# Kinetics of Strain-Induced fcc→hcp Martensitic Transformation

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The kinetics of the strain-induced  $\gamma$  (fcc)  $\rightarrow \epsilon$  (hcp) transformation *i.e.* the amount of phase transformation *vs* applied strain were determined by density measurements at various temperatures. The transformation curve has a sigmoidal shape and approaches saturation below 100 pct transformation. Assuming that  $\epsilon$ -platelets form from stacking faults, the volume fraction can be expressed as an implicit function of strain. The saturation value is constant and can be evaluated from quantitative metallography. The approach to saturation is determined by only one temperature-dependent parameter related to the stacking fault energy. Good agreement with experimental results was obtained. The model was also applied to transformation kinetics after a prestrain inducing both slip and twinning. The prestrain stabilizes austenite with respect to the strain-induced transformation through a block-refining of austenite by the substructure. In addition the nucleation is enhanced through the introduction of stacking faults. This effect vanishes at high applied strains but causes the shape of the transformation curve to become parabolic. It is concluded that decreasing the size of the  $\epsilon$  platelets provides a simple means for reducing the temperature dependence of the transformation kinetics.

MANY experimental studies were devoted to the kinetics of the strain-induced  $\gamma(fcc) \rightarrow \alpha'(bcc)$  martensitic transformation in stainless steels since their yield strength has been shown to increase with the amount of phase transformed.<sup>1,2</sup> In the asquenched fcc alloys the volume fraction of bcc martensite is a sigmoidal function of strain at a given temperature.<sup>1,2</sup> A number of empirical relations were proposed to account for these results.<sup>1,3,4</sup> Recently Olson and Cohen<sup>5</sup> gave a theoretical equation which agrees with Angel's data at different temperatures between  $M_s$  (temperature for spontaneous) transformation) and  $M_d$  (temperature for strain-induced transformation). The fundamental basis of their model is that nucleation of  $\alpha'$  laths occurs only at shear-bands intersection, *i.e.* twins or  $\epsilon$  (hcp) martensite platelets as shown convincingly by transmission electron microscopy observations.<sup>2</sup>

On the other hand the kinetics of the  $\gamma(\text{fcc}) \rightarrow \epsilon(\text{hcp})$ strain-induced martensitic transformation are still poorly documented. Recent work by Sipos *et al*<sup>6</sup> was carried out on a high-manganese steel at only one temperature. First it was shown that the amount of  $\epsilon$ martensite *vs* plastic strain exhibits a sigmoidal shape in the as-quenched condition. The alloy was also studied at the same temperature in the prestrained condition; the prestrain was carried out at a temperature where deformation proceeds mainly by twinning. After prestraining the  $\epsilon$  martensite volume fraction becomes a parabolic function of strain.

The present study is concerned with the kinetics of strain-induced fcc  $\rightarrow$  hcp martensitic transformation for the alloy studied by Sipos *et al*<sup>6</sup> over a wide temperature range between  $E_s$ -temperature for spontaneous transformation-and  $E_d$ -temperature for strain-induced transformation-in the as-quenched condition. The influence of the fraction transformed on the mean

 $\epsilon$  platelet size was also studied by quantitative metallography, since it is a crucial point in any nucleation model as shown for the spontaneous  $\gamma \rightarrow \alpha'$  martensitic transformation.<sup>7</sup> The kinetic measurements are discussed in terms of a simple nucleation model for the as-quenched condition. Then this model is used to explain the results of Sipos<sup>6</sup> in the prestrained condition.

# EXPERIMENTAL PROCEDURE

The composition of the alloy used in this study is Fe-26 pct Mn-0.2 pct C (wt pct). Tensile specimens were solution-treated at 1323 K for 30 min and then air-cooled, which gives a grain size about 100  $\mu$ m. Spontaneous  $\gamma \rightarrow \epsilon$  transformation occurs on cooling at  $E_s \simeq 120$  K (Ref. 8). The  $\gamma \rightarrow \epsilon$  transformation was strain-induced by different amounts of deformation at temperatures between  $E_s$  and  $E_d \simeq 328$  K (Ref. 8). The transformation kinetics were investigated by means of density measurements accurate to 3 Kg/m<sup>3</sup>. The  $\epsilon$  martensite volume fraction f was estimated from the density using lattice parameter measurements of Sipos (6;  $a_{\gamma}$  = 0.3605 nm;  $a_{\epsilon}$ = 0.2541 nm and  $c_{\epsilon}$  = 0.4099 nm) with an accuracy of  $\pm 3$  pct. The experimental error on lattice parameters give a systematical error  $\Delta f/f = 6$  pct.

The average volume of the  $\epsilon$  plates was determined by quantitative metallography. Optical micrographs (magnified 380 times) were taken on the cross-section of tensile specimens after polishing and etching with "Bupac" electrolyte. An example of the  $\epsilon$  martensite structure is shown in Fig. 1.

Fullman<sup>9</sup> has shown for thin circular disks having any distribution of radii and thicknesses that the number of plates per unit volume  $N_V$  was related to the number of plates per unit area  $N_A$  in any plane by:

$$N_V = \frac{8}{\pi^2} \overline{E} N_A$$
 [1]

where  $\overline{E}$  is the average of inverse length of plates.

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Fig.  $1-\epsilon$  martensite in the alloy strained 17 pct at 293 K (f = 0.46 on the entire sample). The marker refers to 50  $\mu$ m.

The average plate volume  $\overline{V}$  for a volume fraction of martensite f is:

$$\overline{V} = \frac{f}{N_V} = \frac{\pi^2}{8} \frac{f}{\overline{E} N_A} \,.$$
<sup>[2]</sup>

Therefore to get  $\overline{V}$  the length of every plate and the number of plates were measured in numerous grains, the area of which was determined by planimetry (2.5 times enlargements were used) and the corresponding volume fraction was measured by density.

#### EXPERIMENTAL RESULTS

## 1) Strain-Induced Transformation Kinetics in the As-Quenched Condition

Results are reported in Fig. 2 for the various temperatures studied. At temperatures below 313 K fracture occurs without necking. Since in these conditions microcracks may appear at large deformations, corrections were made by reaustenitizing these specimens at 1323 K for 10 min and remeasuring the apparent density of austenite. Corrections are only important at 293 K and at 253 K.

The transformation curves in Fig. 2 have a sigmoïdal shape as it was previously shown for the  $\gamma \rightarrow \alpha'$  transformation in stainless steels.<sup>1,2</sup> At low temperatures the transformation seems to saturate at a volume fraction of about 0.6, which is in agreement with X-ray measurements.<sup>8</sup> This level seems lower than that observed for the  $\gamma \rightarrow \alpha'$  transformation in stainless steels of 0.8–0.9 (Ref. 1).

## 2) Quantitative Metallography

Measurements of the average volume of platelets has been carried out on specimens listed in Table I. The influence of the amount of transformation f has been examined between f = 0.1 and f = 0.55. The influence of temperature was examined with specimens deformed at 293 K and at 193 K respectively.

The results reported in Table I are averages calculated from all the measurements (and not average



Fig. 2—Variation of the volume fraction of  $\epsilon$  martensite with applied tensile strain at various temperatures for the Fe-26Mn-0.2C alloy. Experimental data are indicated by points. Solid curves represent the best fit of Eq. [9] for each temperature.

of averages on every area examined). The values of  $\overline{V}$  are smaller than for the  $\gamma \rightarrow \alpha'$  transformation on cooling by at least one order of magnitude.<sup>10,11</sup> The variation of  $\overline{V}$  with the volume fraction is plotted in Fig. 3. A linear decrease of  $\overline{V}$  is observed and seems temperature insensitive.

Thus the average volume of platelets  $\overline{V}$  is related to the volume fraction transformed f by:

$$\overline{V} = V_0 \left( 1 - kf \right) \tag{3}$$

where  $V_0$ , the volume of the first generation of plates, is 3.9  $10^{-16}$  m<sup>3</sup> and k = 1.38. It is to be noted that according to Eq. [3] the volume fraction cannot exceed 1/k = 0.725. As metallographic observations have been carried out up to f = 0.55 and since experimentally observed values of f are lower than 0.63, one may infer the validity of Eq. [3] for any volume fraction.

#### DISCUSSION

A nucleation model is described for the kinetics of the strain-induced fcc  $\rightarrow$  hcp martensitic transformation in the as-quenched condition. Then it is compared to the present experimental results and with those of Sipos in the prestrained condition. At last some guidelines are suggested for the development of high-strength austenitic alloys.

Table I. Average Volume of $\epsilon$ -Platelets Determined by Quantitative Metallography for Different Amounts of $\epsilon$ Martensite						
 Т, К	f	N	$N_A$ , 10 <sup>10</sup> m <sup>-2</sup>	E, 10 <sup>2</sup> m <sup>-1</sup>	$N_{\rm p},$ 10 <sup>16</sup> m <sup>-3</sup>	$\tilde{V}$ , 10 <sup>-16</sup> m <sup>3</sup>
293	0.12	522	0.73	619	0.37	3.2
	0.31	661	2.36	715	1.37	2.3
	0.46	704	3.52	1067	3.05	1.5
193	0.19	252	1.38	599	0.67	2.8
	0.55	446	4.63	1932	7.25	0.75

T = Deformation temperature,

f = volume fraction of  $\epsilon$  martensite,

N = number of platelets observed,

 $N_A$  = number of platelets per unit area,

E = average of inverse platelet lengths,

 $N_V$  = number of platelets per unit volume calculated by Eq [1],

 $\overline{V}$  = average platelet volume calculated by Eq. [2].



Fig. 3-Variation of mean platelet volume with volume fraction of  $\epsilon$  martensite obtained by quantitative metallography at 193 K and at 293 K.

#### 1) Model for the Strain-Induced fcc $\rightarrow$ hcp Martensitic Transformation

The relation between stacking faults and hcp martensite has been emphasized by many authors either for spontaneous or strain-induced transformation (see e.g. Ref. 12 for stainless-steels and Ref. 13 on cobalt). The Shockley transformation dislocations were recently identified by electron microscopy on a cobalt-base alloy.<sup>14</sup> Experimental evidence was given by Fujita and Ueda<sup>15</sup> supporting the view that the nucleation of an  $\epsilon$  platelet is achieved by the propagation of stacking faults in neighboring planes.

Therefore one may consider that for a strain increase  $d\epsilon$  the number of plates which form *per unit* volume of austenite is proportional to the number of stacking faults present. The probability that a stacking fault could form an  $\epsilon$  plate for this strain increase  $d\epsilon$  is  $pd\epsilon$ ; p is assumed to be independent of the strain level  $\epsilon$  and to depend only upon the stacking fault energy (the lower the stacking fault energy, the higher the probability  $pd\epsilon$  of a stacking fault forming an  $\epsilon$  plate).

The faults in the austenite are both the  $\rho_0$  faults present before deformation and the  $\rho_{e}$  faults introduced by strain. Thus for a strain increase  $d\epsilon$  the number of martensite platelets per unit volume of specimen increases from  $N_v$  to  $N_v + dN_v$  and one may write:

$$dN_v = (1 - f) \left(\rho_0 + \rho_\epsilon\right) p d\epsilon.$$
<sup>[4]</sup>

The number of faults introduced by straining is assumed to be proportional to the macroscopic strain  $(\rho_{\epsilon} = K\epsilon)$  since it has been shown for the dislocation density in fcc alloys.<sup>16</sup> Accordingly the initial number of faults per unit volume of austenite can conveniently be written  $\rho_0 = K \epsilon_0^*$  with  $\epsilon_0^*$  a fictitious strain. As p, Kthe rate of formation of stacking faults in course of straining is supposed to be a decreasing function of stacking fault energy. Therefore Eq. [4] may be written as:

$$dN_{v} = (1 - f)(\epsilon^{*} + \epsilon)Ad\epsilon$$
<sup>[5]</sup>

where  $A = p \cdot K$  is a decreasing function of stacking fault energy as both p and K.

Let V be the mean volume of the  $dN_v$  platelets which form when the volume fraction of martensite increases from f to f + df, then df is given by:

$$df = V \, dN_V. \tag{6}$$

The volume V of this generation of platelets is related to the average platelet volume  $\overline{V} = f/N_V$  measured by metallography according to:

$$V = \frac{df}{dN_V} = \frac{df}{d(f/\overline{V})} .$$
<sup>[7]</sup>

Combining Eqs. [7] and [3] yields the following result:

$$V = V_0 (1 - kf)^2.$$
 [8]

Eqs. [5] and [8] can be substituted into Eq. [6]. We can further suppose that the number of initial stacking faults is negligible before those introduced by plastic deformation. This is a reasonable assumption for the as-quenched condition except perhaps in the vicinity of  $E_s$  temperature. Therefore setting  $\epsilon_0^* = 0$  the following relation is obtained after integration:

$$\alpha \epsilon^2 = \Phi_k(f)$$

with

$$\Phi_k(f) \equiv \frac{1}{(k-1)^2} \log \frac{1-kf}{1-f} + \frac{k}{k-1} \cdot \frac{f}{1-kf}$$

and

$$\alpha=\frac{AV_0}{2}.$$

Thus the volume fraction of  $\epsilon$  martensite is an implicit function of strain which depends upon a constant k and upon a parameter  $\alpha$ . This parameter is a decreasing function of stacking fault energy; therefore it is temperature dependent through the temperature dependence of the stacking fault energy.

#### 2) Comparison of Model with Experimental **Results** in the As-Quenched Condition

Eq. [9] has been fitted to the data of Fig. 2. The value k = 1.38, determined by quantitative metallography from data at 293 K and 193 K, gives good agreement with the experimental results at all temperatures. The calculated curves do have a sigmoidal shape. Strictly speaking a saturation level of 0.725 is predicted for an infinite strain. However the transformation rate becomes very slow for volume fractions higher than about 0.60 in good agreement with experimental results and then transformation curves are nearly parallel.

The corresponding values of the  $\alpha$  parameter are given in Fig. 4. It decreases linearly with increasing temperature. According to the model  $\alpha$  would decrease with stacking fault energy. Unfortunately there are no experimental data available for this alloy; however recent experiments on a Fe-20Mn-4Cr-0.5C alloy suggest a strong increase of stacking fault energy with temperature near room temperature<sup>17</sup> as previously established for stainless steels.<sup>18,19</sup> This is in close agreement with the variation of  $\alpha$  in Fig. 3.

In spite of fairly good agreement between our sim-



Fig. 4-Temperature dependence of the  $\alpha$  parameter derived from the data of Fig. 1.



Fig. 5-Transformation curves for Sipos' data at 253 K on the Fe-26Mn-0.2C alloy obtained after 0.05 and 0.30 prestrains at 373 K respectively. Experimental data of Sipos are indicated by points. Solid curves represent Eq. [9] for no prestrain and the best fit of Eq. [10] for each prestrain. Dotted curves represent Eq. [10] for each prestrain when nucleation enhancement is neglected.

ple model and the experimental data, errors may arise from the strain value used in Eq. [9] as previously pointed out by Olson and Cohen<sup>5</sup> for the straininduced  $\gamma \rightarrow \alpha'$  transformation in stainless steels. On one hand, the transformation is oriented by the stress and can give a contribution to the macroscopic strain.<sup>20,21</sup> On the other hand hcp martensite is likely more difficult to deform than austenite. Because of its low stacking fault energy (not very different from that of austenite) deformation of hcp martensite would proceed mainly by basal slip as in hexagonal cobalt.22,23 Thus macroscopic strain is, for the most part, the result of phase transformation and austenite deformation. Unfortunately their relative importance is difficult to estimate except at low temperatures where the contribution of the transformation to the overall strain should be dominant at least up to moderate strains.

#### 3) Application of Nucleation Model to the Prestrained Condition

The model derived in the first paragraph can be extended to account for Sipos' experiments<sup>6</sup> which were performed on the same alloy at 253 K after prestrains of 5 pct and 30 pct at 373 K. These prestrains introduce numerous stacking faults and twins, *i.e.* nuclei for the strain-induced  $\gamma \rightarrow \epsilon$  transformation and obstacles to the propagation of  $\epsilon$  platelets. The model extension is given in the Appendix; the role of obstacles was accounted for in a phenomenological way by considering the substructure as equivalent to some volume fraction of platelets. The relation between the amount of  $\epsilon$ -martensite and the plastic strain is somewhat complicated (see Eqs. [A5] and [A6]):

$$\alpha'[\epsilon^2 + 2\epsilon_0^*\epsilon] = \Phi_k^*\left(\frac{f}{1-f_T}\right)$$
[10]

with  $k^* = k(1-f_T) \sqrt{\alpha^1/\alpha}$ .

Therefore the volume fraction of strain-induced martensite after a prestrain  $\epsilon_0$  above  $E_d$  is a function of applied strain which depends upon three parameters:  $(1 - f_T)$  is the volume fraction of untwinned austenite after prestrain;  $\alpha'$  similar to  $\alpha$  in the asquenched condition is a decreasing function of stacking fault energy and the ratio  $\alpha'/\alpha$  is simply the size reduction of the first  $\epsilon$  platelets due to the prestrain;  $\epsilon_0^*$  is a fictitious strain which accounts for the stacking faults introduced by prestrain.

Eq. [10] is fitted to Sipos' data in Fig. 5. The  $\alpha$ parameter obtained in the as-quenched condition is divided by 1.5 and 7 after 5 pct and 30 pct prestrain respectively, which implies the size of the first  $\epsilon$ platelets is reduced in the same ratio. The volume fraction of twins introduced by these prestrains is about 0.05 and 0.25 respectively, which are believed to be realistic values. The  $\epsilon_0^*$  parameter is respectively 2.5 pct ( $\simeq \epsilon_0/2$ ) and 10 pct ( $\simeq \epsilon_0/3$ ). According to the assumption that the number of stacking faults per unit volume of austenite is proportional to the strain, this parameter  $\epsilon_0^*$  must be proportional to prestrain  $\epsilon_0$  and lower than  $\epsilon_0$  (the case  $\epsilon_0^* = \epsilon_0$  would correspond to identical prestrain and final strain temperatures). The observed values agree fairly well with this condition.

The calculated curves do exhibit a parabolic shape instead of a sigmoidal one as in the as-quenched condition and the saturation level decreases with the amount of prestrain. To illustrate the influence of the  $\epsilon_0^*$  parameter, dotted curves have been plotted in Fig. 5 assuming that no stacking fault was introduced by the predeformation (*i.e.*  $\epsilon_0^* = 0$ ). Thus the two competing consequences of predeformation can be so separated from each other: on one hand, growth of  $\epsilon$  platelets is impeded by the substructure, which stabilizes austenite with respect to the  $\gamma \rightarrow \epsilon$  transformation (as shown by the dotted curves); on the other hand nucleation is enhanced by the stacking faults introduced as illustrated by the difference between the solid and dotted curves. For small applied strains the latter effect is very important since the overall kinetics can be enhanced with respect to the as-quenched condition (see e.g. after 0.05 prestrain). At high strains nucleation enhancement becomes of minor importance.

The stabilizing effect which prevails at higher strains, is due to the block-refining of austenite by the substructure associated with the predeformation. This block-refining of austenite causes a size reduction of the first  $\epsilon$ -platelets (in the ratio  $\alpha/\alpha'$ ) as well as altering the growth of all later generation platelets, which lowers the saturation level (in the ratio  $\sqrt{\alpha/\alpha'}$ , see [Eq. 10]). The prestrain substructure consists of dissociated dislocations, faults and numerous twins. The volume fraction of twins is rather high as inferred by the present estimates as well as qualitative observations by optical and electron microscopy.<sup>8</sup> Therefore in order to elucidate this stabilizing effect it is necessary to know how a growing  $\epsilon$  platelet interacts with a twin. Since the  $\epsilon$  platelet growth is accomplished by the propagation of Shockley dislocations on every second close-packed plane of austenite, this problem can be reduced to incorporation of the Shockley dislocations into the obstacle twin. It was shown that the incorporation of twinning partials into the obstacle twin can always be carried out but energetically unfavorable dislocation reactions are involved, which need very important stress concentrations.<sup>24,8</sup> Thus by the same way twins can provide effective barriers to the growth of  $\epsilon$ -platelets. Slip dislocations must also constitute other obstacles since they are known to impede twin growth.<sup>25</sup>

## 4) Application to the Development of High-Strength Austenitic Alloys

This strain-induced fcc  $\rightarrow$  hcp transformation kinetics model can provide some guidelines to design high-strength austenitic alloys. The temperature dependence of the transformation kinetics does not seem to be as critical as for TRIP steels exhibiting the  $\gamma$  $\rightarrow \alpha'$  transformation: there is neither inhomogeneous deformation nor anomalous temperature dependence of the 0.2 pct yield strength associated with stressinduced transformation in alloys which were given an austenite predeformation.<sup>26,27</sup> However premature brittle fracture may occur when the  $\epsilon$ -martensite volume fraction becomes too high both in the prestrained and as-quenched conditions.<sup>8,27,28</sup>

Moreover it was shown for the Fe-26Mn-0.2C alloy that in the as-quenched condition both the yield stress and the  $\epsilon$  volume fraction exhibit a similar variation with strain.<sup>8</sup> In particular a close correspondence was observed between the maximum workhardening rate  $(d\sigma/d\epsilon)$  and the maximum transformation rate  $(df/d\epsilon)$ . In order to obtain high-strength alloys without brittle failure, the transformation must be slow and progressive to yield a high work-hardening rate but limited to moderate amounts of  $\gamma \rightarrow \epsilon$ transformation.

This goal can be obtained with alloys having a low  $\alpha$ parameter. One way is to choose a deformation temperature near  $E_d$ , which may be rather difficult in alloys exhibiting a high temperature dependence of the  $\alpha$ parameter as in the iron-base alloy studied. Another way is to reduce the size  $V_0$  of the first  $\epsilon$ -platelets. Two methods are available to accomplish this: firstly reduction of the austenite grain size through appropriate annealing treatments; and secondly predeformation of the austenite. The latter seems more efficient, since it gives in addition a reduction of the saturation level. This has been clearly demonstrated on a Fe-20Mn-4Cr-0.5C alloy<sup>27</sup> after a 30 pct prestrain at 373 K, which induces abundant twinning and reduces by a half the maximum amount of  $\gamma \rightarrow \epsilon$  transformation produced by tensile strain as evidenced by X-rays measurements. However both grain-size reduction and austenite predeformation have the same beneficial influence: alloys can be used in a rather large temperature range below  $E_d$  since the temperature dependence of  $\alpha$  is reduced with  $V_0$ .

#### CONCLUSIONS

1) The kinetics of the strain-induced fcc  $\rightarrow$  hcp martensitic transformation of a Fe-26Mn-0.2C alloy exhibit a sigmoidal variation in the as-quenched condition as is the case for the fcc  $\rightarrow$  bcc martensitic transformation. The kinetics can be described by a relation which depends upon one parameter and a constant. The parameter ( $\alpha$ ) is a decreasing function of the stacking fault energy and is proportional to the volume of the first  $\epsilon$  platelets. The constant describes the decrease of platelet size when the amount of martensite increases and is determined by quantitative metallography.

2) The above kinetics model can be extended to results previously obtained after a prestraining which induces an abundant twinning. This prestrain has two influences: first the substructure (twins, dislocations) hinders the growth of  $\epsilon$  platelets and gives a block-refining effect which stabilizes austenite with respect to the strain induced martensitic transformation. Secondly stacking faults provide nuclei for the transformation and therefore enhance the kinetics. This latter effect which vanishes at high strains, is responsible of a shape change of the kinetics curve from sigmoidal to parabolic.

3) The design of high-strength austenitic alloys can be improved by reducing the temperature sensitivity of strain-induced fcc  $\rightarrow$  hcp transformation kinetics, which in turn can be achieved by controlling the  $\epsilon$  platelet size.

#### APPENDIX

## Extension of the Nucleation Model to the Predeformed Condition

Prestraining the alloy studied at 373 K introduces numerous twins and stacking faults<sup>6,8</sup> *i.e.* obstacles to the propagation of  $\epsilon$  platelets and nuclei for the strain-induced  $\gamma \rightarrow \epsilon$  transformation respectively. The latter effect can be easily taken into account through the parameter  $\epsilon_0^*$  defined in the text (see Eq. [5]). As the amount of prestrain  $\epsilon_0$  induces a volume fraction of twins  $f_T$ , Eq. [5] can be simply changed into:

$$dN_V = (1 - f_T - f)(\epsilon_0^* + \epsilon)Ad\epsilon.$$
 [A1]

The role of the obstacles introduced by prestrain can be analyzed by analogy with  $\epsilon$  platelets. Let us suppose that instead of twins the prestrain  $\epsilon_0$  induces a volume fraction  $f_0$  of  $\epsilon$  platelets. The volume of platelets will then be related to the total volume fraction of  $\epsilon$  martensite formed *i.e.* the fraction *f* induced by the final deformation  $\epsilon$  plus the fraction  $f_0$ introduced by the prestrain  $\epsilon_0$ :

$$V = V_0 (1 - k f_0 - k f)^2.$$
 [A2]

Hence setting  $V'_0 = V_0(1 - kf_0)^2$  the volume of the first generation of platelets induced in the course of the final deformation, the volume of the platelets is related to the final volume fraction f by:

$$V = V_0' (1 - k'f)^2$$
 [A3]

with

$$k' = k \sqrt{\frac{V_0'}{V_0}} \, .$$

The influence of the true prestrain substructure on the size of  $\epsilon$  platelets can be accounted for by Relation [A3] but  $V'_0$  is no longer a known function of the twinned austenite volume fraction. Hence substitution of Eqs. [A1] and [A3] into Eq. [6] gives:

$$df = (1 - f_T - f) (1 - k'f)^2 (\epsilon + \epsilon_0^*) A V_0' d\epsilon.$$
 [A4]

Defining  $f^*$  as the ratio of the volume fraction of martensite to the initial austenite volume available *i.e.*  $f^* = f/(1 - f_T)$ , this relation becomes:

$$df^* = (1 - f^*) (1 - k^* f^*)^2 (\epsilon + \epsilon_0^*) 2\alpha' d\epsilon$$
 [A5]

with

 $\alpha' = A V_0'/2$ 

and

$$k^* = k(1-f_T) \sqrt{\frac{\alpha'}{\alpha}}.$$

After integration of this relation one obtains:

$$\alpha'[\epsilon^2 + 2\epsilon_0^*\epsilon] = \Phi_{k*}(f^*)$$
[A6]

where  $\Phi(f)$  has the same meaning as in Eq. [9]. Therefore the volume fraction of strain-induced martensite is a function of the applied strain and depends upon the constant k and upon three parameters:  $(1 - f_T)$  volume fraction of untwinned austenite,  $\alpha'$  similar to  $\alpha$  in the as-quenched condition which is a decreasing function of stacking fault energy and  $\epsilon_0^*$  which accounts for the stacking faults introduced by prestrain.

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