Determination of Interfacial Heat Transfer Coefficients in Casting and Quenching Using a Solution Technique for Inverse Problems Based on the Boundary Element Method

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Both casting and quenching are processes during which several physical phenomena like heat transfer, fluid flow, phase transformation, *etc.* interact in a complex manner. To obtain a numerical model which is capable of accurately simulating the actual process, one has to be able to quantify all the parameters affecting the process. One parameter which substantially influences heat transfer in these processes is the heat transfer coefficient at the interface between the mold and the metal in casting and that between the metal and the quenchant in quenching. The heat transfer coefficient could vary on the surface of a casting or a quench metal both spatially and with time. Its accurate determination is imperative for a realistic simulation of these processes. In this work, an algorithm based on the boundary element technique is proposed to solve for the interface heat transfer coefficient. The problem is cast as one of inverse heat conduction in two dimensions where some of the boundary conditions, namely, the previously mentioned heat transfer coefficients, are unknowns. Since it is the boundary properties that are being determined, the boundary element method (BEM) is the most suitable technique to use. The algorithm uses experimentally measured temperature data inside the domain to determine the interface heat transfer coefficient. The technique is outlined in detail and some casting and quenching examples are presented to demonstrate its capability.

I. INTRODUCTION

FOR any casting simulation model to be successful in predicting the solidification rate and time, it is important that the overall heat transfer coefficient at the metal-mold interface be known accurately. The resistance to heat transfer at the metal-mold interface significantly influences the solidification rate and the total solidification time, especially in permanent mold, investment, and die castings. There are several factors which cause this resistance and affect the heat flux at the interface. These include (a) imperfect contact at the interface resulting in a discontinuity of temperature there, (b) effect of releasing agents, often sprayed on the die walls to ensure proper release of the solidified casting from the die, and (c) formation of air gaps between the interfaces due to contraction of the casting during solidification. Thus, the overall heat transfer coefficient at the interface can be defined as a function of the contact, gap, and spray resistance. It is very difficult, and probably not feasible, to determine the exact contribution of each of these factors. It is important, however, to determine the overall heat transfer coefficient, or in other words, the combined effect of all the different factors affecting the heat transfer at the interface. This heat transfer coefficient may vary from one point to another on the casting surface and also change with time as the air gap thickness changes.

Quenching, an important component of metal heat treatment, comprises of rapid cooling of components from a high temperature when the hot metal is dipped into a bath of cold fluid. The phase transformation that occurs in a quenched part is dependent on the local cooling rate at any point in the quenched metal. This local cooling rate is dependent on the rate at which heat is transferred from the surface of the metal to the surrounding quenchant. There are three stages of this heat transfer process listed in the order of their occurrence as (a) radiation and conduction through the vapor layer formed on the metal surface, (b) boiling, and (c) natural convection. Of these three stages of the cooling process, the maximum amount of heat transfer occurs during the boiling phase. The overall heat transfer coefficient changes from a low value during the vaporization stage to a very high value in the boiling stage and comes down to a comparatively lower value in the convection regime. This variation of the overall heat transfer coefficient at the interface is quite complex in nature and is strongly dependent on time, surface temperature, and spatial coordinates.

The problem of determining the interface heat transfer coefficient can be posed as the inverse problem of determining a boundary condition, as will be discussed in more detail in Section II. Thus, the determination of the interface heat transfer coefficient is equivalent to the determination of the heat flux through the interface. This problem belongs to the larger class of problems better known as inverse heat conduction problems (IHCP). Inverse heat conduction problems have generated a lot of interest in the last decade and substantial work has been done in this area. Since inverse problems are often ill-posed, obtaining analytical solutions of these problems is quite difficult. Thus, numerical techniques have usually been employed. Beck *et al.*[1] have made a significant contribution to the area of IHCP. They published a

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monograph containing an exhaustive bibliography and significant details of past research. Similar work has been done by Alifanov and Egorov.[2] Lazuchenkov and Shmukin,[3] Alifanov and Kerov,[4] Hensel and Hills,[5] Blackwell,[6] Imber,[7,8] and many others. In the specific area of interface heat transfer coefficient determination in casting and quenching, contributions have been made by Ho and Pehlke,[9] Beck,[10,11] Mahapatra *et al.*,[12] and others.

The most common technique that has been used in the solution of these problems is based on the finite difference method and was first proposed by Beck *et al.*[1] Most of the previously mentioned researchers have used Beck's method, or some variation of it, to investigate the problem of determination of the heat flux at a boundary where none of the boundary conditions are known *a priori*. This method is based on optimization, and for regularization purposes, temperature data from future time steps are required to calculate the heat flux at the present time step. However, use of too many future time steps introduces errors in the results calculated.[11] In most of these efforts, the unknown heat transfer coefficient has been assumed to be a function of time but independent of space variables. In the few studies where space variation has been considered,[13] the calculation algorithm is quite primitive and cumbersome. Locally, at any particular point in space and at any time step, the problem is solved assuming space independence. Space dependency of the solution at several such points is then determined iteratively using the initially calculated space-independent solution at all the locations. This iteration process continues until the solution converges at all the points. At the next time step, this cumbersome and time consuming process is carried out all over again.

The approach mentioned earlier, based on optimization, is necessary for many inverse problems. However, for the class of inverse problems that are being studied in this work, this approach is not necessary. In the inverse problem of interface heat transfer coefficient determination, the unknowns to be calculated are boundary unknowns. It is, therefore, natural that a boundary method will be more suited for the problem. Thus, the boundary element method (BEM) has been chosen in the present analysis. In this technique, the boundary flux and temperature are represented as primary unknowns and, unlike finite differences, are calculated separately from a set of linear equations requiring no iterative minimization techniques. The restriction of the flux being independent of spatial variables, imposed by some of the previous investigators, is also removed. Further, temperature data for future time steps is not necessary for the current time step calculations since regularization techniques are not being used. The use of boundary integral equation technique for solving inverse problems is not new. Zabaras *et al.*[14] have used this technique to determine the location of the freezing front in a solidifying liquid using temperatures measured at the boundary.

II. THE PROBLEM

The problem of interface heat transfer coefficient determination in casting is quite similar to that in quenching. The casting problem in which two domains are considered namely, the metal and the mold, is presented first. It is followed by the quenching problem where only one domain, the metal, is considered.

The schematic shown in Figure 1(a) represents the problem of metal-mold interfacial heat transfer for castings. Heat transfer in both the metal and the mold is represented by the Fourier law. The differential equations for heat transfer in the mold and the solidifying metal can be written as

$$(\rho C_p)_{\text{mold}} \frac{\partial T_{\text{mold}}}{\partial t} = k_{\text{mold}} \nabla^2 T_{\text{mold}} \qquad [1]$$

$$\rho_{\text{metal}} \frac{\partial H_{\text{metal}}}{\partial t} = k_{\text{metal}} \nabla^2 T_{\text{metal}} \qquad [2]$$

respectively, where ρ is density, C_p is specific heat, k is thermal conductivity, T is temperature, H is enthalpy, and t is time. The energy equation for the metal has been written in terms of enthalpy rather than temperature to incorporate the latent heat release during solidification.[15] The boundary conditions on the surfaces AB, AD, CD, EF, EH, and HG are assumed to be specified (namely, isothermal, adiabatic, or a combination). At the interface (*i.e.*, BC and FG), such conditions are unknown. The only relation that holds at this interface is

$$-k_{\text{mold}} \frac{\partial T}{\partial n} = -k_{\text{metal}} \frac{\partial T}{\partial n} = h(T_{\text{metal}} - T_{\text{mold}}) \qquad [3]$$

where $\partial/\partial n$ represents the normal derivative of the temperature. Equation [3] ensures that the entire heat lost by the metal is transferred to the mold. Figure 1(b) represents the problem of heat transfer from hot metal in a quenchant. Heat conduction within the metal is governed by the Fourier law (Eq. [2]).

The boundary conditions on the surfaces of the metal (IJ, JK, KL, LI) may or may not be known in a given situation. However, the relation that holds at all these interfaces is

$$-k_{\text{metal}} \frac{\partial T}{\partial n} = h(T_{\text{metal}} - T_{\text{bulk}}) \qquad [3']$$

The goal of this work is to determine the interface heat transfer coefficient h which controls the heat flux at the interface. Toward this end, the casting problem is broken up into two domains, the mold and the metal. The governing differential equation for each of these domains is written in the boundary integral form. The heat transfer problem in the mold is posed as an inverse problem where some of the boundary conditions (namely, the flux and temperature at the interface) are unknown. The solution of this inverse heat transfer problem provides the flux at the interface and also the temperature in the mold at the mold/metal interface. In contrast, the heat transfer problem in the metal is posed as a direct problem where the heat flux at the interface, already known from the solution of the mold heat transfer problem, is used as the interface boundary condition. At every time step, thus, the heat transfer problem in the mold is solved and the heat flux at the interface is determined. This heat flux is then used as a boundary condition for

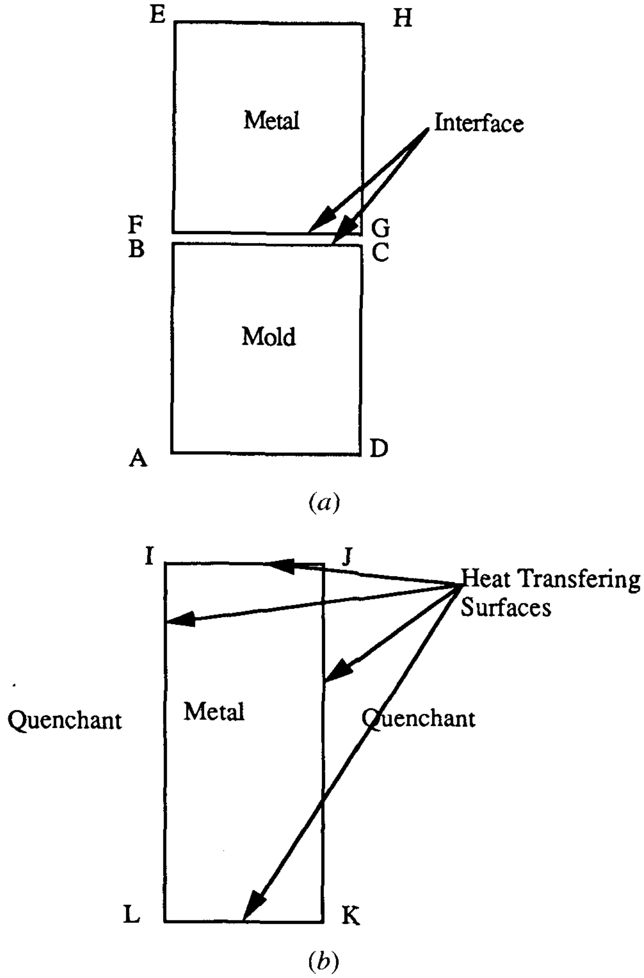

Fig. 1—(*a*) Schematic for (*a*) the metal-mold interface and (*b*) hot-metal—quenchant heat transfer.

the heat transfer problem in the metal. This solution provides the temperature on the metal surface at the interface. The interface heat transfer coefficient is then calculated using Eq. [3].

Determining the heat transfer coefficient in the case of quenching is even simpler. The problem does not have to be broken into two domains. The inverse problem is solved for the metal that is being quenched. Temperature values measured at several points in the metal are used in this solution procedure to calculate the surface heat flux anywhere on the surface of the quenched specimen. This heat flux and the bulk temperature of the quenchant is used in Eq. [3′] to calculate the heat transfer coefficient. The details of the boundary integral representation and the solution strategy is presented in the next section.

III. THE BOUNDARY INTEGRAL REPRESENTATION

In this section, the solution strategy for the inverse problem is described in detail. The procedure is very similar for the direct problem with some differences. At the end of this description, the main differences will be highlighted. For the purpose of convenience, the boundaries where some of the conditions are known (*e.g.*, AB, AD, and CD on the mold in Figure 1(a)) will be referred to as natural boundaries and the interface (where none of the boundary conditions are known) will be referred to as an unnatural boundary. Using the fundamental solution of the Laplace equation and Green's second identity,[16] the equation for heat transfer can be converted into the integral equation

$$\eta T(p) = \int_S (G'T(q) - GT'(q))ds + \int_D \frac{\partial T(q)}{\partial t} \frac{k}{\rho C_p} G dD \quad [4]$$

which relates the temperature at any point p with the boundary and internal data. The variable D is the domain, enclosed by the boundary S, in which this equation is valid. Points p and q are known as the source and the field points, respectively, and the (′) denotes the partial derivative $\partial/\partial n_q$ at point q. The coefficient η is given in the following:

$$\eta = 0, \text{ if } p \text{ lies outside } D + S; \quad 2\pi, \text{ if } p \text{ lies inside domain } D; \quad \alpha, \text{ if } p \text{ lies on } S \quad [5]$$

where α is the included angle between two adjacent tangents at p. The function G is the fundamental solution of the Laplace equation in two dimensions, and is given by $\ln |p - q|$. Using the simple backward-difference representation for the time derivative[17] in Eq. [4], we can write the following:

$$\eta T^n(p) = \int_S (G'T^n(q) - GT'^n(q))ds + \int_D \frac{T^n - T^{n-1}}{\Delta t} \frac{k}{\rho C_p} G dD \quad [6]$$

where the superscript n denotes the nth time step. Thus, the variables on the boundary are temperature and its normal derivative (the heat flux). Inside the domain, the variable is temperature.

The integral equation (Eq. [6]) is discretized by approximating the boundary S by straight-boundary elements. On each such boundary element, the variables (*i.e.*, temperature and flux) are approximated by a quadratic function as

$$X = X_1^{b(n)} S_1 + X_2^{b(n)} S_2 + X_3^{b(n)} S_3 \quad [7]$$

where $X^{b(n)}$ denotes the value of the variable (temperature or flux) at a boundary node and at the nth time step, and S's denote the Lagrangian polynomials. The details of such a discretization can be found in any standard text on BEM.[16]

There are N number of equations that can be written for N number of boundary nodes. For every node on the natural boundary, one of the variables (temperature or the flux) is known and the other unknown. However, for all the nodes on the unnatural boundary, both the variables are unknown. As a result, for C number of nodes on the unnatural boundary, the total number of unknowns is $N + C$.

The domain D is approximated by constant triangular elements. On the jth domain element, the temperature at the nth time step is

$$T_j^n = X_j^{i(n)} \quad [8]$$

It should be noted from Eqs. [7] and [8] that the subscripts of X denote the node number, the superscript n denotes the nth time step, the superscript b signifies that X belongs to a boundary node, and the superscript i signifies that X belongs to a domain or internal node.

After inserting the boundary and domain approximations given in Eqs. [7] and [8] into Eq. [6], performing the integrations analytically over each boundary and domain element, the N equations for the N boundary nodes can be written in matrix form as

$$[A]\{\mathbf{X}^{b(n)}\} + [B]\{\mathbf{X}^{i(n)}\} = [C]\{\mathbf{X}^{i(n-1)}\} + \{\mathbf{P}\} \quad [9]$$

The known boundary data is included in vector $\{\mathbf{P}\}$ after appropriate matrix multiplications. The vector $\{\mathbf{X}^{b(n)}\}$ contains $N + C$ unknowns from the nodes on the natural and unnatural boundaries. If the number of interior nodes is M, the vector $\{\mathbf{X}^{i(n)}\}$ contains M unknowns from the interior nodes. The vector $\{\mathbf{X}^{i(n-1)}\}$ is also a vector of length M, but it is known from the initial condition or from the calculations performed in the previous time step.

By writing the discretized form of Eq. [6] for M number of interior nodes, one gets M number of equations of the following type:

$$[D]\{\mathbf{X}^{b(n)}\} + [E]\{\mathbf{X}^{i(n)}\} = [F]\{\mathbf{X}^{i(n-1)}\} + \{\mathbf{Q}\} \quad [10]$$

where the known boundary data is included in the vector $\{\mathbf{Q}\}$.

Thus, one could write a set of $M + N$ equations relating $M + N + C$ unknowns by writing the boundary integral equation for all the boundary and interior nodes. Since the total number of unknowns is greater than the number of equations, this set is not sufficient to solve for all the unknowns.

The extra conditions that are required for the solution are provided as measured temperature data at a set of interior points; Eq. [6] can be written for all these points as

$$[J]\{\mathbf{X}^{b(n)}\} + [T]\{\mathbf{X}^{i(n)}\} = [W]\{\mathbf{X}^{i(n-1)}\} + \{\mathbf{R}\} \quad [11]$$

where all the known boundary data and the measured data for the interior points are included in the vector $\{\mathbf{R}\}$. Equations [9] through [11] can now be solved simultaneously for $M + N + C$ number of unknowns.

The coefficient matrices on the left-hand side of the linear Eqs. [9] through [11] are dependent only on the geometry of the problem and are all constant with respect to time. Thus, they have to be formed only once and can be stored in an LU decomposed form. The vectors on the right-hand side have to be updated at every time step. Thus, the solution algorithm at every time step merely comprises of some matrix vector multiplications, vector additions, and back substitutions.

The solution strategy for the quenching problem is quite simple. The heat flux and temperature that is calculated in the inverse problem and the bulk temperature of the liquid is directly used in Eq. [3′] to calculate the heat transfer coefficient.

For the casting problem, the differential equation for heat transfer in the metal can be written in the boundary integral form in a manner very similar to Eq. [6] using the enthalpy formulation instead of temperature. The domain and the boundaries are discretized and the integral equations for their nodes are written in a manner similar to the mold. In this problem, however, if the number of boundary nodes are N^* and the number of domain nodes are M^*, the number of unknowns is $N^* + M^*$. The extra unknowns (the heat flux) at the interface have already been calculated from the inverse problem and are part of the known boundary conditions for the direct problem. The set of $N^* + M^*$ equations (which are similar to Eqs. [9] and [10]) can thus be solved to determine the enthalpy and, thus, the temperature[15] on the boundary and inside the freezing metal at all time steps. The temperatures calculated at the metal mold interface can then be used to calculate the interfacial heat transfer coefficient from the relation

$$h = \frac{q}{T_{\text{metal}} - T_{\text{mold}}} \quad [12]$$

Thus, the solution strategy is as follows. The inverse problem of heat conduction in the mold is solved first. The extra conditions needed come from the experimentally measured values of temperature at some preselected points within the mold. The number of points at which the temperature has to be measured should be equal to C, the number of nodes on the unnatural boundary. However, if the flux is known to be independent of spatial coordinates, only a single measuring point is necessary. The heat flux at the interface and the temperature on the mold at the interface is obtained as part of the solution. The heat flux is used in the direct problem of heat transfer in the metal as a known boundary condition. The solution of the direct problem provides the temperature on the metal surface at the interface in addition to temperatures everywhere else in the metal. The two temperatures at the interface and the interface heat flux are used to calculate the heat transfer coefficient. This solution strategy is applied to several problems in the next section.

IV. EXAMPLES

Several example problems have been chosen to demonstrate the capabilities of the algorithm. The first problem is a model problem, with a known analytical solution to validate the accuracy of the numerical scheme. The second is a comparison of the heat flux calculated by the present method with that published in the literature using a different scheme. The third and the fourth examples demonstrate the applicability of the present method for realistic situations where thermocouple data is available from experiments. Also, in these two examples, the effect of gravity in the formation of air gaps is highlighted. The last example is a quenching example where the inverse technique is used to calculate heat transfer coefficients that vary spatially as well as temporally.

A. *Example 1*

In this example, a simple geometry for the mold and the casting are chosen. Both are represented by a unit square (Figure 2(a)).

The temperature variation in the mold is assumed to be given by

$$T = 1 - e^{-\pi^2 t/4} \cos\left(\frac{\pi Y}{2}\right) \qquad [13]$$

whereas in the metal, the variation of the enthalpy is assumed to be given by

$$H = 1 + X - e^{-\pi^2 t/4} \cos\left(\frac{\pi Y}{2}\right) \qquad [14]$$

where X and Y represent the horizontal and vertical directions in a cartesian coordinate system. The previously mentioned expressions have been chosen since they belong to a class of functions that satisfy the thermal diffusion equation. The boundary conditions on all the boundaries except the interface are determined from these relationships and are used in the calculations. Both the temperature and the flux at the interface can be determined from the same relationships. However, these values are only used for testing the accuracy of the numerical scheme. The temperature values of the five points located 0.1 units inside the mold as measured from the surface (Figure 2(a)) are used in the inverse algorithm to calculate the flux and the temperature at the interface. For the sake of convenience, the material properties like the coefficient of thermal conductivity, density, specific heat, *etc.* are assumed to be unity in both the regions. Each side of the two square boundaries are divided into two quadratic boundary elements and the domain is divided into 32 triangular interior elements (Figure 2(a)). On each of these elements, temperature is assumed to be constant. As outlined in the previous section, the inverse technique is used in the mold to solve for the heat flux and temperatures at the interface. The interface flux value is then used in the metal to solve for temperatures everywhere inside and also at the interface on the metal. Analytical value of the flux at the interface and the numerically obtained values with two boundary elements per side and with one boundary element per side are compared in Figure 2(b). The distribution obtained using one boundary element per side has been included here to show how the calculated flux distribution

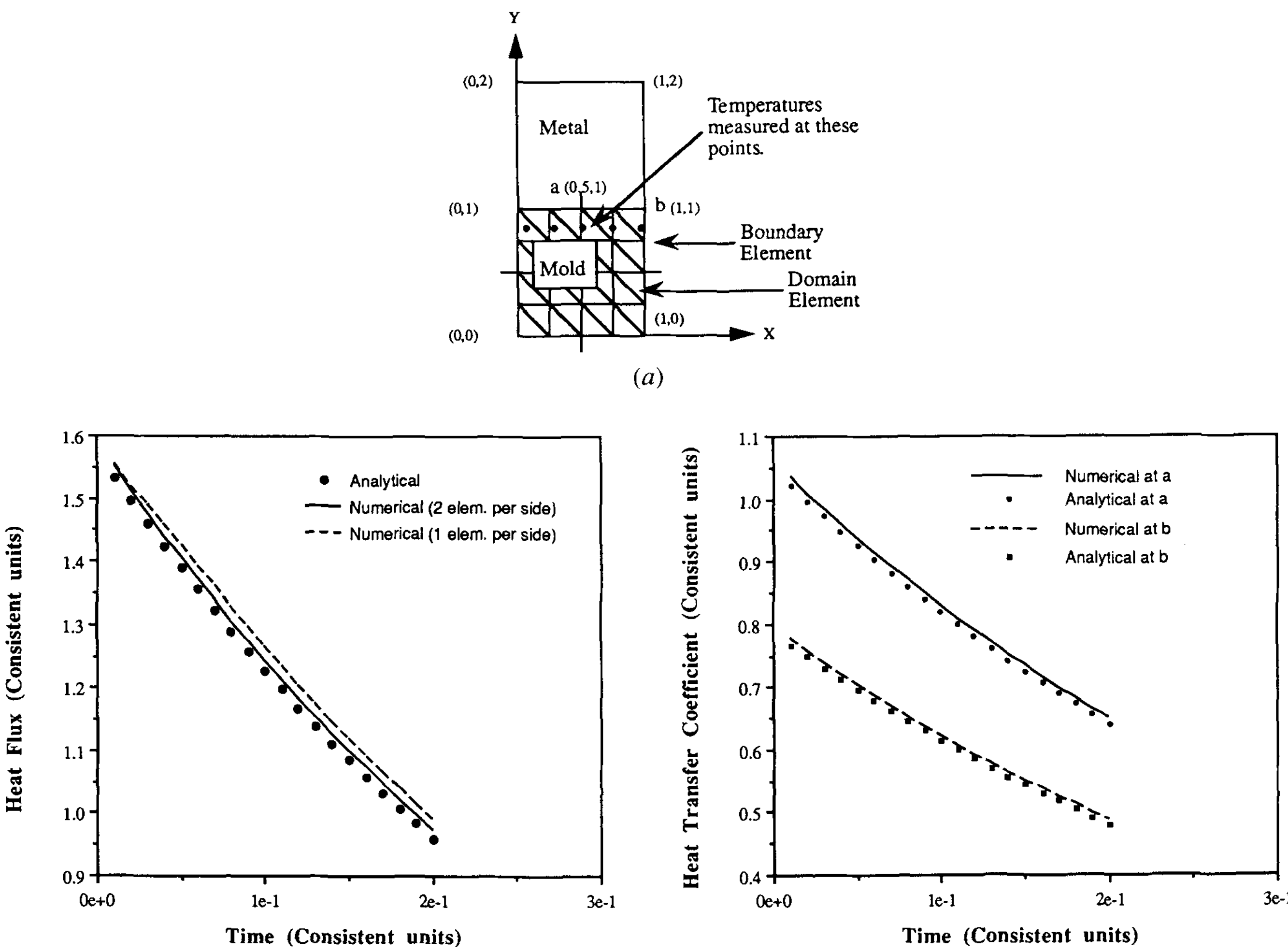

Fig. 2—(*a*) Schematic for Example 1. Comparison of numerically and analytically calculated values of (*b*) heat flux at the interface and (*c*) the heat transfer coefficient at two points on the interface.

approaches the analytical value from the higher side as the number of boundary elements are increased. The temperatures calculated on the metal-mold interface can be used to calculate the heat transfer coefficient at the interface. A comparison of the numerical and the analytical values of the heat transfer coefficient as calculated at two different points (marked as *a* and *b* on Figure 2(a)) on the interface is shown in Figure 2(c). From these comparisons, it is quite clear that the technique outlined in this article can compute both the interface heat flux and the interface temperatures and, thus the heat transfer coefficient, very accurately. The comparison in Figure 2(c) shows that the algorithm can easily handle both the spatial and temporal dependencies.

B. *Example 2*

In this example, the proposed BEM solution technique is compared with the technique proposed by Beck *et al.*[1] A simple problem has been chosen from published literature.[11] Figure 3(a) shows a schematic of the problem. A triangular heat flux is introduced on one surface of the plate. The other three surfaces are insulated. Using the temperature history recorded by a probe within the plate, the heat flux introduced is calculated using the inverse technique.

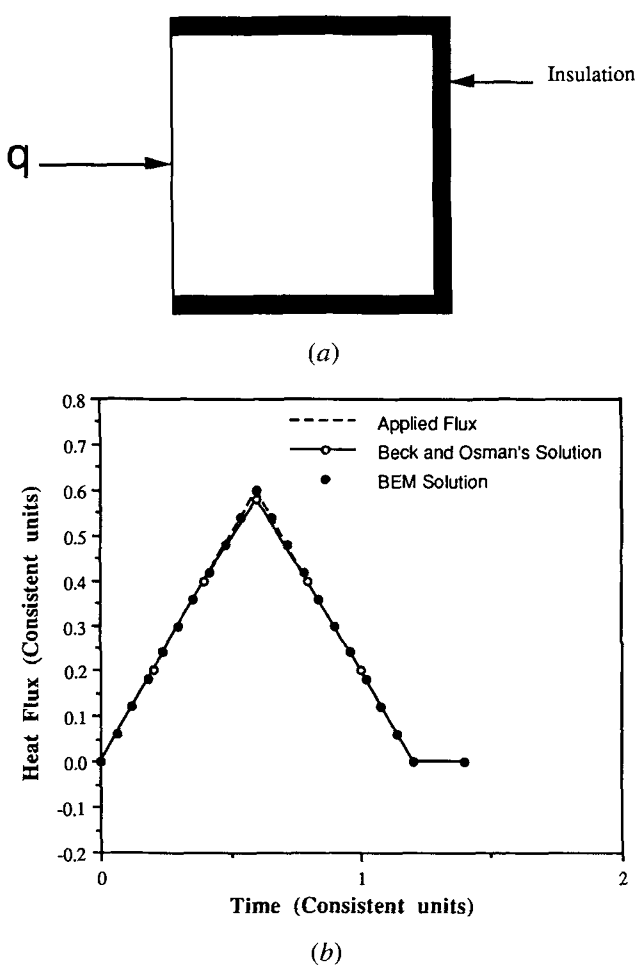

Fig. 3—(*a*) Schematic for Example 2. (*b*) Comparison of BEM solution and Beck and Osman's solution[11] for a triangular heat flux.

The results obtained using a time step of 0.06 are compared with the published results. The BEM solution matches identically with the actual flux. Figure 3(b) shows a comparison of the actual flux: the BEM solution and the solution from Beck and Osman's work.[11] Beck and Osman calculated this flux using two future time steps.

C. *Example 3*

In this example, thermocouple data obtained from actual experiments are used to calculate the heat transfer coefficient at the interface. These data are taken from the work of Ho and Pehlke.[9] A schematic for the experimental setup is shown in Figure 4(a).

The casting is made of aluminum with a water-cooled copper chill. Constant material properties have been assumed for both the materials. The water-cooled boundary of the chill is assumed to have a constant temperature

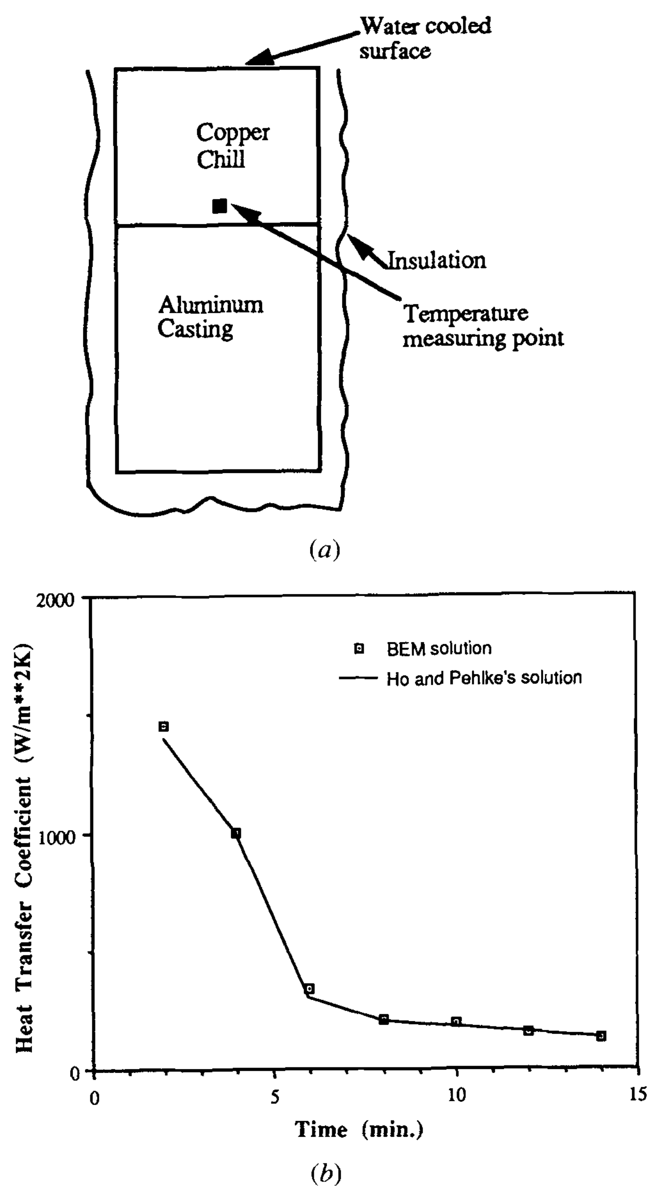

Fig. 4—(*a*) Schematic for Example 3. (*b*) Interface heat transfer coefficients calculated by BEM and by Ho and Pehlke.[9]

boundary condition. The other two sides of the chill are insulated. Three of the four surfaces of the casting are insulated. Heat loss occurs in the casting only through the chill. The heat transfer problem here is, thus, a one-dimensional problem and only temporal variations of heat transfer coefficient at the interface are expected. Thus, temperature data from one thermocouple located inside the chill are enough to determine the heat transfer coefficient. The location of this thermocouple is shown in Figure 4(a). It should be pointed out that Beck's method,[1] which Ho and Pehlke[9] used, requires temperature readings from two thermocouples, one each in the mold and the metal to determine the heat transfer coefficient.

As in the previous example, a similar calculation is done to determine the heat flux and, thus, the interface heat transfer coefficient at the metal-chill interface. The interface heat transfer coefficients, as calculated by the present algorithm and that calculated by Ho and Pehlke[9] are shown in Figure 4(b). The two sets of data compare very well. In the initial stage of the cooling process, the heat transfer coefficient has a high value. After a while, there is a sharp drop in its magnitude, induced by the contraction in the casting. The drop in the coefficient after that is slow.

D. *Example 4*

This is also an example where actual experimental data are used to calculate the heat transfer coefficient at the interface. These data were obtained in an experiment conducted for a separate study. A schematic for the experimental setup is shown in Figure 5(a). The metal that was used in the casting experiment is a copper alloy (C95800). Cement-bonded sand was used as the mold material. As shown in the figure, thermocouples were used to measure the temperature at five different points. During solidification, the shrinkage in the metal creates a gap between the upper surface of the casting and the sand mold. This is shown, although in an exaggerated way, in the figure. Due to gravity effects, the bottom surface of the casting and the adjacent mold surface remain in near-perfect contact. These differences in the contact resistances between the upper and the lower surfaces of the casting are reflected in the temperature values recorded by the thermocouples in the sand above the casting and the corresponding ones in the sand below it.

As in the previous example, the inverse problem is first solved in the two mold regions (the upper and the lower) to obtain the heat flux escaping from the casting surfaces. The dimensions of the casting are such that the heat transfer problem is primarily a one-dimensional one. The thermal data from the thermocouple B are used in the calculation of the heat flux into the mold above the casting, and data from thermocouple D are used for the sand below the casting. The heat flux data obtained as a solution of the two inverse problems are then used in the direct problem of metal solidification as known boundary conditions. The solution of the direct problem provides temperature data everywhere in the casting at all time steps.

The calculated temperature at the center of the casting has been compared with the values recorded by thermocouple C in Figure 5(b). Even though the trends shown by the two curves are identical, there is some discrepancy between the two sets of values. This can be attributed to the fact that constant thermophysical properties have been used in the calculations, the latent heat release has been assumed to be varying linearly in the region between the solidus and the liquidus, and the solidus and the liquidus temperature assumed in the calculation may be slightly higher than the ones actually observed.

The temperatures obtained for the outer surface of the casting is used in Eq. [12] to calculate the interfacial heat transfer coefficient at the top and the bottom of the casting (Figure 5(c)). The two curves clearly show the influence of the large air gap at the top and the absence of such a gap at the bottom. The heat transfer coefficient at the top increases at the early stages of cooling. However, as freezing occurs, the contraction in the metal creates a large air gap. The formation of this gap is reflected by the sharp drop in the heat transfer coefficient. At the bottom surface no such change is observed. The heat transfer coefficient remains high throughout the cool down period and starts decreasing only toward the end when the total amount of heat released decreases.

E. *Example 5*

This is an example of heat transfer coefficient calculation for a quench specimen. For the lack of suitable experimental data, temperature data were generated by solving a direct problem of quenching of a hollow cylinder. Figure 6(a) shows a rectangular geometry that represents a slice taken from the hollow cylinder. For convenience, the top and the bottom surfaces (BC and AD) are assumed to be insulated. Heat transfer occurs from the outer and the inner surface. The heat transfer coefficient variation for the outer surface (AB) is assumed to be

$$\begin{aligned} h &= 25{,}000\, Yt, \quad t \leq 3 \\ &= \frac{25{,}000\, Y}{t}, \quad t > 3 \end{aligned} \qquad [15]$$

and for the inner surface (CD) it is assumed to be

$$\begin{aligned} h &= 25{,}000\, Yt, \quad t \leq 7 \\ &= \frac{25{,}000\, Y}{t}, \quad t > 7 \end{aligned} \qquad [16]$$

where Y is the distance measured in the vertical direction from the origin located at the bottom-left corner of the section.

Using the previously mentioned parameters and the material properties for steel, the direct problem is solved using the commercial finite element code ABAQUS.* Temperature history obtained at 10 predefined points (Figure 6(a)) is then used in the inverse solver to calculate the heat transfer coefficient. Two boundary elements per side and 32 interior elements are used in the calculation. Some of the results obtained are shown in

*ABAQUS is a trademark of Hibbitt, Karlsson and Sorensen, Inc., Providence, RI.

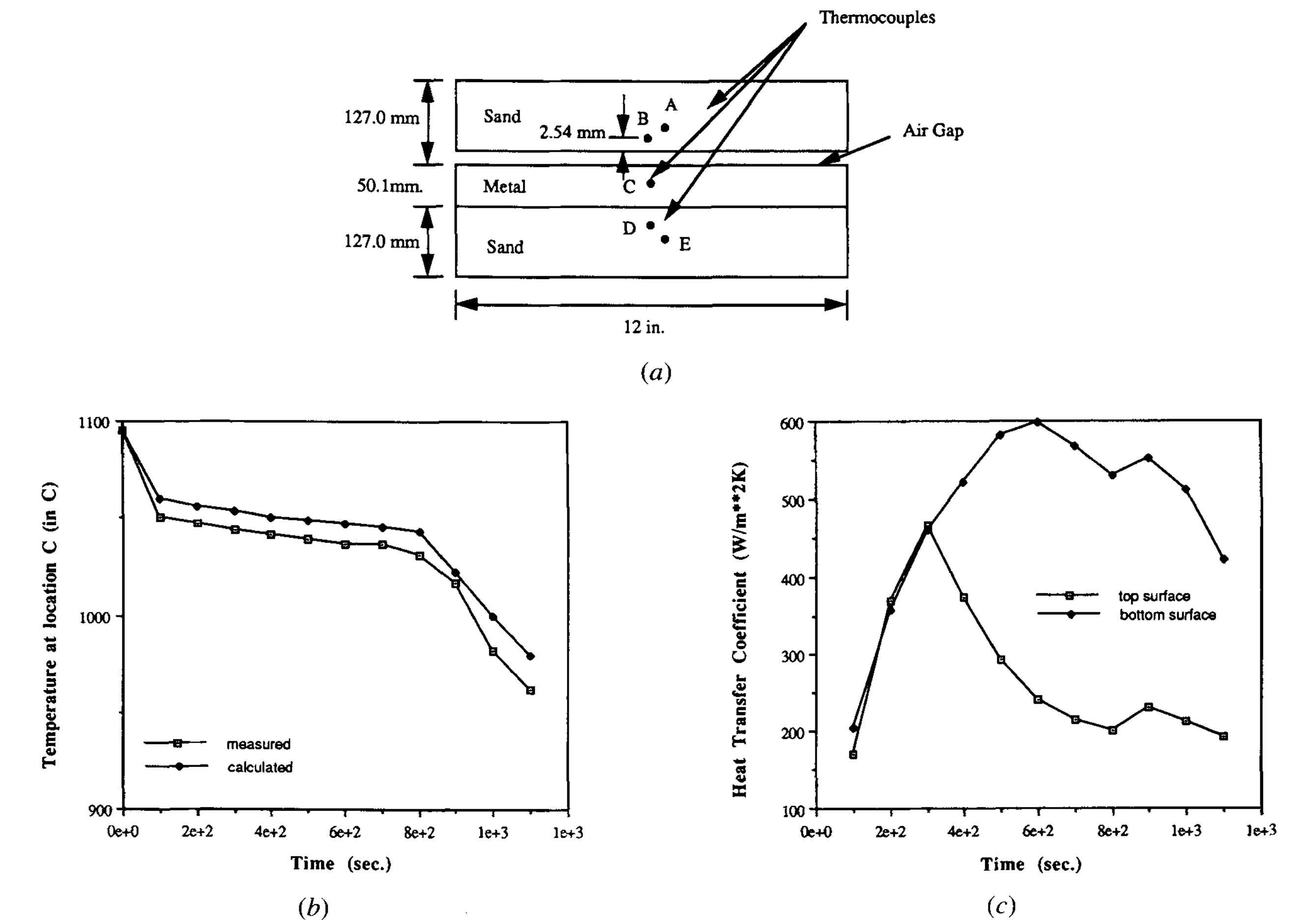

Fig. 5—(*a*) Schematic for Example 4. (*b*) Measured and calculated temperature at location C. (*c*) Interface heat transfer coefficients calculated at the top and bottom surface of the casting.

Figures 6(b) through (d). Figure 6(b) shows the variation of the heat transfer coefficient with time at the top-right corner of the section (marked A in Figure 6(a)).

The calculated value matches quite closely with the applied heat transfer coefficient. Beyond 3 seconds, the heat transfer coefficient drops to a much lower value. In this region, the calculated value shows some fluctuations from the applied heat transfer coefficient. Probably, a smaller time step is necessary in this region to calculate the value accurately. Figure 6(c) compares the calculated and the applied heat transfer coefficient on the outer surface at $t = 3$ seconds. This is the time when the heat transfer coefficient at the outer surface reaches the maximum value. Figure 6(d) compares the calculated and the applied value of the coefficient on the inner surface at $t = 7$ seconds. This is the time when the heat transfer coefficient reaches the maximum on the inner surface.

V. DISCUSSION AND CONCLUSION

A numerical technique, based on the BEM, for obtaining the interface heat transfer coefficient in castings has been described. Even though only rectangular boundaries have been used in the examples presented here, the algorithm is valid for any shape of the boundary.

Over the last decade, Beck *et al.*[1] have been responsible for developing, testing, and implementing algorithms for inverse heat conduction problems. Through their pioneering contribution in the field of IHCP they have successfully raised the general awareness about the importance of inverse problems among the scientific community. There are a number of drawbacks to the finite difference based technique proposed by Beck *et al.* The heat flux at the interface and thus the interface heat transfer coefficient has to depend only on time and be independent of spatial coordinates. Since this technique is based on finite differences, heat flux is calculated indirectly from temperature data. Experimental data from the future time step are needed to regularize the flux at the present time. The heat flux at the interface has to be calculated by some iterative minimization technique.

Some of the disadvantages of Beck's technique[1] can be overcome by using the boundary element representation of the problem. The heat flux at the interface is treated as a primary variable in the boundary integral representation and is calculated by simple matrix operations. This algorithm accounts for both space and time dependencies of the interface heat transfer coefficient. For castings, experimental measurement of temperature is required only inside the mold which is much easier to obtain than temperatures inside the solidifying metal. If only time dependency is assumed, the temperature value

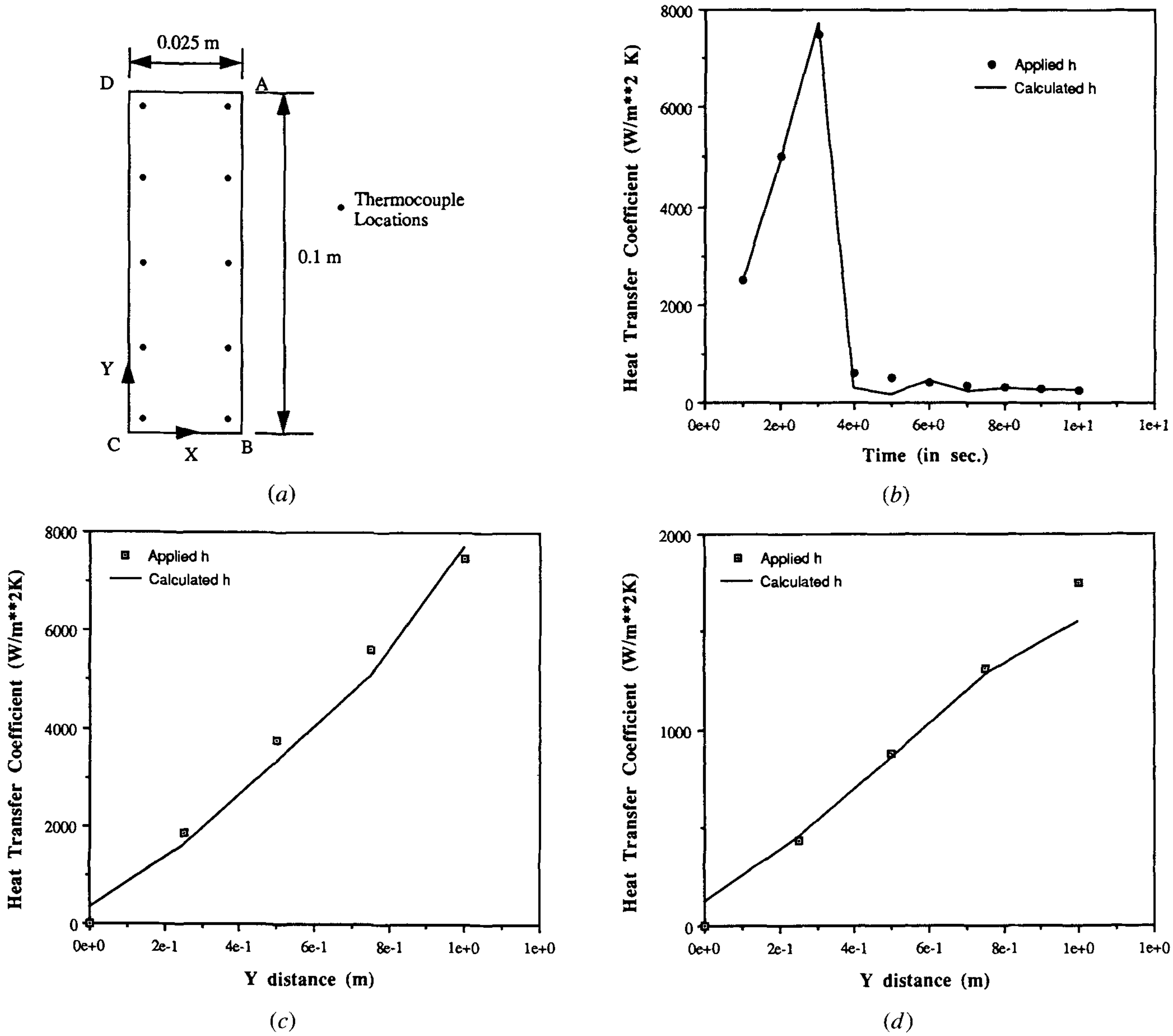

Fig. 6—(*a*) Schematic for Example 5. (*b*) The variation of the heat transfer coefficient at point A with time. The calculated and applied heat transfer coefficient on the (*c*) outer surface at t = 3 s and (*d*) inner surface at t = 7 s.

is needed only at one point inside the mold for all calculations. This method will still work if the heat transfer formulation for the solidifying metal is made more rigorous by including fluid flow *etc*. In that case, the inverse problem will be tackled in a manner identical to the earlier description. The direct problem solution strategy will be replaced by a more involved strategy but the calculated flux at the interface can still be used as a boundary condition for the direct problem.

However, there are some disadvantages of the proposed method and several of the inherent difficulties that arise from the nature of the inverse problem still remain. In the present formulation it has been assumed that all the material properties are constant. This, of course, is far from reality. The material properties both in the mold and the metal change with temperature. The present formulation can be easily altered to accommodate the variation of material properties with temperature. The extra terms that arise in the differential equation due to the variation of material properties can be treated as domain integrals in the same way as the term on the right-hand side of Eq. [4]. The proposed technique is as susceptible to experimental errors as any other technique. If the measurement error masks the actual change in the variables, the difficulties involved in solving the inverse problem is the same no matter what technique is used. This issue has been addressed, albeit indirectly, in examples 3 and 4 where actual thermocouple data have been used to solve for the heat transfer coefficient.

In all the examples presented here, the number of boundary elements used in the solution procedure has been quite low (usually two per straight edge). The rationale for doing this rests on the fact that the number of boundary unknowns are equal to the number of boundary nodes. So, in an inverse algorithm, to solve for x number of boundary unknowns, x number of thermocouples have to be used. In most practical situations, the number of thermocouples that can be used will not be very large. In this work, it has been shown that the proposed algorithm works very well even with a few thermocouples. The algorithm presented will in fact provide more accurate results if the number of thermocouples and thus the number of boundary elements is increased. Two sets of interesting problems, however, arise when the number of thermocouples is unequal to the number of boundary nodes. If the number of thermocouples is greater than the number of boundary unknowns, the system of equations become overdetermined. This system can be solved using a least-squares technique. If the number of unknowns is less than the thermocouples (which is a more realistic situation), the system of equations become underdetermined. This set can be approximately solved for by regularizing the matrix.[18]

In the near future, this method will be applied to solve for the heat transfer coefficients of more complex geometries. A study can be performed to record the variation of the interface heat transfer coefficient with the variation of the shape of the casting. This will create an useful information base containing realistic values of the heat transfer coefficients for different combinations of materials and for different geometrical shapes.

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REFERENCES

1. J.V. Beck, B. Blackwell, and C.R. St. Clair: *Inverse Heat Conduction: Ill-posed problems*, John Wiley and Sons, New York, NY, 1985.
2. O.M. Alifanov and Yu.V. Egorov: *J. Eng. Phys.*, 1985, vol. 48, pp. 489-96.
3. N.M. Lazuchenkov and A.A. Shmukin: *J. Eng. Phys.*, 1981, vol. 40, pp. 223-28.
4. O.M. Alifanov and N.V. Kerov: *J. Eng. Phys.*, 1981, vol. 41, pp. 1049-53.
5. E.C. Hensel and R.G. Hills: *J. Heat Transfer*, 1986, vol. 108, pp. 248-56.
6. B.F. Blackwell: *Numer. Heat Transfer*, 1981, vol. 4, pp. 229-39.
7. M. Imber: *5th Int. Heat Transfer Conf.*, Tokyo, 1974.
8. M. Imber: *AIAA J.*, 1975, vol. 13, pp. 114-15.
9. K. Ho and R.D. Pehlke: *Metall. Trans. B*, 1985, vol. 16B, pp. 585-94.
10. J.V. Beck: *Modeling of Casting and Welding and Advanced Solidification Process V*, M. Rappaz *et al.*, eds., TMS, Warrendale, PA, 1991, pp. 503-14.
11. J.V. Beck and A.M. Osman: *1st Int. Conf. on Quenching and Control of Distortion*, G.E. Totten, M. Rappaz, M.R. Özgü, and K.W. Mahin, eds., ASM INTERNATIONAL, Metals Park, OH, 1992, pp. 147-53.
12. R.B. Mahapatra, J.K. Brimacombe, I.V. Samarasekera, N. Walker, E.A. Paterson, and J.D. Young: *Metall. Trans. B*, 1991, vol. 22B, pp. 861-74.
13. S. Segerberg and J. Bodin: *1st Int. Conf. on Quenching and Control of Distortion*, G.E. Totten, eds., ASM INTERNATIONAL, Metals Park, OH, 1992, pp. 165-70.
14. N. Zabaras, S. Mukherjee, and O. Richmond: *Trans. ASME*, 1988, vol. 110, pp. 554-61.
15. V. Voller and M. Cross: *Int. J. Heat Mass Transfer*, 1981, vol. 24, pp. 545-56.
16. P.K. Banerjee and R. Butterfield: *Boundary Element Methods in Engineering Science*, McGraw-Hill, New York, NY, 1981.
17. M.S. Ingber and A.K. Mitra: *Numerical Methods in Thermal Problems*, R.W. Lewis, K. Morgan, and W.G. Habashi, eds., Pineridge Press, Swansea, United Kingdom, 1988, vol. 5, pp. 231-38.
18. S. Das: Ph.D. Thesis, Iowa State University, Ames, Iowa, 1991.