The Thermal and Metallurgical State of Steel Strip during Hot Rolling: Part III. Microstructural Evolution

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A mathematical model has been developed to compute the changes in the austenite grain size during rolling in a hot-strip mill. The heat-transfer model described in the first of this series of papers has been employed to calculate the temperature distribution through the thickness which serves as a basis for the microstructure model. Single- and double-hit compression tests have been conducted at temperatures of 900 °C, 850 °C, 950 °C, and 875 °C on 0.34 and 0.05 pct carbon steels to determine the degree of recrystallization by metallographic evaluation of quenched samples and by measuring the magnitude of fractional softening. The Institut de Recherches de la Sidérurgie Francaise, (IRSID) Saint Germain-en-Laye, France equation has been found to yield the best characterization of the observed recrystallization kinetics. The equations representing static recrystallization kinetics, recrystallized grain size, and grain growth kinetics have been incorporated in the model. The principle of additivity has been invoked to permit application of the isothermal recrystallization data to the nonisothermal cooling conditions. The model has been validated by comparing predicted austenite grain sizes with measurements made on samples quenched after one to four passes of rolling on the CANMET pilot mill. The austenite grain size evolution during rolling of a 0.34 pct carbon steel on Stelco's Lake Erie Works (LEW) hot-strip mill has been computed with the aid of the model. The grain size decreased from an initial value of 180 μm to 35 μm in the first pass due to the high reduction of 46 pct. The changes in austenite grain size in subsequent passes were found to be small in comparison because of the lower per pass reductions. It has been shown that the equation employed to represent grain growth kinetics in the interstand region has a significant influence on the computed final grain size. Altering the rolling schedule had a negligible influence on the final grain size for a given finished gage. A 200 °C increase in entry temperature to the mill resulted in a 20-μm increase in final grain size, which is significant. This can be attributed to increased grain growth at the higher temperature.

I. INTRODUCTION

MICROSTRUCTURAL changes that occur during hot rolling of steel strip are a result of the complex interplay between thermal, mechanical, and metallurgical phenomena. Processes have been operated for decades, with the operating practices being established by trial and error to ensure that dimensional tolerances and mechanical property specifications are met. Advances in process control driven by the development of more accurate sensors and burgeoning computer power have allowed greater control to be exercised over thermomechanical processes and the resulting microstructure and mechanical properties of the products. However, models for process control are in an evolutionary stage, moving away from the fully empirical, which are based on accumulated data bases, to the knowledge intensive, which are rooted in the fundamental principles of transport phenomena, continuum mechanics, and physical metallurgy. This transition is important to the industry, because knowledge-based models provide a more scientific framework within which to effect process control and produce material with the desired shape, dimensions, and mechanical properties.

II. PREVIOUS WORK

Sellars and co-workers,[1-5] pioneers in the field of modeling microstructural evolution during hot rolling of steel strip and plate, have placed emphasis on the importance of quantifying the thermal field in the material and the associated metallurgical phenomena. Their model developed for thermomechanical processing of strip has been applied to the rolling of C-Mn steel, and the predicted austenite grain size compares favorably with the measurements. More recently, researchers at Nippon Steel,[6,7,8] Kawasaki Steel,[9,10] and Institut de Recherches de la Sidérurgie Francaise (IRSID)[11] have described their respective efforts in modeling the microstructure and mechanical properties of steel during hot rolling. Each research group has been successful in predicting the austenite grain size following finish rolling and the mechanical properties of the finished steel, particularly for plain C-Mn grades. However, it is important to note that although the equations employed by each research group to characterize dynamic and static recrystallization kinetics, recrystallized grain sizes, and grain growth kinetics bear some similarities, they also exhibit significant differences. A more detailed discussion of these differences together with the approaches adopted to model temperature and strain in the roll bite follow.

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Table I is a compilation of the equations employed by the different research groups to develop models to predict microstructural evolution during hot rolling of steel strip for plain C-Mn steels. During hot rolling, the first metallurgical phenomenon that must be accounted for is dynamic recrystallization, since it occurs during the course of deformation. When the dislocation substructure due to hot deformation becomes dense and inhomogeneous, new recrystallized grains may be nucleated. The mechanism of nucleation of recrystallized grains depends on the strain rate for a particular temperature. At a low strain rate, nucleation occurs by bulging of existing grain boundaries,[12] while at high strain rates, a cellular substructure of tangled dislocations develops throughout the grains giving rise to nuclei.[13] Dynamic recrystallization initiates at a critical strain, ε_c, which has been found to be slightly less than the peak strain. Sellars[1] and Saito et al.[10] have employed a relationship relating the critical strain to the Zener-Holloman parameter and the initial grain size to determine the onset of dynamic recrystallization, as can be seen in Table I. Through a series of measurements, Senuma and Yada[7] have found the critical strain to be independent of both the initial grain size and the strain rate; the relationship employed in their model is also given in Table I. McG. Tegart and Gittins[14] have suggested that the initial rolling passes may be conducive to dynamic recrystallization because of the high temperature and low strain rate.

The dynamic recrystallization kinetics for single-peak flow curves can be represented by an Avrami expression. Senuma and Yada[7] have employed an equation of this type (Table I) to track the progress of this phenomenon. Researchers at Kawasaki Steel[9,10] and IRSID,[11] following the earlier work of Sellars,[1] did not incorporate the kinetics of dynamic recrystallization in their models. The Kawasaki steel group assessed whether there was sufficient strain to initiate the event, and if so, the grain size was assigned a value equal to the dynamically

Table I. Summary of Reported Recrystallization and Grain Growth Relationships

University of Sheffield Sellars and Co-Workers[1-5]	Nippon Steel Yada et al.[6,7,8]	Kawasaki Steel Saito et al.[9,10]	IRSID Laboratories Perdrix[11]

Dynamic Recrystallization

University of Sheffield Sellars and Co-Workers[1-5]

$\varepsilon_p = 4.9 \cdot 10^{-4} d_0^{1/2} Z^{0.15}$

$\varepsilon_c = a\varepsilon_p$

$Z = \dot{\varepsilon} \exp \dfrac{Q}{RT}$

$Q = 312$ kJ/mol

(metadynamic)

$X = 1 - \exp\left(-0.693\left(\dfrac{t}{t_{0.5}}\right)\right)$

Nippon Steel Yada et al.[6,7,8]

$\varepsilon_c = 4.76 \cdot 10^{-4} \exp\left(\dfrac{8000}{T}\right)$

$d_{dyn} = 22,600 Z^{-0.27}$

$Z = \dot{\varepsilon} \exp \dfrac{Q}{RT}$

$Q = 267.1$ kJ/mol

$X_{dyn} = 1 - \exp\left(-0.693\left(\dfrac{\varepsilon - \varepsilon_c}{\varepsilon_{0.5}}\right)^2\right)$

$\varepsilon_{0.5} = 1.144 \cdot 10^{-5} d_0^{0.28} \dot{\varepsilon}^{0.05} \exp\left(\dfrac{6420}{T}\right)$

Kawasaki Steel Saito et al.[9,10]

$\varepsilon_c = 3.68 \cdot 10^{-4} Z^{0.19} d_0^{0.44}$

$d_{dyn} = 2.82 \cdot 10^4 Z^{-0.24}$

$Z = \dot{\varepsilon} \exp \dfrac{Q}{RT}$

$Q = 312$ kJ/mol

IRSID Laboratories Perdrix[11]

not incorporated

Static Recrystallization

University of Sheffield Sellars and Co-Workers[1-5]

$X = 1 - \exp\left(-0.693\left(\dfrac{t}{t_{0.5}}\right)^2\right)$

$\varepsilon < 0.8\varepsilon_p$

$t_{0.5} = 2.5 \cdot 10^{-19} \varepsilon^{-4} d_0^2 \exp\left(\dfrac{Q_s}{RT}\right)$

$\varepsilon > 0.8\varepsilon_p$

$t_{0.5} = 1.06 \cdot 10^{-5} Z^{-0.6} \exp\left(\dfrac{Q_s}{RT}\right)$

$Q_s = 300$ kJ/mol

$d_{rex} = 0.5 d_0^{0.67} \varepsilon^{-1}$

$(\varepsilon < \varepsilon^*)$

$d_{rex} = 1.8 \cdot 10^3 Z^{0.15}$

$(\varepsilon > \varepsilon^*)$

$\varepsilon^* = 0.57 d_0^{0.17} \varepsilon_p$

Nippon Steel Yada et al.[6,7,8]

$X = 1 - \exp\left(-0.693\left(\dfrac{t}{t_{0.5}}\right)^2\right)$

$t_{0.5} = 2.2 \cdot 10^{-12} S_v^{-0.5} \dot{\varepsilon}^{-0.2} \varepsilon^{-2} \exp\left(\dfrac{30,000}{T}\right)$

$d_{rex} = \dfrac{5}{S_v \varepsilon^{0.6}}$

$S_v = \dfrac{24}{\pi d_0}(0.491e^\varepsilon + 0.155e^{-\varepsilon} + 0.1433e^{-3\varepsilon})$

Kawasaki Steel Saito et al.[9,10]

$X = 1 - \exp\left(-0.693\left(\dfrac{t}{t_{0.5}}\right)^2\right)$

$t_{0.5} = 2.5 \cdot 10^{-19} \varepsilon^{-4} d_0^2 \exp\left(\dfrac{Q_s}{RT}\right)$

$Q_s = 300$ kJ/mol

$d_{rex} = 0.5 d_0^{0.67} \varepsilon^{-1}$

IRSID Laboratories Perdrix[11]

$X = 1 - \exp\left(-0.693\left(\dfrac{t}{t_{0.5}}\right)^{n_r}\right)$

$t_{0.5} = 3.67 \cdot 10^{-14} \varepsilon^{p''} \dot{\varepsilon}^{-0.28} \cdot d_0^{0.14} \exp\left(\dfrac{Q_s}{RT}\right)$

$n_r = 272 d_0^{-0.155} \varepsilon^{-0.5} \cdot \exp\left(\dfrac{-37,492}{RT}\right)$

$d_{rex} = 18.51 \ln\left(\dfrac{T}{973}\right) \cdot d_0^{0.374} \varepsilon^u \dot{\varepsilon}^{-0.1}$

$u = -0.5 d_0^{0.267}\left(\dfrac{973}{T}\right)^{3.933}$

$p'' = -0.86 d_0^{0.24}$

$Q_s = 301$ kJ/mol

Grain Growth

University of Sheffield Sellars and Co-Workers[1-5]

$d^{10} = d_0^{10} + At \exp\left(\dfrac{-Q_{gg}}{RT}\right)$

$T > 1100$ °C

$A = 3.87 \cdot 10^{32}$

$Q_{gg} = 400$ kJ/mol

$T < 1100$ °C

$A = 1.31 \cdot 10^{52}$

$Q_{gg} = 914$ kJ/mol

Nippon Steel Yada et al.[6,7,8]

$d^2 = d_0^2 + At \exp\left(\dfrac{-Q_{gg}}{RT}\right)$

$A = 1.44 \cdot 10^{12} \quad \dfrac{Q_{gg}}{R} = 32,100$

Kawasaki Steel Saito et al.[9,10]

$d^{10} = d_0^{10} + At \exp\left(\dfrac{-Q_{gg}}{RT}\right)$

$T > 1100$ °C

$A = 3.87 \cdot 10^{32}$

$Q_{gg} = 400$ kJ/mol

$T < 1100$ °C

$A = 1.31 \cdot 10^{52}$

$Q_{gg} = 914$ kJ/mol

IRSID Laboratories Perdrix[11]

$d = d_{rex}\left(1 + \alpha \ln \dfrac{t}{t_{rex}}\right)$

$\alpha = 0.195$

(C-Mn-Al steels)

$\alpha = 0.098$

(Nb grades)

recrystallized grain size determined from an equation of the form

$$d_{dyn} = BZ^{-a} \qquad [1]$$

It is important to note, however, that if the equilibrium grain size for the given conditions is greater than twice the initial grain size, grain coarsening occurs instead of grain refinement.[15,16] Dynamic recovery occurs during hot deformation and accounts for the steady-state flow stress observed in steels that do not dynamically recrystallize and for the prepeak softening in dynamically recrystallizing steels. Static recovery initiates as soon as deformation is complete and extends into the incubation period of static recrystallization. Although the dislocation density within the subgrains is reduced, there is little change in the size or shape of the subgrain and effects on retained strain have been found to be small.[17] At the present time, static recovery has been ignored by all the modelers, except the Kawasaki researchers.[9,10,18]

Static recrystallization, on the other hand, is a process by which a large number of dislocations are simultaneously annihilated. The recrystallization kinetics are well described by the Avrami equation, as indicated in the models summarized in Table I. Based on an analysis of several C-Mn and low-alloy steels, Sellars[5] has reported that a time exponent, $n = 2$, is satisfactory for strains less than the critical value for dynamic recrystallization. However, for strains in excess of the critical strain, metadynamic recrystallization occurs after deformation.[19] For this condition, Glover and Sellars[20] have shown, with the aid of direct metallographic observations, that the time exponent n decreases to a value of 1.

An examination of Table I shows that in the models developed to date, with the exception of the IRSID model, the static recrystallization time exponent n has been assigned a constant value. The IRSID researchers[11] have derived an expression for n as a function of initial grain size, strain, and temperature.

From Table I, it is also evident that the time for 50 pct recrystallization, $t_{0.5}$, is different in each of the microstructure models. The principal difference is that Sellars' equation,[5] used also by researchers at Kawasaki, does not contain a strain-rate term in the expression for $t_{0.5}$, while the others do. Yada[6] accounts for the effects of initial grain size on $t_{0.5}$ with a term S_v, representing the grain boundary area per unit volume, a parameter influenced by the degree of deformation.

There are also significant differences in the expressions employed by different researchers to calculate the recrystallized grain size and in their treatment of partial recrystallization. Partial recrystallization can occur if there is insufficient time at temperature or if the strain level at a given temperature is inadequate to promote full recrystallization. Perdrix,[11] at IRSID, has advocated the characterization of the partially recrystallized structure as a mean grain size equal to the recrystallized grain size; he found the mean grain size to be independent of the softening ratio. For instance, for softening ratios of 0.2, 0.6, and 1, the mean grain sizes were found to be 55, 54, and 53 μm, respectively, with the distribution being more nonuniform for low softening ratios due to the presence of a few large unrecrystallized grains. During subsequent deformation, the strain is assumed to distrib-

ute homogeneously throughout the matrix and is related to the applied strain, ε_1, and the fraction recrystallized, X, through the following expressions:

$$\varepsilon_{eff} = 0.5\varepsilon_1(1 - X) \qquad X \geq 0.1 \qquad [2]$$

$$\varepsilon_{eff} = \varepsilon_1 \qquad X < 0.1 \qquad [3]$$

Yada[6] has related the recrystallized grain size to the deformed grain boundary area, S_v, as shown in Table I. In the event of partial recrystallization, a mean grain size is computed according to the expression

$$d = d_{rex}(0.2 + 0.8X) \qquad [4]$$

He computes a residual strain, based on changes to the dislocation density, which is added to the nominal strain in subsequent deformation steps.

The equations proposed for recrystallized grain size by Sellars[5] are employed by the researchers at Kawasaki Steel for their model. In the event of partial recrystallization, the structure is considered to be mixed. On subsequent deformation, it was assumed that the strain was distributed equally to both recrystallized and unrecrystallized regions, with the unrecrystallized regions retaining the strain imparted in the previous deformation.[21,22] It has been acknowledged that this treatment is complicated, since after successive deformation passes, the number of different components in the structure becomes significant and treating each component independently is unrealistic. More work is clearly required to understand the behavior of partially recrystallized structures.

Although recrystallization of austenite removes the relatively high internal energy imparted by the hot deformation, the structure is still metastable. Further reduction in the overall internal energy occurs by a reduction of the total austenite grain boundary area. Sellars[1] deduced the following equation for grain growth based on experimental data:

$$d^m = d_0^m + Ct \exp\left(\frac{-Q_{gg}}{RT}\right) \qquad [5]$$

Theoretically, m is expected to be 2 for a pure metal. Senuma and Yada[7] at Nippon Steel have reported experimental agreement with this equation for a grain size exponent, $m = 2$, as shown in Table I. However, many other investigators have obtained values of m ranging from 6 to 10. For instance, Campbell et al.[23] obtained a value of 6 for m, 8.2×10^{24} for C, and 400 kJ/mol for Q by fitting Eq. [5] to torsion data. This is in accordance with Licka and Wozniak[24] and similar to the findings of Hawbolt et al.[25] ($m = 7.5$, $C = 4.2 \times 10^2$, and $Q = 400$ kJ/mol). These high values for the exponent and the exponential dependence of grain size on temperature suggest that grain sizes below a certain limit cannot be maintained, even for very short times. This may be because the grain size equation, Eq. [5], has been tested using grain growth data measured over relatively long times equal to or in excess of 100 seconds. Equivalent grain growth data are not available to accurately characterize the initial stages of grain growth for times up to 2 seconds, which corresponds to the interstand times in a strip mill. Until such data become available, grain growth equations of the form described in Eq. [5] have to be employed.

In addition to characterizing the metallurgical phenomena, it is important that the temperature field, strain distribution, and strain rate are accurately known, since they influence the metallurgical state. Less attention has been devoted to this aspect of modeling. Most of the models[6–11] ignore the through-thickness thermal gradients, particularly in the roll bite, and are based on the assumption of homogeneous strain.

III. SCOPE AND OBJECTIVES

Modeling microstructural evolution during hot-strip rolling is a relatively new activity for the research community. The efforts to date have been directed toward incorporating a selected set of equations characterizing the metallurgical phenomena in a relatively simple model describing the thermal state of the steel in the roll bite.

A research program directed at predicting the microstructural evolution during the hot rolling of steel was initiated at the Centre for Metallurgical Process Engineering at the University of British Columbia. The objectives of the overall program were as follows.

(a) To determine the roll/strip interface heat-transfer coefficient and utilize it to determine the thermal field in the roll bite. The results of this study were included in the first of this three-part series.[26]
(b) To develop techniques to compute the roll forces based on the calculated thermal field. This is the emphasis of the second part of this three-part series.[27]
(c) To develop a model incorporating heat flow and structure-modifying phenomena to evaluate the influence of processing parameters on austenite grain size evolution. This was the objective of this third part.

Existing correlations for recrystallization and grain growth, many of which are presented in Table I, have been evaluated by comparing predictions with experimental measurements made on 0.34 and 0.05 pct carbon steels. Equations describing the isothermal kinetics that best fit each of the metallurgical phenomena have been selected on this basis and incorporated in the heat-transfer model described in the first of this series of papers.[26] The principle of additivity has been invoked to apply isothermal kinetics to the nonisothermal conditions that were obtained in a strip mill. Finally, the validity of the model has been examined by comparing model predictions of austenite grain size with measurements made on samples rolled on a pilot mill for conditions simulating Stelco's Lake Erie Works (LEW) hot-strip mill. The model has been employed to examine the influences of pass scheduling and rolling temperature on austenite grain size evolution in the finishing mill.

IV. METALLURGICAL PHENOMENA

A. Static Recrystallization

The following techniques were employed to determine the extent of recrystallization that occurs after hot deformation. The microstructures of test samples deformed and quenched under conditions simulating hot-strip mill processing were examined and quantified. In addition,

multihit strip mill simulation compression tests were conducted on the cam plastometer at CANMET and on the Gleeble at the University of British Columbia to measure the static recrystallization behavior as revealed by fractional softening.

1. Determination of recrystallization by metallography
To determine the degree of recrystallization from microstructural evaluation, single-hit Gleeble compression tests were conducted on a 0.34 pct carbon steel* at

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*Similar tests were not conducted on 0.05 pct carbon steel because of the difficulty of revealing prior austenite boundaries in the low-carbon steel.

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two temperatures, 900 °C and 850 °C, at a strain rate of 1 s^{-1}. Following deformation, individual test samples were helium quenched (30 °C/s to 40 °C/s) after delay times of 0.5, 2, 5, and 10 seconds. The helium quench enhanced the visibility of the prior austenite grain boundaries by encouraging proeutectoid ferrite growth on these boundaries during quenching. Although it has been demonstrated by Kopp et al.[28] that axial compression tests give rise to strain inhomogeneity, they have also shown that the local strain approaches the nominal strain in certain regions of the compression sample. Thus, in this study, microstructure evaluation was conducted in those regions where the local strain approximates the nominal strain. The polished surfaces of quenched samples were etched in a modified aqueous picric acid etchant at 80 °C for 10 to 45 minutes. After etching, the samples were examined or lightly polished and re-etched with 2 pct nital. The microstructures obtained for the 0.34 pct carbon steel initially deformed to a strain of 0.3 at 900 °C, at a strain rate of 1 s^{-1}, then held for delay times of 0.5 and 10 seconds are shown in Figure 1. In Figure 1(a), large grains characteristic of the deformed, unrecrystallized austenite are apparent with a few smaller recrystallized grains often located at three grain intersections on prior austenite grain boundaries. For the 10-second delay time (Figure 1(b)), the area fraction of the recrystallized equiaxed grains has increased. The degree of recrystallization visible in the microstructure was determined using a point grid area counting procedure.[29] The corrected metallographically observed average percent recrystallization for the 0.34 pct carbon steel at the two test temperatures, 850 °C and 900 °C, for different delay times has been compared using the predicted equations proposed by Sellars[1] and IRSID,[11] as shown in Table II. It is clear for these two tests that the IRSID equation[11] yields better agreement with the metallographic observations.

2. Determination of recrystallization by fractional softening tests
Double-hit compression tests conducted on the Gleeble at the University of British Columbia and on the cam plastometer at CANMET were used to determine recrystallization kinetics from fractional softening. A typical example of the true stress–true strain curves obtained during the first and second stages of deformation for the 0.34 pct steel deformed at 850 °C, with an interhit time of 0.5 second, is shown in Figure 2. The flow stress at

(a)

% Recrystallized = 8

(b)

% Recrystallized = 33
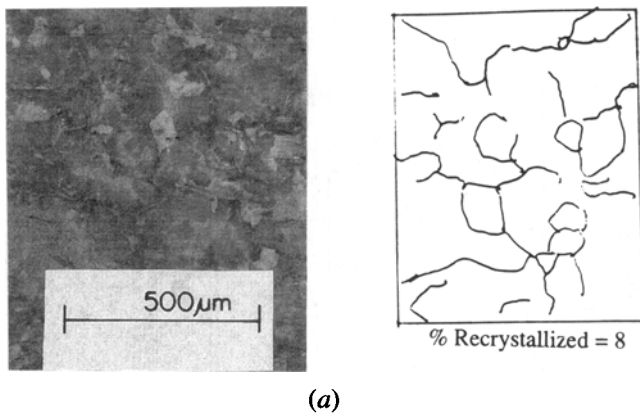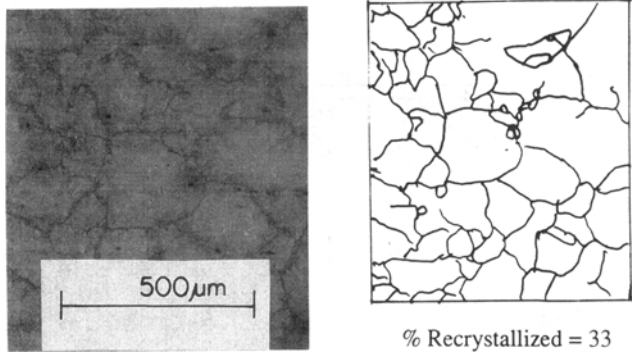
Fig. 1—Microstructure of a 0.34 pct carbon steel obtained after a single deformation at 900 °C to a strain of 0.3 at a rate of 1 s^{-1} with delay times of (a) 0.5 s and (b) 10 s.

a strain of 0.2 pct is termed the yield stress and is denoted by σ_I in Figure 2. At the instant the test is interrupted, the corresponding flow stress is denoted by σ_{II}. On reloading, after the 0.5- second delay, the measured 0.2 pct yield stress σ_0 attains a value between σ_I and σ_{II} due to softening. The restoration parameter, R_y, given by the equation

$$R_y = \frac{\sigma_{II} - \sigma_0}{\sigma_{II} - \sigma_I} \qquad [6]$$

Fig. 2—Flow curves obtained by a double-hit test on 0.34 pct carbon steel at 850 °C at an average strain rate of 1 s^{-1} and an interhit delay time of 1 s.

is a measure of the combined effects of static recovery and recrystallization. According to Perdrix,[11] translating the initial stress-strain curve onto the second deformation curve, as shown by the dotted line, to make it coincide with the stress-strain curve of the second deformation defines a new stress value, σ_{III}, which can be employed to determine the extent of restoration which excludes static recovery. This method, which is termed the back-extrapolation method, gives a restoration index, R_b, defined as follows:

$$R_b = \frac{\sigma_{II} - \sigma_{III}}{\sigma_{II} - \sigma_I} \qquad [7]$$

However, Jonas[30] has reported that the back-extrapolation method accounts for only a small portion of the recovery component. For plain-carbon steels, recrystallization is found to start at softening ratios, R_b, of 10 and 15 pct at temperatures of 1000 °C and 900 °C, respectively. Thus, in the current work, to extract recrystallization data from restoration data obtained by the back-extrapolation method, it is assumed that recovery occurs at the beginning of restoration and recrystallization commences at

Table II. Comparison of the Degree of Recrystallization Obtained Using Metallography with That Predicted by the IRSID and Sellars' Relationships

Test Temperature (°C)	Strain Rate (s^{-1})	Strain	Delay (s)	Average Pct Recrystallized Obtained on 5 Areas Equivalent to Figure 1	Pct Recrystallized Metallographic Measurements Corrected for Cooling Rate	Additional* Recrystallization due to Helium Cooling at $\approx$30 °C/s (Pct)	IRSID Pct Recrystallized	Sellars Pct Recrystallized
850	1	0.3	0.5	4	3	1	0.0	0.0
			2	5	4	1	1.3	0.0
			5	6	4	2	3.5	0.0
			10	11	8	3	7.3	2.0
900	1	0.3	0.5	11	9	2	0.0	0.0
			2	18	15	3	2.3	0.0
			5	27	20	7	7.0	0.0
			10	31	19	12	16.4	2.5

*This value represents the difference in observed recrystallization between water-quenched and helium-cooled samples. The etching procedure revealed only random patches of prior austenite boundaries in the water-quenched samples, making it difficult to determine the percent recrystallized.
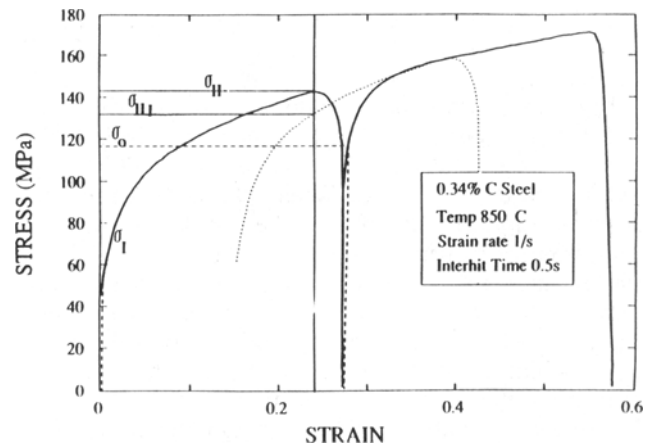
softening ratios of 12.5, 15, 16.25, and 17.5 pct at temperatures of 950 °C, 900 °C, 875 °C, and 850 °C, respectively.

The above procedure was employed to analyze the multihit cam plastometer tests conducted on the 0.34 and 0.05 pct carbon steels at temperatures of 875 °C and 950 °C, respectively, for the strain rates and interhit times given in Table III. Samples of the test data for the two grades are shown in Figures 3 and 4. It can be shown that an increase in the delay times and/or temperature increases fractional softening; i.e., with increasing delay time, the flow stress on the second hit approaches that of the initial hit. It should be emphasized that these test temperatures and interhit delay times were selected on the basis of available literature to show the effect of recrystallization and to test and obtain the necessary empirical coefficients describing the recrystallization kinetics.

The recrystallization data have been plotted against time in Figures 5 and 6 for the 0.34 and 0.05 pct carbon steels, respectively, and are seen to correspond to the sigmoid-shaped curves that can be described by the Avrami equation given in Table I. The Avrami pre-exponential and time exponent constants have been determined from the data for each grade, using the appropriate $t_{0.5}$ relationships listed in Table I. The results have been employed to calculate the recrystallization curves shown in Figures 7 and 8, using both the Sellars and IRSID analyses (Table I). It can be seen that the Sellars equations tend to overestimate the time required for recrystallization, while the IRSID equations give good agreement with the measurements. The principal difference between the IRSID and Sellars equations lies in the expression for $t_{0.5}$. The IRSID equations contain a strain-rate term which indicates that an increase in strain rate decreases the time for 50 pct recrystallization. The recent work of Laasraoui et al.[31] is in concurrence with the IRSID work. This effect would be expected to be more significant in strip rolling operations where the strain rates are high.

3. Recrystallized grain size

Although the equations to predict recrystallization behavior, as formulated by Sellars[1] and IRSID,[11] yielded

Table III. Cam Plastometer Multihit Deformation Tests for 0.34 Pct and 0.05 Pct Carbon Steel

Steel (Pct C)	Temperature (°C)	Strain Rate (s^{-1})	Interhit Time (s)
0.34	875	1	0.5
		9.5	0.5, 1, 9
		12	0.5, 1, 9, 10
	950	9.5	0.5, 1, 3, 10, 30
		12	0.5, 1, 3, 10, 30
		21	1.1
		28	1.1, 1.6
		32	1.1, 1.6, 9
		50	1.1, 1.6
0.05	875	9.5	0.5, 1
		12	0.5, 1, 3, 10, 30
	950	9.5	0.5, 1, 3, 10, 30
		12	0.5, 1, 3, 10, 30
		32	0.4, 1.1, 1.6, 3, 10
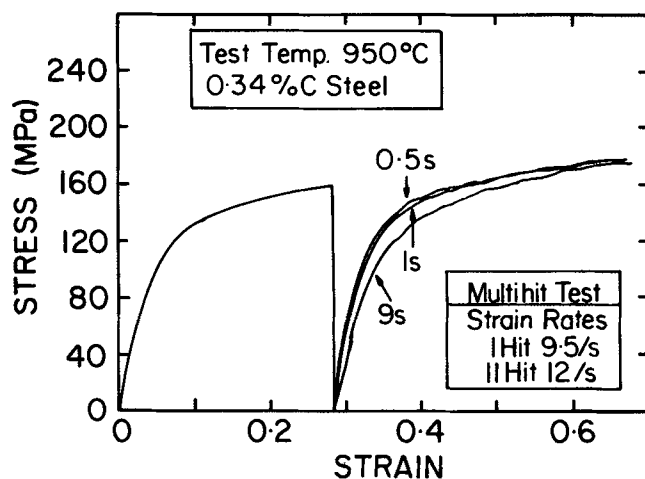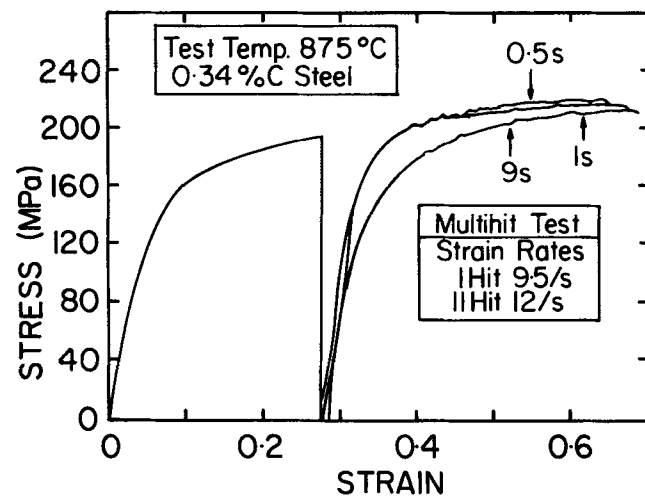
Fig. 3—(a) Stress-strain curves obtained by conducting double-hit compression tests on a 0.34 pct carbon steel at 875 °C for holding times of 0.5, 1, and 9 s. (b) Stress-strain curves obtained by conducting double-hit compression tests on a 0.34 pct carbon steel at 875 °C for holding times of 0.5, 1, and 9 s.
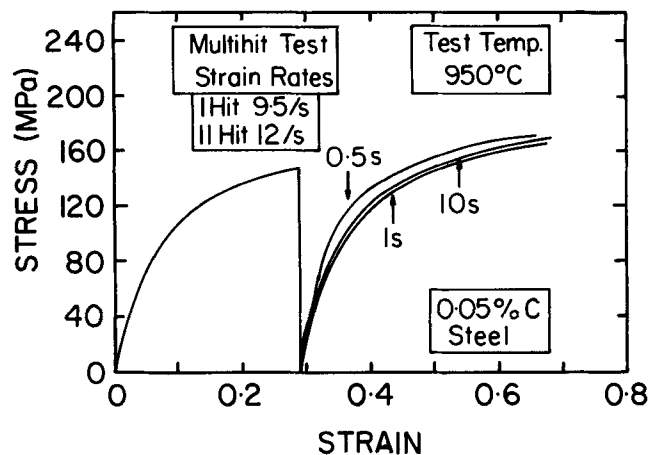

Fig. 4—Stress-strain curves obtained by conducting double-hit compression tests on a 0.05 pct carbon steel at 950 °C for holding times of 0.5, 1, 3, 10, and 30 s.
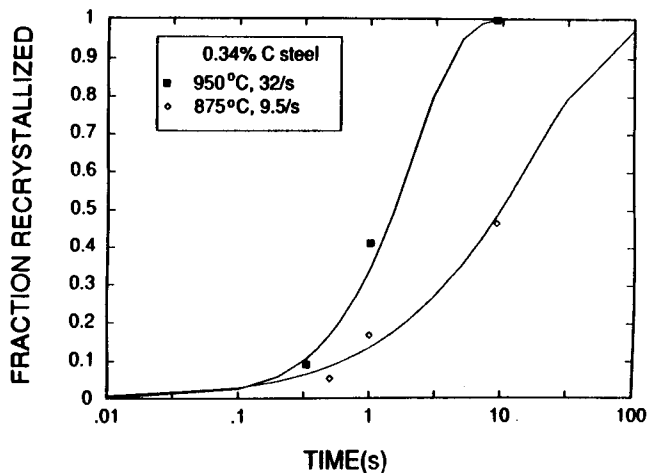
Fig. 5—Recrystallization kinetics for a 0.34 pct carbon steel obtained by fractional softening measurements, using the back-extrapolation procedure corrected for recovery.

Fig. 7—Comparison of measured fractional softening and predicted recrystallization kinetics for the 0.34 pct carbon steel.

different results, the corresponding predictions of recrystallized grain sizes for plain-carbon steels were found to be very similar. Since the IRSID equations were chosen to represent the static recrystallization kinetics of the 0.34 and 0.05 pct carbon steels under investigation, for the sake of consistency, these equations were also employed to predict recrystallized grain size.

To assess the relative merits of the different methods proposed to treat partially recrystallized structures, the grain size predicted by each method was compared with the measurements. Details of three multihit tests conducted on the cam plastometer for a 0.34 pct carbon steel, which would result in partial recrystallization after the first hit, are given in Table IV. Following the final test and after sufficient delay time to ensure complete recrystallization, the samples were quenched for subsequent grain size determination. The final measured grain size was compared with predictions based on Sellars'[1] early strain partitioning approach, the IRSID method,[11] and the treatment by Anelli et al.[32] developed for plates; the results are shown in Figure 9. The grain sizes ob-
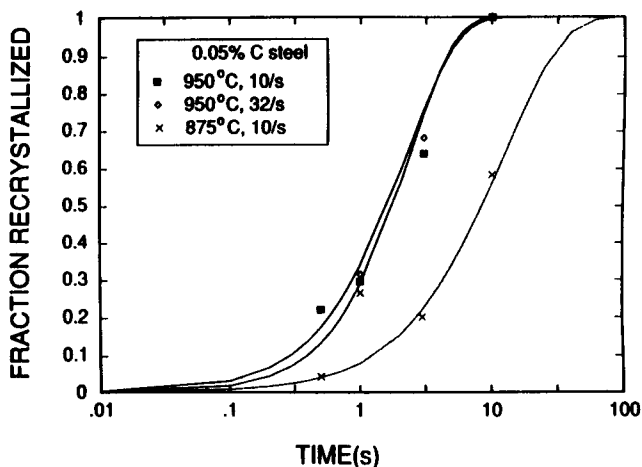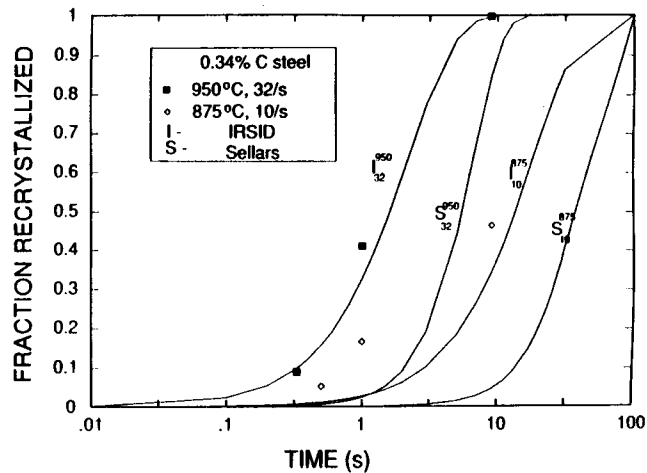
served in the quenched sample are reasonably uniform and, therefore, are represented by a single point on the graph. It is evident that both IRSID's and Sellars' approaches gave grain sizes in agreement with the measurements. The results based on the method adopted by Anelli et al.[32] did not compare as well. In this work, IRSID's approach of utilizing an average grain size and an effective retained strain has been adopted.

B. Dynamic Recrystallization

From Table I, it is evident that dynamic recrystallization is treated less rigorously than static recrystallization in the various models. No experimental work on dynamic recrystallization was conducted as part of this study. In the current model, the instantaneous strain in the roll bite is compared to the critical strain calculated from the expression given by Sellars[1] to assess whether dynamic recrystallization is likely to initiate. If it is, then metadynamic recrystallization kinetics are employed in place of static recrystallization kinetics in the interstand region. The metadynamic recrystallization kinetics have

Fig. 6—Recrystallization kinetics for the 0.05 pct carbon steel obtained by fractional softening measurements using the back-extrapolation procedure corrected for recovery.
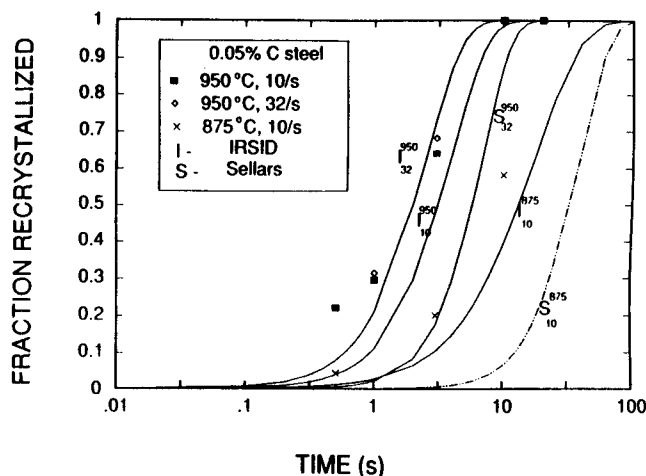

Fig. 8—Comparison of measured fractional softening and predicted recrystallization kinetics for the 0.05 pct carbon steel.

Table IV. Deformation Conditions for Multihit Tests Conducted on the 0.34 Pct Carbon Steel to Yield Partial Recrystallization between Hits

Temperature (°C)	Strain Rate (s^{-1})	Interhit Time (s)
1000	21, 28	1.1
950	32, 50	1.1
950	28, 32, 50	1.6, 1.1

been defined by Sellars to have a form similar to that for static recrystallization with a time exponent equal to 1 and $t_{0.5}$ corresponding to a strain greater than $0.8\varepsilon_p$, as shown in Table I.

C. Grain Growth

From the literature, it is apparent that high-temperature austenite grain growth data immediately following recrystallization in the short time intervals of 0 to ~2 seconds, corresponding to interstand times on a hot-strip mill, are not available. The empirical correlations proposed by various researchers are generally based on grain growth data corresponding to longer times. In this study, the equation

$$d^{7.5} = d_{rex}^{7.5} + 4.2 \times 10^{27}\, t \exp\left(\frac{-400.0}{RT}\right) \qquad [8]$$

has been employed to calculate grain growth. The constants in this equation were obtained by performing a regression analysis on data obtained by Hawbolt and co-workers[25,33] for a eutectoid plain-carbon steel isothermally annealed at temperatures of 800 °C to 1100 °C for growth times extending to 600 seconds.

V. MODEL OF MICROSTRUCTURAL EVOLUTION

The temperature of the strip changes continuously as the steel passes through the finishing mill. The kinetic equations for recrystallization and grain growth have all been determined under isothermal conditions. The principle of additivity is extremely useful in modeling, because it allows isothermal kinetic data to be employed for the determination of structural changes occurring during continuous cooling or heating. The applicability of the principle of additivity to recrystallization and phase transformations has been successfully demonstrated;[34,35,36] recent unpublished research[33] has also validated its applicability to austenite grain growth. There is no principle equivalent to additivity to deal with the effects of continuously varying strain rate on dynamic recrystallization or the subsequent static recrystallization. The equations given in Table I are for the isostrain-rate condition, which is not the case in the roll bite. Thus, it is difficult to treat varying strain-rate conditions.

To model the microstructure evolution in the finishing mill, the following procedure was adopted. The roll bite was discretized into a series of slices, each vertical slice containing nodal volumes, as shown in Figure 10. Each slice is assumed to move in plug flow through the roll bite.

(a) The temperature distribution in the roll bite at each stand was computed with the heat-transfer model described in the first of this series of papers.[26] The strain and strain rate have been computed from the reduction; the former has been assumed to be uniform for each vertical slice and the latter constant in the roll bite, owing to the difficulty of characterizing the associated effect on metallurgical phenomena.

(b) Within the roll bite, the instantaneous strain is compared to the critical strain to assess whether dynamic recrystallization is likely to initiate. If it is, the metadynamic recrystallization equations are employed on exit from the roll bite to determine the degree of recrystallization.
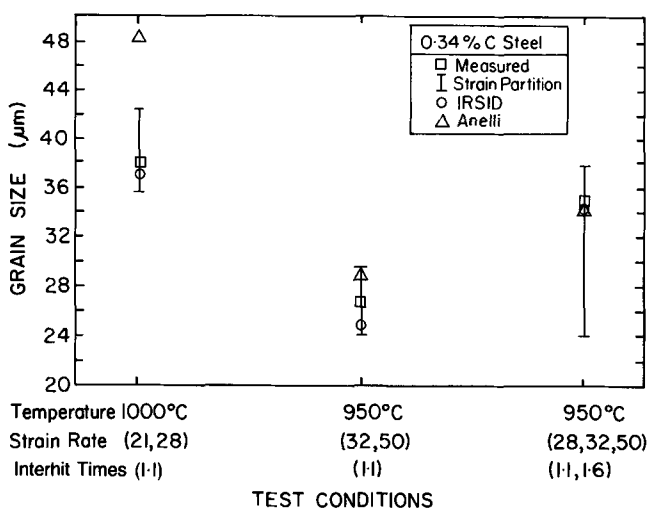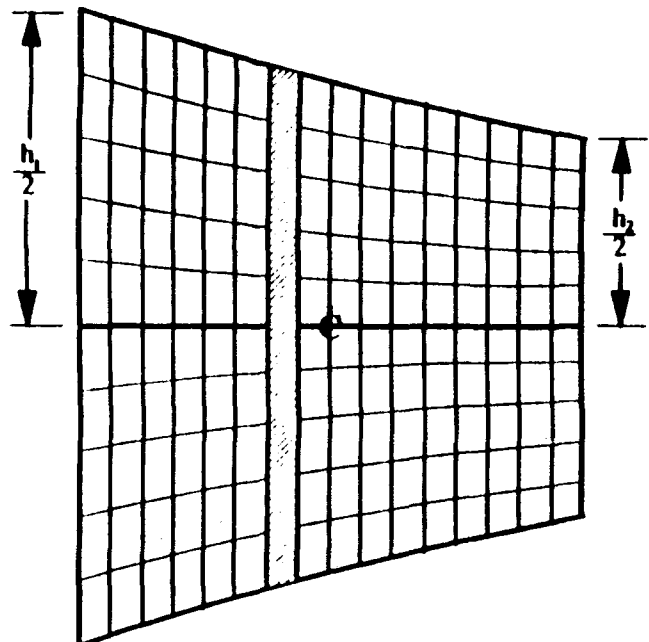

Fig. 9—Comparison of measured and predicted grain sizes for a 0.34 pct carbon steel for three different isothermal multihit tests which showed partial recrystallization during the course of the tests.

Fig. 10—Schematic diagram of roll-bite discretization.

(c) If insufficient strain is realized for dynamic recrystallization, then static recrystallization is allowed to proceed in the interstand region. The temperature at each nodal volume is computed by the heat-transfer model.
(d) If recrystallization is partially complete, then an effective strain and a mean grain size are computed according to the method proposed by Perdrix,[11] which serve as input conditions for the computations for the next stand.
(e) If recrystallization is complete prior to reaching the next stand, grain growth equations are invoked to determine the grain size prior to entry to the next stand.
(f) The procedure is repeated for successive stands.

An important difference between the current work and many of the earlier models, with the exception of Sellars,[2] is that the temperature distribution through the steel both within and outside the roll bite is accurately characterized for inclusion in the model. All of the microstructural phenomena—dynamic and static recrystallization and grain growth—are thermally activated; temperature appears in an exponential term and, hence, any inaccuracy in computing temperature will have a strong influence on the predictions. The effects of strain and strain rate are second order as compared with temperature and, as a first step, have been assumed to be uniform through a vertical slice in the roll bite.

Currently, finite element models are under development to predict the strain rate and strain distribution through the roll bite during hot rolling of steel strip. The models have been applied, to a limited extent, to industrial rolling operations[3,37,38] but have not yet been applied to the development of microstructures. Preliminary analysis has shown that there is a region of high strain

rate near the entrance just beneath the roll due to redundant shear resulting from friction;[3] the strain rate locally approaches 5 to 10 times the average strain rate, based on a limited number of studies. This is unlikely to significantly alter the final microstructure since it is localized, but will have to be assessed in the next generation of models. It has also been shown that strain inhomogeneity is limited to a narrow shear zone just beneath the roll in the chill zone, with over 70 pct of the central thickness subjected to the nominal strain.[3,39] This will likely affect the subsequent static recrystallization kinetics in the shear zone, but over most of the thickness, the assumption of uniform strain is reasonably valid.

VI. MODEL VALIDATION

To validate the microstructural model, a series of tests was conducted on the pilot mill at CANMET to determine the changes to austenite grain size as a result of one through four passes of rolling, each pass simulating an individual stand of Stelco's LEW hot-strip mill; the details are given in Table V. The pilot mill at CANMET's Metals Technology Laboratory consists of a single two-high reversible hot-rolling stand. The feeding of the workpiece into the mill is done manually for the initial pass and all subsequent reversing passes and all mill data being acquired on the mill computer.

Prior to rolling, each sample was reheated to the desired temperature in the programmable reheating facility. A total of four tests was conducted for each grade, the conditions being given in Table VI. The first test is a simulation of the first stand, the second a simulation of stands I and II, the third a simulation of stands I through III, and the fourth a simulation of all four stands. The rolled samples were quenched in an ice bath within a period of 5 seconds following the last deformation. A comparison of Tables V and VI shows that the rolling temperature and degree of reduction experienced in successive stands of the LEW mill are effectively reproduced in the CANMET simulation tests. However, it is apparent that on this reversing pilot mill, it is impossible to simulate the short interstand times and the strain rates that are realized in the production mill for stands III and IV.

Table V. A Typical Rolling Schedule at Stelco's Lake Erie Works Mill

Pass Number	Reduction (Pct)	Average Strain Rate (s⁻¹)	Center Temperature (°C)	Interstand Time (s)
I	43	17.58	1077	3.04
II	41	38.03	1060	1.8
III	25	49.99	1020	1.24
IV	17	57.1	970	—

Table VI. Simulation Tests Conducted on the CANMET Pilot Mill for Assessing Microstructural Change in a 0.34 Percent Carbon Steel

Test Number and Number of Passes	Reheating Temperature (°C)	Reduction (Pct)	Strain Rate (s⁻¹)	Center Temperature (°C)	Interstand Time (s)	Quenching Time (s)
1	1100	43	17.5	1060	—	5
2	1100	43	17.5	1067	10	5
		41	33.3	1040		
3	1100	43	17.4	1060	11	5
		41	33.1	1030	9.5	
		25	30.4	986		
4	1120	43	17.4	1070	11	6
		41	33.2	1057	8	
		25	30.3	1009	10	
		17	27.4	923	—	

The quenched samples were etched to determine the prior austenite grain size. The microstructures obtained from each test are shown in Figure 11. Each structure consisted of uniform, equiaxed grains, indicating that complete recrystallization had occurred. This is to be expected, since a delay time of approximately 5 to ~6 seconds occurred after rolling prior to water quenching.

The microstructural model was employed to simulate the tests, the model-predicted grain sizes being compared with measurements on samples quenched following one or more deformation passes, as shown in Figure 12. The centerline and surface temperatures computed by the heat-transfer model are also shown in the figure. The starting grain size in the simulation tests was 180 μm, corresponding to prior austenite grain size measured on transfer bar samples. A dramatic decrease in grain size from 180 to 67 μm occurred after the first pass, which compares favorably with model predictions, as can be seen in Figure 12. This decrease in grain size can be associated with the large reduction in pass I. The measured average grain sizes after two and three passes were observed to be 34 and 25 μm, in good agreement with model predictions. The reduction in magnitude of the austenite grain size after the third pass is significantly less than after the first and second passes. This is due
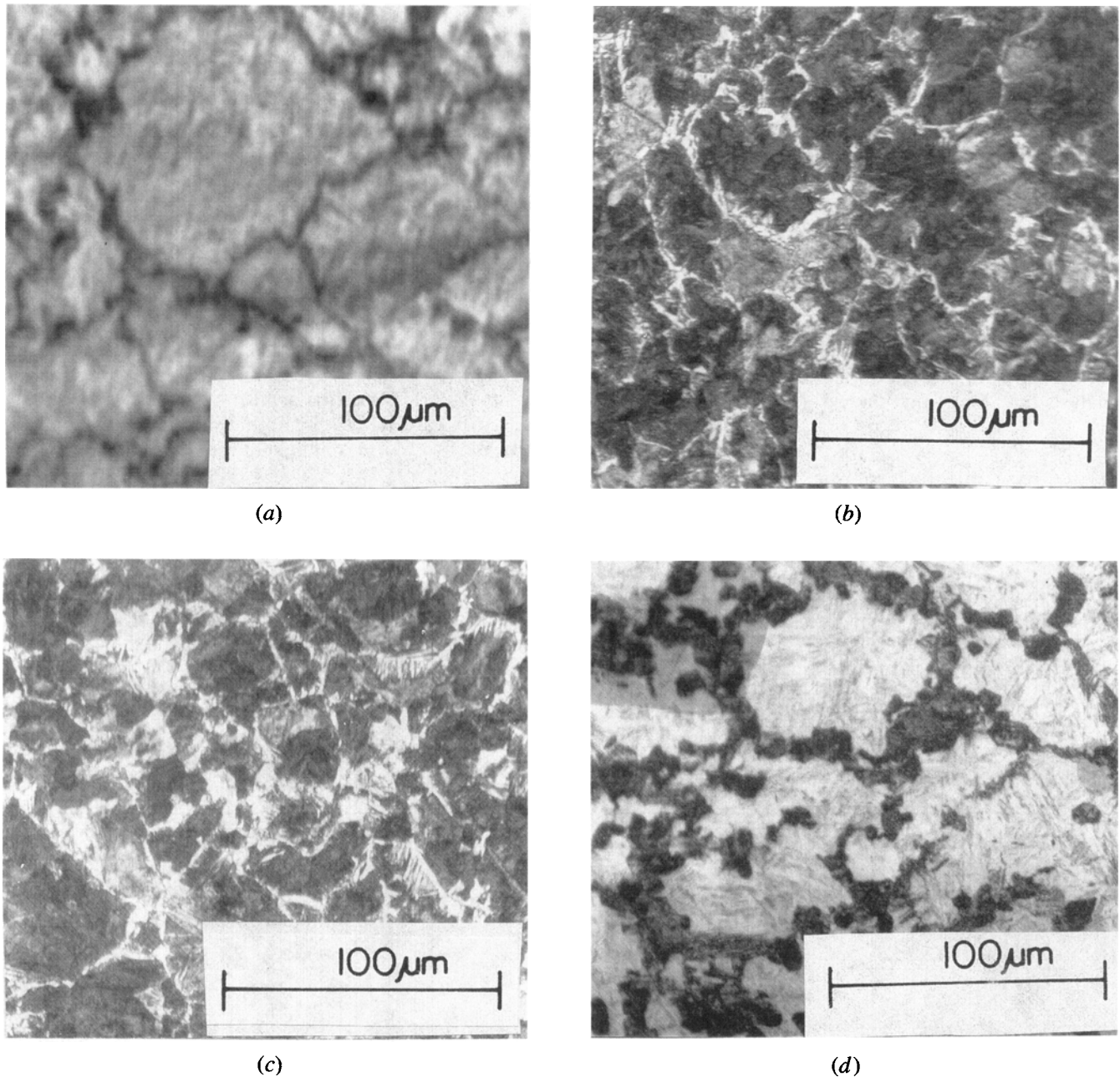

Fig. 11—Microstructures obtained after hot rolling and quenching for different numbers of passes: (a) one pass at 1060 °C; (b) two passes at 1060 °C and 1040 °C; (c) three passes at 1060 °C, 1030 °C, and 986 °C; and (d) four passes at 1070 °C, 1057 °C, 1009 °C, and 923 °C.
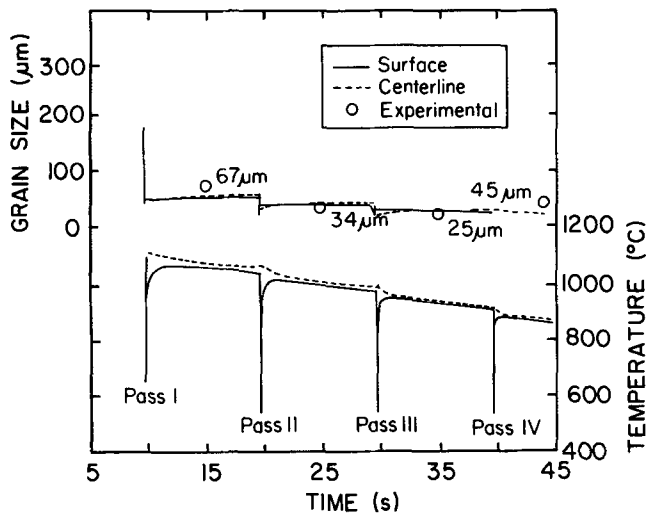
Fig. 12—Comparison of the predictions of the microstructural model with the experimentally measured grain size obtained by pilot mill simulation at CANMET on a 0.34 pct carbon steel.

to the lower reduction in pass III. Following the fourth pass, an increase in the measured grain size was observed contrary to predictions. This could have been due to an error in the temperature measurement, resulting in a higher rolling temperature for that test.

It is evident that the model is capable of predicting grain size evolution on the pilot mill for conditions similar to the industrial mill. However, this simulation did not test the capability of the model to describe partial recrystallization between stands and its effects on microstructural evolution in subsequent stands because of the long interpass times obtained on the reversing pilot mill.

VII. APPLICATION OF THE MODEL TO INDUSTRIAL ROLLING PRACTICE

To examine the capability of the model to simulate an industrial rolling schedule, the model has been employed to compute the changes in austenite grain size during the rolling of 0.34 pct carbon steel at Stelco's LEW mill, according to the schedule given in Table V. The tem-
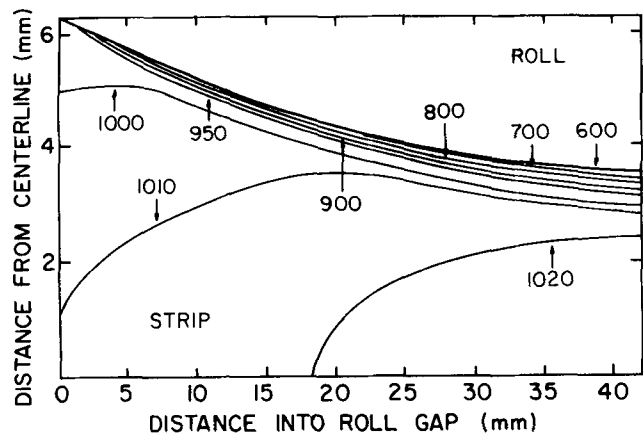

Fig. 13—Predicted temperature contours in the roll bite of the second stand for a 0.34 pct carbon steel strip.
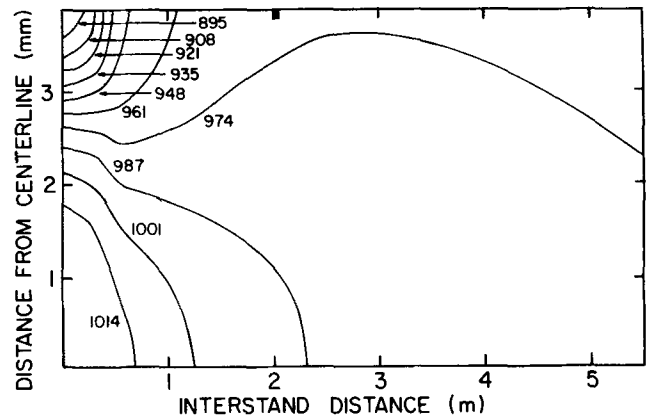

Fig. 14—Predicted temperature contours in the 0.34 pct carbon steel strip between stands II and III.

perature distribution in the steel within the roll bite and in the interstand region has been calculated with the heat-transfer model[26] described earlier. Figures 13 and 14 show contours of constant temperature within the roll bite of stand II and in the interstand region, respectively. The steep thermal gradients in the strip due to chilling of the rolls rapidly disappear in the interstand due to conduction from the interior.

An initial grain size of 180 μm was assumed for the transfer bar, which was determined from measurements on crop shear samples. The surface and centerline temperature variations as the strip passes through the finishing mill together with corresponding changes in austenite grain size are shown in Figure 15. There is a significant decrease in grain size from 180 to 35 μm after the first pass followed by some grain growth. The difference between the centerline and surface grain size was approximately 9 μm. As was mentioned earlier, this decrease can be attributed to the large reduction generally taken on the first pass of a strip mill. The subsequent decrease in grain size in the second, third, and fourth passes was minimal owing to the lower reduction associated with later stands, and the surface and centerline grain sizes
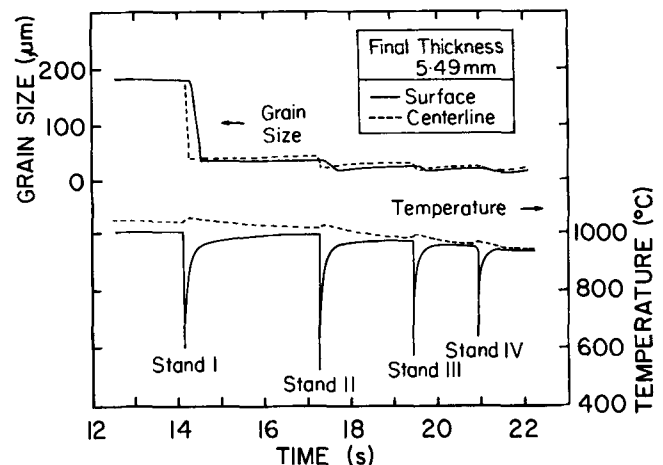

Fig. 15—Predicted evolution of the austenite grain size for 0.34 pct carbon steel rolled according to schedule 1 (Table VII). The surface and centerline temperatures and the grain size evolution are shown.

approached the same values. The model predicts complete recrystallization between stands and a final grain size of 27 μm.

The steep reduction in austenite grain size during the first pass has also been predicted by Sellars,[2] Yada,[6] and Saeki et al.[18] in their respective simulations of the hot-strip rolling process. The final grain size predicted by the various models for four through six stand operations lies in the range of 15 to ~35 μm. In examining the results of the different models, it appears that an important difference is the extent of grain growth predicted between stands. Sellars'[2] and Saeki et al.'s[18] models predict significant grain growth between stands, because they employ a grain growth equation with a grain diameter exponent of 10. Yada's[6] model predicts very little grain growth between stands and lower final grain sizes, because the corresponding grain size exponent is 2. In this work, an exponent of 7.5 has been employed. Although some of the differences in the model predictions could be ascribed to the rolling schedule, especially the total reduction and variations in temperature, it is evident that the grain growth kinetics in the interstand can significantly influence the final grain size. This finding underscores the importance of obtaining correlations for grain growth immediately following recrystallization (0 to ~2 seconds).

Figure 16 is a contour plot of the temperature distribution in the strip on exit from the finishing mill. It is apparent that although gradients in temperature exist on exit from the fourth stand due to roll chilling, the surface temperature rapidly rebounds due to conduction from the interior. Figure 17 shows the corresponding recrystallization contours, from which it is evident that recrystallization is initially retarded at the surface owing to the lower temperature. In plain-carbon steels, owing to the rapid rate of recrystallization between stands, recrystallization is complete in the interstand times and the effects of gradients in temperature on structure appear to be minimal. In microalloyed grades, where recrystallization is retarded, thermal gradients may be of greater significance.
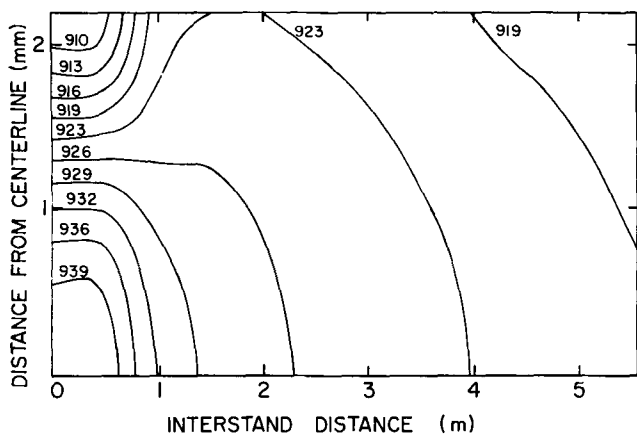

Fig. 16—Predicted contour plot of the temperature at the finish mill exit for a 0.34 pct carbon steel rolled according to schedule 1 (Table VII).
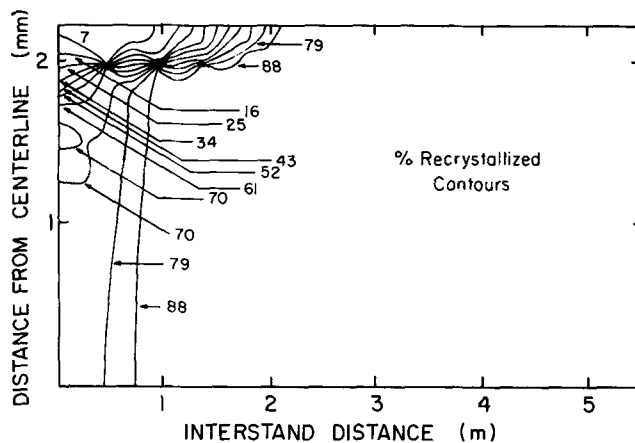

Fig. 17—Predicted contour plot of the degree of recrystallization at the finish mill exit for a 0.34 pct carbon steel rolled according to schedule 1 (Table VII).

A. Influence of the Rolling Schedule on Austenite Grain Size

Schedule 1 in Table VII corresponds to a schedule typical of the practice at Stelco's LEW for a 0.05 pct carbon steel. Schedule 2 is a variation of schedule 1, giving the same final gage. Figure 18 compares the corresponding changes in austenite grain size. The recrystallized grain size after the first pass was 32 μm for schedule 1 with a 44 pct reduction and 45 μm for schedule 2 with a 34 pct reduction, illustrating the strong influence of strain on recrystallized grain size. Further reduction in stands II through IV eliminates this difference in grain size, the exit grain size being comparable for the two cases. Figure 19 compares the grain size at the center and surface during rolling according to schedule 1, illustrating that the exit differences are small. Figure 19 also shows that recrystallization is complete in the interstand times after the first, second, and third passes and almost complete after the fourth pass.

From this analysis, it can be concluded that although percent reduction, or strain, has a strong influence on recrystallized grain size, the distribution of strain among the passes has less of an influence provided the overall reduction is constant. This finding is valid only if recrystallization is complete between stands, as is the case in the simulations performed. If the strain in a given pass is insufficient to promote full recrystallization between passes, then altering the reduction schedule could have a greater influence.

B. Influence of Rolling Temperature on Austenite Grain Size

The entry temperature of a 0.05 pct carbon transfer bar was varied in steps of 50 °C in the model to examine the effect of this variable on the final grain size. A 200 °C variation in entry temperature from 1214 °C to 1014 °C resulted in a 20 μm reduction in final grain size, as can be seen from Figure 20. The increased grain size with higher rolling temperatures is largely due to grain growth, which becomes significant at higher temperatures.

Table VII. Rolling Schedules Employed to Study the Effect of Percent Reduction on Microstructure

Schedule Number	Pass Number	Surface Temperature (°C)	Reduction (Pct)	Speed (rpm)
1	I	1098	34	52.2
	II	1013	36	87.6
	III	1006	36	120.0
	IV	987	21.5	150.3
2	I	1104	44	52.2
	II	1013	36	87.6
	III	1006	28	120.0
	IV	987	17	150.3

VIII. SUMMARY AND CONCLUSIONS

The mathematical models developed by researchers at Sheffield University, Nippon Steel, Kawasaki Steel, and IRSID Laboratories to predict microstructural evolution during hot rolling have been critically evaluated.

In this study, the static recrystallization kinetics equations developed by Sellars[1] and Perdrix[11] have been tested by comparison with experimental measurements.
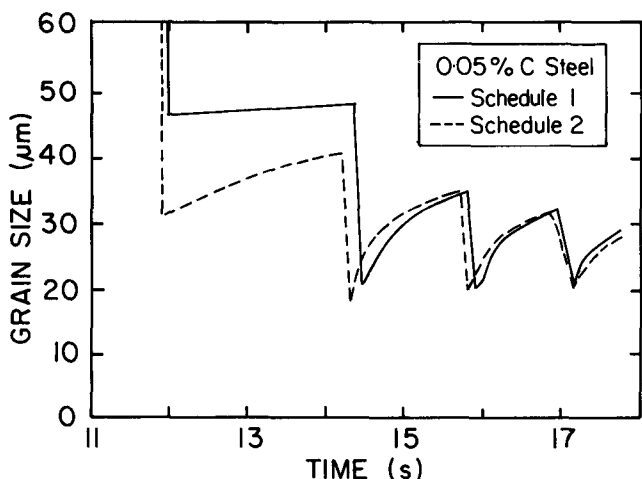

Fig. 18—Predicted effect of two different rolling schedules on the evolution of the austenite grain size for a 0.05 pct carbon steel.

Single-hit tests have been performed on the Gleeble thermomechanical simulator at temperatures of 900 °C and 850 °C at a strain rate of 1 s^{-1} on a 0.34 pct carbon steel. The percent recrystallized has been determined metallographically from test samples quenched after delay times of 0.5, 2, 5, and 10 seconds. Double-hit tests have also been performed on the cam plastometer at temperatures of 875 °C and 950 °C with varying interhit times to determine the extent of fractional softening. The recrystallization kinetics obtained metallographically and by the fractional softening tests for the 0.34 and 0.05 pct carbon steels tested are best represented by the equations proposed by Perdrix at IRSID.[11] The equations proposed for recrystallized grain size and for the treatment of partial recrystallization by the IRSID researchers have also been adopted in this study. Grain growth has been predicted using the equation form adopted by Sellars and co-workers,[1-5] Yada and co-workers,[6,7,8] and Saito et al.[9,10]

The equations representing static recrystallization, grain growth, and critical strain for dynamic recrystallization have been incorporated in the heat-transfer model described in an earlier paper. The principle of additivity has been invoked to apply the isothermal kinetic data to nonisothermal conditions. The model has been employed to predict the austenite grain size distribution through the thickness of strip during rolling in the finishing mill.
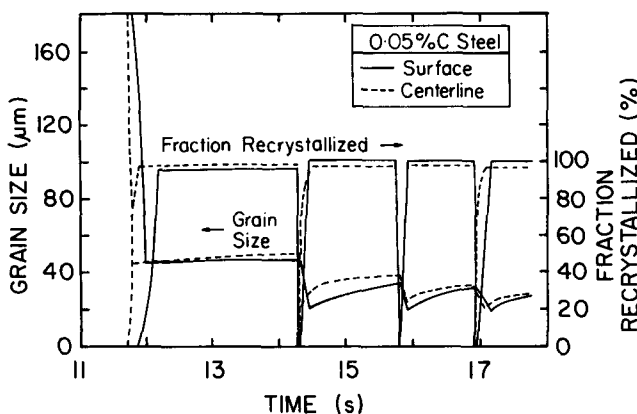

Fig. 19—Predicted evolution of the austenite grain size and the fraction recrystallized during rolling of a 0.05 pct carbon steel in Stelco's LEW hot-strip mill.
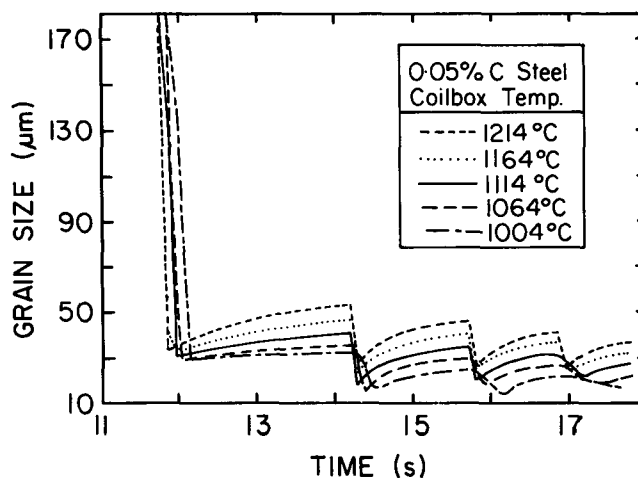

Fig. 20—Predicted effect of the rolling temperature on the computed grain size.

To validate the model, predictions of austenite grain size after one to four passes of rolling have been compared with measurements on samples rolled in CANMET's pilot mill. The measured grain sizes after one, two, and three passes were found to be 67, 34, and 25 μm, respectively, which compared favorably with model predictions. The results have shown that percent reduction has a strong influence on the recrystallized grain size.

To examine the capability of the model to simulate an industrial rolling schedule, the model has been employed to compute the changes in austenite grain size for a 0.34 pct carbon steel at Stelco's LEW hot-strip mill. The model predicts complete recrystallization between stands and a final grain size of 27 μm. The latter is comparable with Sellars' prediction of 35 μm for a four-stand hot-strip mill. A comparison of final grain sizes predicted by the different models suggests that the equations employed to simulate interstand grain growth have a strong influence on the results. The rolling schedule appears to have a negligible influence on the final grain size, provided the overall reduction is maintained as constant and full recrystallization occurs between stands.

NOMENCLATURE

A'	empirical constant
A	empirical constant
A_1	inverse of constant A
d	instantaneous grain diameter, μm
d_0	initial grain diameter, μm
d_{dyn}	dynamically recrystallized grain size, μm
d_{rex}	statically recrystallized grain size, μm
m	grain size exponent
n	exponent in Avrami equation
n'	Zener-Holloman exponent
Q_{gg}	activation energy for grain growth, kJ/mol
Q_R	activation energy for recovery, kJ/mol
Q_s	activation energy for static recrystallization, kJ/mol
R	gas constant, kJ/mol °C
S_v	grain boundary area
t	time, s
$t_{0.5}$	time for 50 pct recrystallization
T	temperature, °C
X_{dyn}	fraction dynamically recrystallized
X_R	fraction statically recovered
X	fraction recrystallized
Z	Zener-Holloman parameter
ε	strain
$\varepsilon_{0.5}$	50 pct of total strain
ε_c	critical strain for dynamic recrystallization
ε_p	peak strain
$\dot{\varepsilon}$	strain rate

ACKNOWLEDGMENTS

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