set we have following simple formulas

$$r=\left\langle \sum_k c_k e_k \Big/ \left(\sum_k c_k^2 \cdot \sum_k e_k^2\right)^{1/2} \right\rangle \ , \tag{28}$$

$$s=\left\langle \sum_k c_k^2 \Big/ \sum_k e_k^2 \right\rangle \ . \tag{29}$$

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