Learning from comparing
A review and reflection on qualitative oriented comparisons of teaching and learning mathematics in different countries

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Abstract:
The article reviews five qualitative comparative studies in order to reflect on methods and methodology of international comparative research within a qualitative paradigm. The goals, methodologies and theoretical perspectives of these studies are described and compared. Issues such as the different uses of the notion 'culture', and the purpose of comparison, are raised and discussed. The question of how comparative research can contribute to better a understanding of the 'covert culture' that shapes the mathematics education in different contexts is raised.

Kurzreferat:

Methode und Methodologie internationaler Vergleichsstudien, die einem qualitativen Paradigma zugeordnet werden können, werden in diesem Artikel reflektiert, indem fünf qualitative vergleichende Studien vorgestellt werden. Es werden Ziele, Methodologien und theoretische Perspektiven dieser Studien beschrieben und verglichen. Außerdem werden grundlegende Fragen erörtert, wie zum Beispiel das Problem der Verwendung des Begriffs ‚Kultur', oder die inhaltliche Zielsetzung internationaler Vergleichstudien diskutiert. Auch wird die Frage gestellt, wie vergleichende Forschung dazu beitragen kann, ein besseres Verständnis der ‚geheimen Kultur' zu erlangen, die in verschiedenen Kontexten das Lernen und Lehren von Mathematik formt.

ZDM-Classification: D20

1 Introduction

'Learning from comparing', has a double meaning here. On one hand, the title expresses the goal of international comparative research. By comparing educational processes in different cultural contexts we hope to get new insights into these processes and to question some of our implicit understandings about teaching and learning mathematics. But there is very little research that reflects on the different methods and methodologies used in international comparative studies in a qualitative paradigm. For this reason, 'learning from comparing' here also refers to reviewing and comparing existing qualitative research, in order to raise methodological questions and issues related to qualitative comparative research.

In this article I chose five comparative studies to review and compare. Two focus on the perspective of learners (Broadfoot; Osborn; Planel and Sharpe 2000) and teachers (Pepin 1997) and how they perceive mathematical education in their countries. These two studies were selected because they represent studies that are embedded in a general comparative educational research tradition. The other three studies (Kaiser 1999; Huang 2002; Knipping 2003) focus on classroom processes and ask how mathematics topics are taught differently in various countries. These three studies were all presented in a recent special issue of ZDM on '*Comparative Studies in Mathematics Education'*. I first review the studies separately. I describe the goals of the studies, highlight their methodological and theoretical approaches and summarize their results. Then I compare their goals, methodologies and theoretical perspectives in order to identify important similarities and differences and the ways in which these studies help us to understand the 'covert culture' of mathematics classrooms. Finally, I discuss what further directions and goals have to be addressed and what socio-constructivist and socio-cultural research offers for further comparative classroom research.

2 Experiencing learning differently

Everyone who has experienced educational systems in different countries knows that education is not the same everywhere. Differences exist in many respects, on the structural level of the educational system as well as on the level of classroom processes. In particular, teaching and learning processes are not the same in the classrooms of one country or another. Of course one might argue that these differences do not exist between countries only, but that we also find important differences between classrooms in the same country, or even at the same school. Undoubtedly, such differences exist and are of interest and importance. Particularly, when we want to improve our teaching practices and provide students with good opportunities for learning, differences are an extremely valuable basis for change. Nevertheless, there seem to be cultural beliefs and understandings that shape the classroom processes and teaching practices within countries, as well as how students, parents and teachers perceive them. Trying to change an educational system and teaching practices by "importing" the practices of another country generally leads to opposition. And for people who have been brought up in one educational system it is generally hard to learn in a different way than what they experienced in the past. As the American anthropologist Edward T. Hall puts it:

> "Education and educational systems are about as laden with emotions and as characteristic of a given culture as its language. … Learning to learn differently is something that has to be faced every day by people who go overseas and try to train local personnel. It seems inconceivable to the average person brought up in one culture that something as basic as this could be done any differently from the way they themselves were taught. The fact is, however, that once people have learned to learn in a given way it is extremely hard for them to learn in any other way." (Hall 1959/1990, p. 48)

Considering mathematics education specifically, it is not yet clear how differences in the teaching of mathematics in different countries affects its learning in

fundamental ways. Up to now very little research has been carried out to investigate mathematical teaching and learning processes by comparing practices in different countries. But comparative research is attracting more and more interest from mathematics educators. The interest of the International Commission on Mathematical Instruction in a study on "Mathematics education in different cultural traditions" is a clear sign of such an interest (ICMI 2001, Yam and Lau 2002). Comparing teaching and learning processes in different countries promises to help us to become more aware of our own implicit assumptions concerning the learning of mathematics and learning processes in general. Through this we might come to a deeper understanding of mathematics learning and how this takes place in different cultural contexts. But one of the difficulties of such research is that culture exists on two levels, which have been described by anthropologists and psychoanalysts as 'overt culture', visible and easily described, and 'covert culture', hidden and taken as granted by the members of a culture (see Hall 1959/1990, p. 62). It is the 'covert culture', the implicit "colearning" (Bateson 1991) that actually shapes our learning and our learning of any subject matter. This has particularly been highlighted by socio-constructivist and socio-cultural research in mathematics education (Seeger, Voigt and Waschescio 1998; Cobb and Bauersfeld 1995). I will argue that in order to really benefit from comparative research in mathematics education we will have to come to an understanding of the 'covert culture' of mathematics teaching and learning. How this can be done is not yet clear and there is a need for further research and methodological reflection.

The selection of articles I review here is far from being exhaustive and my focus is particularly not on large scale studies, but on smaller studies with a specific research interest. My analysis is meant as first steps in an investigation of qualitative research that compares mathematics education in different countries. I ask: What are the goals, methods and methodological frameworks of these studies? What results do they come to? What theoretical background informs the interpretations of these results? I believe that meta-reflections of this kind are necessary in order to profit from the potential of international comparative research to go beyond the level of descriptions of the 'overt culture', without leaving the reader with the unsatisfying question: "So what?" Interesting, ongoing and very promising studies such as those brought together in the ICMI Comparative Study and the *The Learner's Perspective Study* (LPS), which has been carried out as an international collaboration of researchers in nine countries (Clarke and Gallos 2002; Jablonka 2003; Shimizu 2002) will have to be taken into account in future analyses. As yet these studies have only been partly published.

3 Students' perception of educational differences

In the "Quality of Educational Systems Transnationally" (QUEST) project Patricia Broadfoot, Marilyn Osborn, Claire Planel and Keith Sharpe investigated the cultural bias of pupils' learning outcomes through a two-country comparative study (Broadfoot et al 2000; Osborn and Planel 1999). The project concerned primary education in both England and France and was intended to explore the extent to which culturally located educational differences and students' perception of these differences were significant in explaining the varied outcomes in pupils' learning. The researchers aimed at determining the significance of differences documented in previous research (for example Broadfoot and Osborn 1993) in French and English teachers' priorities and pedagogy, for students' attitudes to school, their motivation to learn, and their learning outcomes. The study asked: "In what ways did the performances of pupils in Language and Math differ in France and England? How far could these differences be traced back to different teacher perceptions, pedagogical approaches, and curriculum content? Were there other patterns of differential performance which were more significant than those between the two countries? (e.g. gender, social class)" (Broadfoot et al 2000, p. 26).

These research goals suggested a methodology of 'contextualised comparisons'. "By conducting a detailed examination of the links between learning outcomes, classroom processes and pupil attitudes in these two countries, it was hoped to explain any national differences identified in terms of both pupil's attitudes to learning and characteristic classroom approaches on the part of teachers" (Broadfoot, et al., 2000, p. 25). In order to probe possible reasons for differences between the two countries the researchers took a quantitative/qualitative approach to data-collection and analysis. A sample of 800 children was chosen for study. These pupils, all within the last stages of their primary education, came from schools of differing geographic and socio-economic backgrounds. Questionnaires concerning perception of teaching, understanding of the purposes of schooling and perspectives on national identity were complemented by classroom observations and focused interviews with pupils. The questionnaires contained fixed-response and open-ended questions. The classroom observations were documented by field-notes and systematic observation, based on a schedule quantifying "different types of pedagogic strategy and curriculum context as well as pupil engagement and interaction" (ibid., p. 28). Each student's behaviour was recorded over a ten-minute period, including six minutes of systematic observation (at ten-second intervals) and four minutes of "contextualizing field notes". In addition photographs, teaching documents and examples of students' work were collected. Furthermore, a small exploratory study was carried out in each country with 10 children who had experienced both English and French primary education. To compare learning outcomes, test items, in language as well as mathematics, were based on national assessment tests. In mathematics, in addition to the questions from the national tests, problems requiring problem-solving skills were given to the students, since the team felt that these were not adequately represented

in the national tests.

The QUEST research revealed that the quality of learning is less a result of the 'objective' features of a learning situation, and more a product of the ways these are interpreted by the people involved in that particular culture and setting. The research underlines the limitations of large-scale quantitative international surveys of pupils' performance and questions the validity of decontextualized measures of educational quality. For example, students' performances in language and mathematics not only showed clear differences, but in general English children used more experimental and individualized approaches, trying to find out their own solutions to problems, whereas French children tended to a more conventional technical and formal approach. On the whole the spread in terms of attitude and achievement of children of different socio-economic status was greater in England than in France; so too was the span in terms of performances in maths and language. The authors point out that French pupils, their parents and the wider society all share the experience of a publicly understood ladder of progress through school and the valuing of educational success; compared to England where social class differences, as well as negative peer group influences opposed to school values, are important. Although students in both countries shared the role and structural position of 'pupil', there were, nevertheless, striking differences in their perceptions of what it meant to be a pupil. French students were significantly more positive about school and more likely to see teaching as helpful and useful to them. They uniformly appeared to be more motivated toward educational success and academic goals. In contrast, English pupils demonstrated more individual variation in their general attitudes toward schooling and performance and were strongly affected by the vagaries of pupil sub-cultures.

The study was intended to not only document and compare learning outcomes, but to explain the reasons and "significance of these in terms of different classroom practices which the pupils studied had experienced" (ibid., 26). Altogether the results of the study demonstrate the crucial role of how 'objective' features of a learning situation are interpreted by those who are involved in that situation. Further, the results demonstrate that these interpretations are culturally shaped. The researchers' assumption that "deep-rooted cultural differences in the educational traditions of the two countries would still be manifest in the day to day realities of classroom life" (Broadfoot 1999, p. 249) was borne out by their findings. Their hypothesis "that the traditional, formal, didactic style of teaching characteristic of French primary schools would tend to foster a passive, authority-dependent, style of learning among pupils, in contrast to England where a more developmental educational philosophy would typically result in pupils who are capable of being more autonomous in their learning and willing to adopt a more individualistic approach" (ibid., p. 249) was also supported by their results.

4 Teachers' beliefs and practices

Birgit Pepin carried out a study comparing beliefs and practices of mathematics teachers in England, France and Germany (Pepin 1997) in order to understand classroom practices in different countries and to find out why teachers create different environments for learning mathematics. She was interested in mathematics teachers' work, in particular in the reasons teachers give for their practices and for their perceptions of teaching and learning mathematics. She wanted to understand teachers' ideas and practice from their point of view.

Pepin carried out an ethnographic study, in which teachers' understandings of their classroom practices, teachers' general tasks and responsibilities, and their working conditions were investigated. The researcher observed teachers in their classrooms for a two week period and interviewed each teacher several times, before, during and after the classroom observations. She asked the teachers about their working conditions, their professional experiences, their teacher education and their view of the subject discipline. But she also tried to focus their attention on what had happened in the classroom and to give them an opportunity to provide explanations for what had happened. Therefore, in the second and third interview during her classroom observations, she started her interviews with the technique of stimulated recall, replaying a videotape of the teacher's lesson in order to give the teacher an opportunity to comment on their practices and thoughts during the lesson. In a final interview she confronted the teacher with her "developing understandings of what she had seen and heard" (ibid., p. 102) and asked the teacher for further comments. She structured her initial descriptions of each case with the following three categories: "teacher's background and beliefs", "the context in which the teacher was working"; and "teacher's classroom practice". Recorded and transcribed interviews (including stimulated recall), videos of the lessons, field notes and further documents such as teachers' preparation notes or official documents, e.g. curricula, constituted the body of data.

Based on these data, initial case studies of every individual teacher were written, and at this point the twelve teachers were "treated as individuals", but "the aim of this part of the analysis was to go beyond the individual cases of teachers and to use the twelve case studies as a start to developing an understanding of teachers' work" (ibid., p. 113). This was attempted through a comparative approach that was in the first instance based on literature describing national cultural traditions of the three countries (Pepin 1999). In the second instance Pepin singled out themes that emerged about teachers' perception of their work and their practices. "… categories were used to formulate hypotheses, which were subsequently tried out and checked against what was mentioned in the cases and those data collected from associated professionals, with the view to seeking supporting or conflicting evidence" (Pepin 1997, p. 116). "From this emerged a set of similarities and differences of teachers across countries which reflected teachers' concerns in terms of their work and what they had told the researcher about the

teaching that she had observed. These concepts were no longer teachers' own concepts, but the researcher's way of representing teachers' concepts." (ibid., p. 116) These similarities and differences became explicable in the third instance when a sort of meta-analysis was carried out. Through these analyses, the influence of cultural traditions on teachers beliefs and practices became visible. Pepin explicitly applied the method of triangulation, by "examining data relating to the same construct from participant observations and interviewing" (ibid., p. 108) and by inviting teachers to comment on the researcher's report of "what she thought she had found out, but from the teacher's point of view" (ibid., p.108)

Pepin describes differences between teachers respecting their views of mathematics, their tasks and responsibilities and their classroom practices. In the following I will focus on the views of mathematics and teachers' classroom practices; for further details see Pepin (Pepin 1997). She presents English teachers as putting the most emphasis on "the view that mathematics is a 'tool' for doing other things. In many way, mathematics appears to be a set of unrelated but utilitarian rules and facts" (Pepin 2002, p. 248). But she also points out that at the same time English teachers view mathematics as dynamic and take a problem oriented approach, where doing mathematics, solving problems and the discovery of patterns is considered as the important focus in teaching. "English teachers spend relatively little time explaining concepts to the whole class … When English teachers use whole-class teaching, they typically explain a concept from the front in a relatively didactic way." (ibid., p. 249) They mostly briefly introduce a concept or a skill to be learnt by an example and then give as much time as possible to students to practice autonomously or in little groups. Teachers provide individual help to students and make choices about which set of exercises each student should attempt. They decide how to teach according to the perceived ability of each individual student, who are grouped in 'sets' according to ability. The set assigned to the students determines "what is taught and to what 'depth', which exercises to leave out and which mathematical language to use" (ibid., p.252).

Pepin observes that what French teachers say about mathematics still shows signs of the 'formal' view that has traditionally been held, but she also recognizes an "overlay of perspectives from a formal to a more dynamic view of mathematics" (ibid., p.248). She finds that the traditional '*cours magistral'* (lecture type lesson) has been replaced by lessons structured by three parts. An activity at the beginning of the lesson introduces a notion or idea that is then studied in the main part of the lesson, where the important content of the unit is written down. In the third part of the lesson students do exercises, but relatively little time is spent on routine procedures. Mathematical knowledge is presented as "open to question, challenge and discussion" and needing to be investigated. In their teaching French teachers focus on "developing mathematical thinking, which includes exploring, developing, and understanding concepts, and mathematical reasoning." (ibid., p. 249). But, Pepin also finds a 'utilitarian' view like that emphasised by English teachers. It should be mentioned here that students in France learn in mixed-ability classes in comprehensive schools, and are not assigned to 'sets' as in England.

German teachers hold, according to Pepin "a relatively formal view of mathematics". In particular teachers of the Gymnasium, a school form that has a distinctive academic tradition and prepares pupils for university entry, hold this view. Mathematics is seen here as "a unified body that needs to be passed on to pupils". She finds that even teachers of the *Hauptschule*, a school form with a vocational orientation, aspire to this formal view of mathematics though they present mathematics to the students more as a 'bag of tools', with facts, rules and techniques to be used. Teachers understand their role as "the explainer who teaches the structure of mathematics through an 'exciting' delivery and by adapting the structured textbook approach meaningfully." She finds that "new solutions or procedures is not usually encouraged and lessons appear relatively formal and traditional in terms of their mathematical content." (ibid., 250) Teachers in the *Gymnasium* spend a lot of their time in a 'conversational style', whereas teachers in German *Hauptschule* and *Realschule* acknowledge that the students in these school forms need more practical and individualised help, though they would also like to teach in a 'conversational style'.

Pepin explains these differences in terms of "national cultural tradition of the three countries" (Pepin, 1999). She traces the fact that English teachers spend little time on whole-class discussions, but rather focus on students practice on exercises, back to individualistic traditions in English humanistic principles. This way of teaching allows giving students individual help. But she also admits that other aspects cannot easily be understood by these traditions; for example, that teachers claim a lack of time for lesson preparation. A reason for the latter might be teachers' opposition in recent years to a more technocratic approach "where teachers are told what and, increasingly, how to teach" (Pepin 2002, p. 254) that is meant to 'reform' the educational system and which conflicts with liberal-humanistic and individualistic traditions where the teachers where "at the centre of decision-making in schools" (ibid., p. 253). The focus on challenging mathematical problems and reasoning in French teaching is, according to Pepin, due to "traditional (encyclopaedic) views of, and high regard for, rationality." (ibid., p. 253) Whereas the fact that "they draw together ideas from the class and write them down in the form of a cours, reflecting their expectation of keeping the whole class together and moving pupils forward together" (ibid., p. 253) is due to an 'egalitarian view' that is also embedded in French traditions. Also the mixed ability teaching and the introduction of all students to the entire curriculum is due to this 'egalitarian tradition'. Concerning German teaching Pepin states that "In terms of educational traditions, the liberal-humanistic customs (of Humboldt) are still detectable and upheld in the pedagogic approaches of German *Gymnasium* teachers

and influences to a large extent the pedagogic culture of perceived 'good' mathematics teaching." (ibid., p. 253). Pepin explains the German 'conversational style' in terms of this tradition, though in particular *Hauptschule* teachers face a reality where this kind of teaching is obviously failing.

5 Mathematics classrooms in comparison

5.1 English and German Mathematics Classrooms

The study of Kaiser on *English and German Mathematics Teaching* aimed "to generate qualitative hypotheses on the differences between teaching mathematics under the educational systems in England and in Germany" (Kaiser 2002, p. 248). The study is described by the researcher as "an ethnographic study embedded in a qualitatively oriented paradigm of social sciences" (ibid., p. 247). Kaiser considered an ethnographical approach appropriate to the goal of her research as it focuses "on descriptions of real life and the construction of social phenomena" (ibid., p. 248). As characteristic of such an approach, she considers first a "partial enculturation" in order to gain an inside perspective; second, a flexible research strategy, which includes finding "a balance between the research interest and the requirements of the situation" (ibid., p. 248), and third, participant observation and field-notes, allowing a "diving into the field" (ibid., p. 248) as the primary data. Embedded in an ethnographic methodology, the study does not intend to make any 'lawlike' statements; "in contrast the study refers to the approach of the 'ideal typus' developed by Max Weber (*Webersche Idealtypen*), and describes idealised types of mathematics teaching reconstructed from the classroom observations in England and Germany" (ibid., p. 248). In this way the study aims at generating empirically grounded theory.

The study included 242 mathematics lessons in England from Year 6 to A-level, mainly focussing on Year 9 to 11, with 102 lessons in Germany from Year 8 to Year 10. This means that the study focuses on the end of the lower secondary level. In Germany the different types of schools from the three-tier system were included, but emphasis was put on the two higher tiers of the German school system. In England the focus was also on higher achieving students; only top set groups of the 14 state-run comprehensive schools and 3 private schools with a selective character were observed. Observations were carried out in England, and mostly in the Federal state of Hessen in Germany. The method of participant observation with detailed field-notes was used primarily. These notes documented classroom observations, discussions with teachers after classroom visits and during lunch in the staff room, as well as participation in school assemblies and discussions with pupils after lessons. For the analysis of teaching and learning processes verbatim statements were considered necessary so audio-tape recordings were made.

Kaiser describes differences between German and English mathematics teaching in terms of the implicit "understanding of theory", which she characterises ideal-typically as 'scientific' oriented in German classes versus 'pragmatic' in England. While theoretical mathematical considerations are of great importance in German mathematics teaching, though graduations exist between different types of schools, "practical and purpose-depending handling of theory" (ibid., p. 249) are described as dominant in English teaching. These different orientations are also interpreted by the way new concepts are introduced, the meaning proofs have in class and the importance of rules and mathematical language.

In German mathematics teaching high importance is given to the introduction of new mathematical concepts and the deduction of new methods in whole class discussion guided by the teacher. In contrast, in English teaching new concepts are often introduced implicitly by exercises or just given by the teacher as information. New methods are demonstrated by examples and students mostly work individually on exercises, which are reflected on in short class discussion sequences. The emphasis on formal proofs for theorems is, according to Kaiser, of high relevance in German teaching, particularly of the *Gymnasium*, and experimental or practical proofs are considered insufficient. In English mathematics teaching theorems are often developed from examples and teachers do refer to example-related explanations as 'proof'. They also accept when students check formulae or solutions by examples without trying to give a general explanation. Examples have a high status in general in English mathematics teaching, which means also that rules and standard algorithms are of minor importance. "Thus solutions are often formulated in connection with examples, and there often do not exist special names for general solution of formulae like in German mathematics teaching." (ibid., p. 252). German teaching is characterised as 'rule-oriented', where exact application of arithmetic and algebraic algorithms is highly important, though there exist important differences between *Gymnasium*, *Realschule* and *Hauptschule*. Also there exist differences in the emphasis on precise mathematical language in German teaching, where formal notations are considered important, and English mathematics teaching, where a collectively accepted terminology is of minor importance. Kaiser also finds that a minor emphasis on real-world and modelling examples is typical for German mathematics teaching, whereas these examples have a fairly high importance in English mathematics teaching. "They serve to introduce and derive new mathematical concepts and methods, but also to impart abilities that enable [them to, C.K.] apply mathematics to solve extra-mathematical problems." (ibid., p. 253).

Kaiser not only recognised different types of teaching according to a subject-oriented or pragmatic understanding of mathematical theory, but she also found significant differences in 'teaching-and-learning styles' influenced by different educational traditions and philosophies in the two countries. She describes German lessons as dominated by teacher-guided discussions where 'joint progress' in the learning process is the goal. Mathematical ideas and tasks are

mostly discussed with the whole class, or students are given the opportunity to discuss with their peers. Individual work is rarely to be found in German classes, whereas students' individual work dominates in English teaching. Students work with individualised learning materials and mostly discover new mathematical contents by themselves, sometimes guided by the teacher or short phases of classroom discussions. Kaiser explains this by the "orientation towards the individual" as the most significant principle of English philosophies of education. She considers pragmatism as nearly as important for the English tradition, which means that theoretical knowledge is considered to be not that important and therefore not taught systematically. German traditions of education are more influenced by academic sciences and therefore emphasise systematic knowledge. In the history of education in Germany two traditions can be found: a 'realistic tradition', which influenced the German *Hauptschule* and *Realschule* and a 'humanistic tradition' going back to Humboldt, influencing the spirit of the German *Gymnasium*. "Both, the realistic tradition of education and the humanistic conception of *Allgemeinbildung* demand the teaching of formal learning objectives, in which learning to learn or the formation of the intellect has a central meaning" (ibid., p. 255).

Kaiser considers national traditions as important for describing mathematics and educational processes in classrooms in different countries. Therefore, she uses as a theoretical background for her comparative study work that reflects on "educational philosophies and their historical development". In particular, she refers to McLean (1990) who develops a classification of different educational traditions in Europe. McLean distinguishes two main European traditions, the 'encyclopaedic', predominantly found in the French educational system, and the 'humanistic', found especially in the educational systems of England and Wales. He also assumes a third tradition, the 'naturalistic' tradition, which has challenged the two dominant traditions all over Europe, for example the 'community-centred' approaches predominantly in Denmark, or 'work-centred' traditions of education in German-speaking regions. Therefore, he argues that the third category can not be described by joint principles, but is a "shorthand for a variety of individual and community oriented views" (quoted in Kaiser 2002, p. 242). Kaiser mostly follows McLean's characterisation of the English 'humanistic' tradition, with emphasis on 'individualism' and an 'empirical pragmatic orientation'. But she considers his descriptions of the educational traditions in Germany as problematic, in particular his categorisations of the tripartite school system. Therefore Kaiser offers a historical investigation into educational philosophies in the German system of education. She confronts two main development lines: the 'realistic education', still found today in Hauptschulen and *Realschulen* in Germany, with a vocational education orientation, and the 'Humanistic Allgemeinbildung', found in particular in the German Gymnasium.

5.2 Pythagorean theorem in classrooms in Hong Kong and Shanghai

This study had a straightforward goal, to try to shed some light on a seeming paradox:

> "The East Asian students have consistently out-performed their counterparts in West in mathematics in many international comparative studies. … Yet, the learning environment in East Asia seems undesirable: larger classes, expository methods, relentless norm-referenced assessments, teacher-centred with passive student learning" (Huang and Leung 2002b, 248).

To do this Huang compared mathematics teaching in Hong Kong and Shanghai, and characterized the various "ways of handling the same topic", the Pythagorean theorem, in grade 8 classrooms (Huang 2002). The object of this comparison was to see whether teaching in East Asia could truly be characterised as "expository methods, relentless norm-referenced assessments, teacher-centred with passive student learning."

In the study eight Hong Kong videos on the Pythagorean theorem were selected from the TIMSS-Repeat Video study. Then, eleven Shanghai lessons on the same theorem were videotaped following the method used in the TIMSS-Repeat study. Further, the videos were complemented by a teacher and a student questionnaire, photocopies of text pages, worksheets, overhead transparencies, and other materials used in the lesson. English transcripts were used for the analysis, therefore the transcripts of the Shanghai lessons were translated, after first being transcribed in Chinese. The analyses focused on the structure of the lesson, "ways of teaching Pythagoras' theorem and patterns of classroom interaction" (Huang and Leung 2002a, 268). Analyses were carried out quantitatively and qualitatively, with particular focus on the teachers' questions.

The analyses reveal that Hong Kong and Shanghai teachers try to explore and justify Pythagoras' Theorem through various activities and methods. Huang and Leung summarize that "teachers in Hong Kong and Shanghai emphasize on exploring the Pythagoras theorem, but [the, C.K.] Shanghai teacher tries to encourage students to make a conjecture through calculating, whilst the Hong Kong teacher prefers helping the students to discover the theorem by visual experiences. Moreover, the Shanghai teacher seems to emphasize the mathematical proofs, while the Hong Kong teacher tends to verify the theorem visually." (Huang and Leung 2002b, 247) The researchers conclude from the results of their study that for both Hong Kong and Shanghai teachers not only drilling and procedural skills are important, but also students' conceptual understanding of mathematics which they try to foster in their teaching.

Huang's interpretation of these findings is that "the paradox of Chinese mathematics teaching" is based on a limited vision of the variety and depth of the teaching taking place in East Asian classrooms. The role of cultural comparison in coming to this conclusions was focussed on similarities as well as differences. "Thus, from macro-level such as the structure of lesson, the Hong Kong lesson and Shanghai lesson are more similar, from micro-level such as the ways of dealing

with justification or practicing, still there are remarkable differences" (Huang and Leung 2002b, p. 252). From the differences found it can be seen that a single description of teaching in China cannot be accurate. From the similarities, especially in the teacher's attention to conceptual understanding, it can be seen that "expository methods, relentless norm-referenced assessments, teacher-centred with passive student learning" is not the whole story of East Asian teaching methods.

5.3 Proving in classrooms in France and Germany

Knipping (2003; 1999) reports on proving processes in everyday classroom practices in French and German geometry teaching. The study asked: What kind of proving processes do we find in everyday teaching of mathematics in French and German classrooms? What value and what role is attributed to proofs? How are mathematical statements justified? Which functions do proving processes have in class?

Proving processes in mathematics classrooms were observed and analysed to obtain differences through systematic and ongoing comparisons. First, all observed mathematical proofs, proving contexts and argumentations were analysed on the basis of epistemological and didactical research in the field of proof and proving. For the analyses of the argumentations Toulmin's functional model of arguments (1958) was used to characterise different kinds of arguments in class, including multi-step formal and informal arguments. This led to the identification of different types of proving processes and functions of proving processes in class, as well as different types of argumentations during these processes.

Then, in a second step, other cases were used to clarify which features of individual cases were typical, and which were particular characteristics of that case. Proofs, contexts and argumentations were compared in order to find prototypes of distinctively different types of proving processes and argumentations in class. Referring to Zerssen (1973) a prototype was understood as a case "in the sense of a concrete model" (ibid., p. 53); it is a case that can apply to a group as representative in the sense that through it special characteristics of a group of cases become clear. Starting from two examples of different proving practices typical characteristics of these prototypes were worked out. Finally these prototypes were again compared with the other observed cases and ideal-typical, i.e. theoretical characterisations of proving processes and their functions were given (see Knipping 2003). The ideal-typical characterisations developed in this way are claimed to have a heuristic function, because "the pure type contains a hypothesis of a possible occurrence" (Gerhardt 1991, p. 437).

Classroom observations and comparative analyses focused on units in which the Pythagorean Theorem was taught. The instructional units were selected according to curricular criteria and covered topics in geometry in grade 8/9 (13-15 year old students), when proof is listed both in French and German curricula as an explicit topic for the first time. Analyses and results from six instruction units on the Pythagorean theorem were presented in the published study. The data collection was carried out at six *Collèges* in the Paris region and three *Gymnasien* and two *Gesamtschulen* in Hamburg. Two of the observed cases in France were classes in a highly selective bilingual stream. Classroom processes were documented with audio tape recordings and photos of the blackboard. In addition, observations were recorded in the form of field notes. The tape recordings were transcribed to make detailed analysis of the classroom discourses during the proving processes possible, which were used for the reconstruction of argumentations in class.

The first type of proving processes, illustrated by a prototype called Nissen, was only found in the German classes. In this type proving processes are linked to a concrete calculation problem; the Pythagorean theorem is presented as a solution of an application problem. The meaning and the justification of the general problem are developed from a special case that is discussed collectively by the whole class. The function of the proving process is in this case to bring out (or reveal) the meaning of a theorem by starting from an application problem and special cases. The practice of the theorem in later lessons, again in the form of application exercises, reinforces this function. The function of this type of proving process is characterized by the expression "to see that", that is, it is described as a heuristic function. Through the proof the relations expressed in the Pythagorean Theorem are discovered, by a collective discussion guided by the teacher. The leitmotif of this type of proving processes is to "see" the relations. The calculations and the visual representations are the basis of this kind of argument, founded essentially on the contemplation of figures.

On the other hand, the second type illustrated by Pascal was only encountered in the French lessons. The function of this type of proof process is characterised by the leitmotif "to explain why". This type of teaching starts with the presentation of the statement of the theorem. To prove means here to be brought back to theorems, definitions and techniques which were already validated in class as official knowledge. The proof of the theorem is conducted at the same time orally and at the blackboard. This type of proving process leads to the establishment of new knowledge. The resolution of exercises in this kind of class is based on the proving processes and is explained in the same discursive way as in the proving processes. What is significant in the resolution of the exercises is not the result which one obtains, but the justification given. It is the discursive or communicative function of proofs that is significant here. This function corresponds to the fact that a proof must be explained by other proofs. According to Knipping, these differences reflect different functions of proving processes.

One basic assumption of the study was that justification and development of knowledge cannot be separated from each other within proving, nor can they be separated from their context of origination. However, Knipping's analyses of the different types of proving processes did not involve attempting to explain

their characteristics by reference to contexts outside the classroom. Her intent was not to explain, but to interpret, which she did by referring to concepts from prior epistemological and didactical research in the field of proof and proving.

6 Reflections on the studies

How might we learn through comparing these studies? First, we might learn from the kinds of questions these studies raise, that is, from the goals of the studies. Second, we might learn from the methods and methodology used in the studies. Third, we might learn from the studies what kinds of background theory are promising for investigating differences in the teaching and learning of mathematics within a comparative approach.

6.1 Goals

The goals of these studies differ in significant ways. The studies of Pepin (1997), Kaiser (1999) and Broadfoot et al. (2000) took seriously Sadler's recognition that "In studying foreign systems of Education we should not forget that the things outside the schools matter even more than the things insides the schools, and govern and interpret the things inside. … A national system of Education … has in it some of the secret workings of national life. It reflects, while it seeks to remedy, the failings of the national character. By instinct, it often lays special emphasis on those parts of training which the national character particularly needs." (Sadler 1964, p. 310, cited in Kaiser 2002, p.241). For Pepin and Kaiser it was their goal to explain what goes on in schools by making reference to differences in the French, English and German 'national cultural traditions'. Specifically Pepin sought to identify differences in teachers' perspectives and teachers' practices that reflected these cultural differences. Broadfoot et al. (2000) had a similar aim, although their focus was more on learner's perspectives. In addition, while they considered cultural influences, they also considered other factors such as socio-economic status, and in fact made the influence of these factors the object of further cultural comparison. The study of Huang examined differences in teachers' practices within one country (China) but in two cities that have different histories and hence different cultural traditions. This is a reminder that 'cultural traditions' do not necessarily follow national borders. It is possible to find 'foreign' systems of education within a country. Comparative studies of regional differences within countries are valuable, as are comparisons of groups of countries that share some historical and cultural characteristics (e.g., the West/East comparisons of which Huang's study is implicitly a part.) It is not absolutely clear from Huang's article that this was his aim to explain the differences within schools in terms of differences in regional cultures, but it seems likely. Knipping's study falls into another category. She compared neither educational cultures between nations nor within nations, but instead compared classroom proving practices, taking an international approach only because it seemed more likely that classrooms in different countries would reveal different proving processes. Her goal was to describe what is happening inside the classroom, and on interpreting that as a closed system rather than explaining it in terms of outside factors.

6.2 Methodological perspectives

The methodologies of the studies are significantly different. The QUEST project (Broadfoot et al 2000) and Huang (2002) both use a mixed quantitative-qualitative approach, but in very different ways. Huang and Leung (2002a) are mostly influenced by a quantitative paradigm, following closely the instructions for the TIMSS video study and trying to analyse the lessons in a standardized way. In contrast Broadfoot et al. (2000) triangulate quantitative and qualitative methods, both in data-collection and analysis. Broadfoot et al. and Pepin both share the assumption that learning is less a result of the 'objective' features of a learning situation, and more a product of the ways these are interpreted by the people involved in that particular culture and setting. Nevertheless reflection on qualitative methodology in the QUEST project is less extensive than in the studies of Pepin (1997), Kaiser (1999) and Knipping (2003), which are strictly embedded in a qualitative paradigm. Pepin and Kaiser, both describe in detail the 'ethnographic' approach they take and the way they intend to come to theoretical descriptions and empirically grounded categories in their study. Both researchers consider a "partial enculturation" as important in order to gain an inside perspective. Therefore participant observation with field-notes is of major importance in these studies. Pepin uses a more restricted, but systematic way of data collection, and explicitly applies the method of triangulation, relating field observations and informal conversations with teachers' interviews. In contrast to this open, ethnographic approach Knipping (2003) has a qualitative sampling strategy, selecting classrooms and periods in the school year where she hopes to see proving processes. Kaiser (1999) and Knipping (2003) both intend to identify different types of classroom teaching. As in Pepin (1997), individual cases are analysed first, and then distinctively different types are singled out by comparison. In Kaiser (1999) and Knipping (2003) this approach is framed by the methodology of ideal types developed by Max Weber (*Webersche Idealtypen*), and by further development of this approach by others (e.g., Zerssen 1973) in terms of 'prototypes'. In Knipping, in contrast to the other approaches, the analyses of the data were guided by theoretical concepts and models, such as the concept of 'functions' of proof, or Toulmin's functional model of arguments, which were used as 'sensitising concepts'.

6.3 Theoretical perspectives

We can learn from comparing these studies that theory is given different relevance and status in qualitative comparative studies, and that there is no common background theory to which these studies refer. Most of the studies aimed not only at documenting differences,

but also at providing explanations for the differences found. The way the researchers tried to reach this goal varied however. Pepin assumed that "it is necessary to study the culture of the participants under study before producing explanations for their behaviour" (Pepin 1997, p. 76). And, she asks "How can we understand teachers' practices in the light of what we know about the different countries? … Where do the cultural and educational traditions stem from, and how do they feed into the classroom?" (Pepin 2002, 246). Therefore, Pepin investigated research on the history and philosophy of education and provided theoretical descriptions of the 'national cultural tradition' of education in England, France and Germany. Kaiser (1999) took a similar approach and this background was used in both studies to provide explanations of differences that came out empirically in their studies. Broadfoot et al. (2000), however, took a different approach. The researchers share the idea of different 'national educational traditions' with Pepin and Kaiser, but they refer primarily to outcomes of previous empirical studies on teachers' views and practices (e.g., Broadfoot; Osborn; Gilly and Brucher 1993) in order to explain differences between students' perceptions and outcomes.

Knipping (2003) seems not to have been interested in the question of how 'national cultural traditions' shape classroom practices. She only referred to comparative research in order to motivate her comparative approach, assuming that French and German classes would provide more diversity in classroom proving processes. She explained differences in terms of different 'functions of proving processes', a theoretical construct that is grounded in her empirical research and her theoretical framework based on epistemological and didactical research in the field of proof and proving. Though her results showed that there are 'cultural differences' in everyday classroom proving processes, she did not provide a theoretical background which would allow her to explain these differences in terms of 'national culture'. Neither did Huang and Leung (2002a), who found that though the content and structure of lessons in Hong Kong and Shanghai are similar, the proving practices in these classes are significantly different. This provokes the questions how and in how far 'national cultural tradition' or 'culture' influence the 'culture' of mathematics classrooms, and how the question of change can be addressed here. These questions ask for an explanation of the concept of 'culture' and it's relation to education. In section 7, I discuss some aspects of this fundamental, but complicated concept.

7 Fundamental questions and problems

7.1 What do we mean by 'culture'?

The notion of 'culture' has become very popular and is used for various purposes in different contexts. The term 'culture' has also been used with different meanings in this article. In this section I address some of the aspects and difficulties of this term in relation to the research I reviewed above. First, I discuss why a differentiation of 'culture' and 'nationality' is necessary. In particular, I argue that we need a more precise notion of 'culture' which makes it possible to acknowledge diversity within societies. Second, I emphasise the need for clarification of how 'culture' and the 'culture of mathematics classrooms' are interwoven.

Clarke challenges a "simplistic identification of culture with nationality" (Clarke 2002, p.8). He illustrates this problem by referring to Asian-American students in the United States who "perform at a level comparable with their high-performing counterparts in schools in Asian countries" (ibid., p. 7). This group of students significantly outperforms other American students, an issue that has become of political interest since these Asian-American students pass university entrance exams more easily than other students. Clarke concludes from this that students' 'cultural affiliation' is at least as important as the influence of the 'cultural alignment' of the particular school system students attend. He criticizes that in comparative research generally learners' perspectives are not sufficiently taken into account: "The agency of the student, the nature of learner practice, and the cultural specificity of that agency and that practice must be accommodated within our research designs." (ibid., p. 9). These considerations motivate *The Learner's Perspective Study* in which Clarke participates. The example of Asian-American students highlights several different problems. First, the fact that students' perspectives can be very different from the values school systems present. Second, that students in most countries are a heterogeneous rather than a homogenous group; although, Broadfoot (1999) shows that values of students with different socio-economic and ethnic backgrounds are more diverse and heterogeneous in England than in France, where students, as parents and teachers, hold more common views about school and learning. And finally, that there is a need for an explanation of what we mean when we talk about 'cultural affiliation' of students or the 'cultural alignment' of a particular school system. If we do not want to naively assert more and more meanings for 'culture', but to use the notion as a core concept in comparative studies in mathematics education, we will have to acknowledge research in other disciplines, like cultural and intercultural studies as well as anthropology and social studies, in our future work.

7.2 Culture and the 'culture of education'?

Another variation of the notion of 'culture' deserves our attention here. In mathematics education this term has been used particularly in socio-constructivist and socio-cultural oriented research, as for example in the volume 'The culture of the mathematics classroom', edited by Seeger, Voigt and Waschescio (1998). But the notion differs in meaning when talking of 'French culture', 'Japanese culture' or 'culture of the mathematics classroom'. Edward T. Hall describes 'culture' from an anthropologist's point of view as follows: "For an anthropologist culture has long stood for the way of life of a people, for the sum of their learned behavior

patterns, attitudes, and material things" (Hall 1959/1990, p. 20). Although learning and 'learning to learn in a certain way' are shaped within a 'culture', 'culture' as 'the way of life of a people' and 'culture' as the ways of communication and interaction in the classroom are two different notions. As Bauersfeld explains:

"A classroom culture clearly cannot exist independently of the culture of the society that maintains this institution. The notion *subculture* would be more adequate then. But another difference is much more important: Cultures have no *curriculum* although an observer can interpret functions, structures, and development. And one can neither *create* nor *reform* a culture, although cultures change (not only but also) through the interventions of their members. Cultures, especially, are not formed through agents. Subcultures also emerge, but they also depend on the influence of explicitly trained agents, particularly in educational settings." (Bauersfeld 1998, p. 384)

How the 'culture' of one society or another shapes students' ways of learning mathematics differently is not yet clear. Empirical studies like the ones reviewed in this article provide initial insights into how the teaching, and therefore learning, of mathematics is different. But as Bauersfeld emphasises "living in a culture is an adaptive as well as a constructive process" (ibid., p. 385), which implies a focus on the student and her or his learning process, as well as on the cultural setting these processes happen in. Bauersfeld quotes Bruner:

"If psychology is to get ahead in understanding human nature and the human condition, it must learn to understand the subtle interplay of biology and culture. Culture is probably biology's last great evolutionary trick." (Bruner 1996, p. 184, cited in Bauersfeld 1998, p. 385).

The micro-perspective of socio-constructivist learning theories has helped to clarify the learning processes of an individual as embedded in and related to social processes. These theories, based on an interactionist view of learning, provide a valuable theoretical framework for describing and explaining learning processes in mathematics classrooms, where a group of students is learning together. These theories also provide the means to describe the implicit 'colearning' happening when students are learning, and learning mathematics in particular. It is this 'colearning' that frames students' attitudes and beliefs towards mathematics, so it is important for all later learning of mathematics. Bauersfeld describes this 'colearning' as follows: "the future member takes over regulations and learns to obey norms for everyday life. Along the way, these formations, emerging with the individual, become habitual and subconscious (if their parts have ever been formed consciously) and this to the extent that the member can act adequately and in accordance with this culture even in situations never experienced before" (ibid., p. 385). Bourdieu describes this 'formation' as 'habitus' (Bourdieu 1972, 1980). But, theoretical descriptions of learning processes from a socio-constructivist learning theory tend to emphasise the local context these processes are embedded in. Comparison of local contexts embedded in different 'cultural contexts' according to national, regional or socio-economic backgrounds is rarely a focus of these kinds of studies. Studies like those of Bourdieu on the other hand, share the idea that learning forms 'habitual and subconscious' patterns, though they are mostly interested in macro-perspectives. They are interested in how far the 'habitus' differs according to national, socio-economic, gender or ethnic background. And as Hall emphasises "once people have learned to learn in a given way it is extremely hard for them to learn in any other way" (Hall 1959/1990, p. 48). As the studies reviewed in this article show macro-perspectives can help to explain the learning and the implicit 'colearning' of mathematics.

7.3 Two goals of comparative research

In reflecting on the comparative studies reviewed in this paper it seems that two approaches are represented. In section 6 different goals, methodological and theoretical frameworks of qualitative comparative research have been identified. Considering these differences again, I classify these studies into two types. The first type (represented by Pepin 1997 and Kaiser 1999) intends to identify differences between 'national cultural traditions' concerning mathematics education in one country or another. In such studies comparisons between countries are the object of the study. These studies follow a logic that is described by Alexander as follows:

"I argue that educational activity which we call pedagogy – the purposive mix of educational values and principles in action, of planning, content, strategy and technique, of learning and assessment, and of relationships both instrumental and affective – is a window on the culture of which it is a part, and on that culture's underlying tensions and contradictions as well as its publicly-declared educational policies and purposes" (Alexander 1999, p. 149)

On the other hand, in the second type of studies the main interest lies in specific aspects of learning. For example Broadfoot et al. (2000) are interested in beliefs and attitudes of students and their learning outcomes, which turn out to be culturally shaped. In this type of study comparisons are not the object, but the method of the study. This becomes even more clear in the study of Knipping (2003). She is primarily interested in a specific topic of mathematics education, the teaching and learning of mathematical proof in school. She investigates French and German classrooms, because she hopes the differences between them will allow her 'to see more'; comparisons are used as a method. Coming back to the question how comparative research can help us to better understand the learning and the implicit 'colearning' of mathematics, the different types of comparative studies identified here offer different insights. The 'object' approach makes clear what differences and similarities exist and which aspects of learning mathematics are possibly involved. The 'method' approach might help to clarify aspects of learning that are already considered as important ones to study in depth.

7.4 Our own taken-for-granted assumptions

The different approaches these studies take, 'object' or

'method', are reflected in their theoretical frameworks. Kaiser (1999, 2002) and Pepin (1997, 2002) take historical studies on different 'national cultural traditions' as their background theories for explaining the differences they find. On the other hand, Knipping bases her analyses of classroom processes on theoretical and empirical research on proof and argumentation in mathematics education and philosophy. Even though the studies reviewed here show important differences in theoretical backgrounds and in goals, they share one common goal: they intend to challenge our own implicit cultural understandings about learning mathematics. They share what Stigler and Perry describe as follows:

"Cross cultural comparison also leads researchers and educators to a more explicit understanding of their own implicit theories about how children learn mathematics. Without comparison, we tend not to question our own traditional teaching practices and we may not even be aware of the choices we have made in constructing the educational process" (Stigler and Perry 1988, p. 199).

But the question is, to what extent do these studies help us to come aware of our own implicit understandings about learning mathematics? To what extent do their theoretical backgrounds help to reveal our own 'taken-for-granted assumptions'? To what extent do their methodological approaches help to reveal these differences? Revealing assumptions of this kind is also the goal of ethnographic approaches. In the following I present a serious criticism by an ethnographer in order to examine how we could get more out of our own analyses and so to reach our goal of revealing new insights through comparative research for mathematics education.

The ethnographer Joseph Tobin, observing the increasing interest in educational ethnography, makes the critique that ethnography in educational research is often reduced to thick description, in the style of Clifford Geertzs' analysis of a Balinese cockfight (Tobin 1999, p. 123). His concern is that by this reduction ethnography becomes synonymous with 'case studies' and other qualitative inquiry methods and loses its key feature. For Tobin the key characteristic of ethnography is the 'insider/outsider dialogic'. "Ethnography is a study of an insider's culture, privileging insider's meanings, told by an outsider, for a readership of other outsiders." (ibid., p. 124). Therefore, "an outsider goes to live for a period of time among a group of exotic others who, viewed as expert informants, are asked to provide insiders' explanations" (ibid., pp. 123-124). Tobin considers that the encounter with another culture, which seems strange, exotic, counter-intuitive, and inexplicable, is important. By asking the insiders to explain their practices and their reasoning, the outsider tries to understand their reasons, not necessarily to agree with them. In order to understand, the outsider has to make the strange familiar. And in doing so, the familiar becomes strange as well. In this process Tobin locates the potential of ethnographic research, which is to "defamiliarise some of our taken-for-granted assumptions about what how we live our own lives in our own cultures" (ibid., 124). Educational research in general does not take an ethnographic approach in a strict sense, but uses ethnographic methods by analogy. But still we have to ask ourselves critically if we are not depriving ourselves in our comparative research, when we do not go far enough to ask: Which of our taken-for-granted assumptions become suddenly strange, once we understand that aspects of teaching and learning mathematics can be very different from what we have experienced so far? I leave it to you, the reader, to decide whether the comparative studies reviewed in this paper have exposed at least some of your implicit assumptions concerning teaching and learning mathematics.

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