A brief historical comparison of tendencies in Mathematics Didactics/Education in Germany and the United States

Günter Törner (Germany)
Bharath Sriraman (USA)

Abstract:
In this paper we outline in macroscopic terms major tendencies in the mathematics education histories of Germany and the United States. In particular, we spell out periodic shifts in focus of mathematics education over the last 100 years and in this process unravel common focal points in the parallel development of the field. In doing so we reflect and hypothesize on why certain trends seem to re-occur, sometimes invariantly across time and geographic location.

ZDM-Classification: D10, D20

You can imagine, Reader, to what lengths I might take this conversation on a subject which has been talked about written about so much for the last two thousand years without getting one step further forward. If you are not grateful to me for what I am telling you, be very grateful for what I am not telling you
- Denis Diderot in *Jacques the Fatalist*

Introduction

It is a positive sign that an international discussion on theories of mathematics education is taking place especially in the wake of TIMSS and PISA. Both the TIMSS and PISA were instrumental in a global re-examination of curricular practices in numerous countries around the globe, especially in the countries that did not fare so well. In the United States, a positive consequence of international assessments has been increased interaction among mathematics education researchers with researchers in other countries such as Japan, Netherlands and Singapore as well as the publication of articles contrasting curricula, comparative video-study analysis of teaching practices in different countries etc. This internationalization of mathematics education is by now means new. Since this special issue of ZDM is dedicated to Hans-Georg Steiner, we find it important to mention that Steiner called for more collaboration between mathematics education researchers around the world and started the *Theories of Mathematics Education group* (TME) to examine and compare questions of research interest, curricular practices (e.g., McKnight et al., 1987), and theoretical traditions used (see Sriraman & English, 2005 for more details on the TME group). However, the community has engaged in collaborative and forward looking initiatives since then as is evidenced in the numerous handbooks published (e.g., Bishop 1996; English et al., 2002; see surveys by Schoenfeld, 1999, 2002) as well as books on cross national comparisons of mathematics education (Kaiser, 1999, 2002). Most recently PME took the initiative to closely examine specific geographic trends in mathematics education research which resulted in a research forum on this topic at the 29th PME in Melbourne. Our original paper reported on tendencies with mathematics didactics in Germany. However in this paper we extend the ideas further and engage in a comparison of trends within Germany to those that occurred in the United States within the last 100 years. Our motivation for doing so is to spell out redundancies in trends that occur locally in other geographic regions. As mathematicians we find redundancies troubling because it implies that considerable energy is being spent on similar research with findings that are somewhat invariant. We realize the danger in comparing research dealing with people, cultures and institutions that involve many uncontrollable variables to research in mathematics which does not involve such parameters, and instead only on agreed upon definitions, theorems and proofs which transfer (or generalize). However the forward looking point we are trying to emphasize is that given the new technology and freely available resources, by mining the literature the mathematics education community would perhaps save both time and effort in research as well as allow the research to move forward if redundancies were slowly eliminated. In this paper we engage in the comparison of mathematics education trends that are concurrently occurring (or occurred) in Germany and the United States. In doing so we reflect and hypothesize on why certain trends seem to re-occur, sometimes invariantly across time and geographic location. Numerous reviews about the state of German mathematics didactics are available in German (see [1], Hefendehl et al., 2004; Vollrath et al., 2004). Numerous reviews of mathematics education history in the United States and Canada are also available (NCTM, 1936, 1970). However there are no extant attempts to trace and analyze the last hundred years of “mathematics didactics” trends in Germany in comparison to what was happening in the United States.

Remarks on terminology and history

It has become standard practice for researchers writing in English to use the term “Mathematikdidaktik” when referring to mathematics education in Germany. However, there is no real comprehensive English equivalent for the term "Mathematikdidaktik". Neither "didactics" nor "math-education" describes the full flavor and the historical nuances associated with this German word. Even the adjective “German” is imprecise since educational research approaches in Germany splintered in the aftermath of World War II, with different philosophical schools of thought developing in the former East (GDR) and the west (FRG) on research priorities for university educators, until the reunification which occurred in 1990. Currently the 16 states in Germany reveal a rich heterogeneity in the landscape of mathematics teaching, teacher training and research methods, which manifests itself to insiders who microscopically examine the TIMSS- and PISA-results. However the reasons for this heterogeneity remain a mystery to outsiders. We outline in macroscopic terms the historical reasons for this heterogeneity. In doing so we do not differentiate explicitly between the alignment (or misalignment!) of theories preferred by university educators in comparison to practices of mathematics instruction in schools. The mutual dependencies between the two is certainly an interesting research question which brings into focus the system wide effectiveness (or ineffectiveness) of educational research (see for example Burkhardt & Schoenfeld, 2003) and particularly shows up in the history of mathematics education in the United States. Analogous to the difficulty of defining the term "mathematikdidaktik", the term mathematics education in the United States has meant different things in different time periods. In broad brush terms in the last 100 years, the term has mutated and carried a combination of meanings affiliated with culture, psychology, philosophy curriculum and specific content. The research focus has been influenced by movements within →behaviorism→ arithmetic→psychology of learning→arithmetical pedagogy→ structural thinking→ problem solving →socio-cultural learning→teacher education→ equity. Schoenfeld (2002) characterized the primary research methods in mathematics education as ranging from associationism /behaviourism → Gestaltism → constructivism →cognitive science → socio-cultural theories. Numerous surveys of the state of education in the United States regularly comment on the periodicity of curricular and research phenomena especially whatever is termed "reform mathematics". Nearly 60 years ago Betz (1936) outlined a list of ten discussion questions for secondary education which are still relevant to the present day. Two of the ten questions are fundamental in nature: (1) Have we a consistent and practicable philosophy of education? (2) Which guiding principles if any may be applied in the appraisal of the newer educational trends? The relevance of these two questions by no means suggests that research to date has be unable to address these issues. On the contrary they suggest that the difficulty inherent in mathematics education research due to the ever changing variables of culture, curriculum, theory or methodology borrowing in vogue, technology and educational policy.

The Pedagogical tradition of mathematics teaching - Mathematics as Educational Value

Reflections on the processes of mathematics teaching and learning have been a long-standing tradition in Germany. The early proponents of these theories of teaching and learning are recognizable names even for current researchers. Chief among these early theorists was Adam Reise "the arithmetician" who in the 16th century itself stressed hand computation as a foundational learning process in mathematics. This emphasis is found in the pedagogical classics of the 19th century written by Johann Friedrich Herbart (1776-1841), Hugo Gaudig (1860-1923), Georg Kerschensteiner (1854-1932) (see Jahnke, 1990; Führer, 1997; Huster, 1981). The influence of this approach echoed itself until the 1960's in the so-called didactics of mathematics teaching in elementary schools to serve as a learning pre-requisite for mathematics in the secondary schools.

In the United States, the concern for pedagogy goes back to the 19th century to schools established in Massachusetts. The 32nd yearbook of the NCTM (1970) includes a history of this time period and the arithmetic textbooks used in schools from the advent of colonization on the North American continent. The structure and delivery of arithmetic in schools in this time period is very similar to that of Germany.

Traditions in didactics research *(early 20th Century)*

In Germany, in the early part of the previous century, mathematicians like Felix Klein (1849-1925) became interested in the complexities of teaching and learning processes for mathematics in schools. The occasionally invoked words "Erlangen program" and "mathematization" are the present day legacy of the contributions of Klein and Freudenthal to mathematics education. Klein characterized geometry (and the teaching of it) by focussing on the related group of symmetries to investigate mathematical objects left invariant under this group. The present day emphasis of using functions (or functional thinking) as the conceptual building block for the teaching and learning of algebra and geometry, is reminiscent of a pre-existing (100 year old) Meraner Program. During this time period one also finds a growing mention in studying the psychological development of school children and its relationship to the principles of arithmetic (Behnke, 1953). This trend was instrumental in the shaping of German mathematics curricula in the 20th century with the goal being to expose students to mathematical analysis at the higher levels. The most notable international development in this time period was the founding of the ICMI in 1908, presided by Felix Klein. One of the founding goals of ICMI was to publish mathematics education books, which were accessible to both teachers and their students. We see this as one of the first attempts to "elementarize" (or simplify) higher level mathematics by basing it on a sound scientific (psychological) foundation. Mathematics educators like Lietzmann (1919) claimed that "didactic" principles were needed in tandem with content to offer methodological support to teachers. This approach mutated over the course of the next 50 years well into the 1970's. The over-arching metaphor for mathematics education researchers during this time period was to be a gardener, one who maintains a small mathematical garden analogous to ongoing research in a particular area of mathematics. The focus of research was on analyzing specific content and use this as a basis to elaborate on instructional design (Reichel 1995, Steiner, 1982). This approach is no longer in vogue and is instrumental in creating a schism between mathematicians and "mathematics-didakters", partly analogous to the math wars in the United States.

In the United States, the attempt to elementarize mathematics and rest it on sound psychological foundations are best seen in the contributions of William Brownell. Brownell (1947) was the chief spokesperson for the "meaningful "arithmetic". Meaningful arithmetic refers to instruction which is deliberately planned to teach arithmetical meanings and to make arithmetic more sensible to children through its mathematical relationships. Brownell categorized the meanings of arithmetic into the following groups. A group consisting of a large list of basic concepts. For example: meanings of whole numbers, of fractions, ratios and proportions etc. A second group consisting of arithmetical meanings which includes understanding of fundamental operations. Children must know when to add, subtract, multiply, and divide. They must also know what happens to the numbers used when a given operation is performed. A third group of meanings consisting of principles that are more abstract. For example: relationships and generalizations of arithmetic, like knowing that 0 serves as an additive identity, the product of two abstract factors remains the same regardless of which factor is use as a multiplier, etc. A fourth group of meanings that relates to the understanding of the decimal number system, and its uses in rationalizing computational procedures and algorithms. (Brownell, 1947).

Meaningful arithmetic is "deliberately planned to teach arithmetical meanings and to make arithm etic sensible to children through its mathematical relationships" and in a sense bears some resemblance to the functional thinking emphasized by the Meraner program. Brownell argued that learning arithmetic through computations requires continuous practice. Without practice, retention of arithmetic skills deteriorates, and asserts that learning the meanings of arithmetic would be cumulative, with better retention, and saves time in the end. Brownell said that teaching meaningful arithmetic would reduce practice time, encourage problem solving, and develop independence in students. He remarked that there was lack of research in 1947, on teaching and learning arithmetic meaningfully. Brownell doubted that quantitative research could address questions in the area of teaching and learning arithmetic.

One could say that Brownell was clairvoyant for his time, emphasizing qualitative research in the age of behaviorism He was noted for his use of a variety of techniques for gathering data, including extended interviews with individual children and teachers, and for his careful, extensive and penetrating analyses of those data (Kilpatrick, 1992). Kilpatrick suggests that meaningful arithmetic anticipated the modern mathematics movement that began in the 1950's. This is addressed in a subsequent section of the paper.

Unlike the strong presence of mathematicians as initiators of didactic traditions in Germany in the early part of the 20th century, during that time period in the United States philosophers such as William such as William James, Charles Sanders Peirce, Oliver Wendell Holmes, George Herbert Mead, and John Dewey. These philosophers are also referred to as the *American Pragmatists*. For example: (a) Dewey and Meade emphasized that conceptual systems are human construct, and that they also are fundamentally social in nature. (b) Pierce emphasized that the meanings of these constructs tend to be distributed across a variety of representational media (ranging from spoken language, written language, to diagrams and graphs, to concrete models, to experience-based metaphors) – each of which emphasize and ignore somewhat different aspects of the constructs they are intended to express and/or the "real life" experiences they are intended to describe. (c) Dewey emphasized that knowledge is organized around experience at least as much as around abstractions – and that the ways of thinking which are needed to make sense of realistically complex decision making situations nearly always must integrate ideas from more than a single discipline, or textbook topic area, or grand theory. (d) James emphasized that the "worlds of experience" that humans need to understand and explain are not static. They are, in large part, products of human creativity. So, they are continually ch anging - and so are the knowledge needs of the humans who created them. (e) Dewey emphasized that, in a world filled with technological tools for expressing and communicating ideas, it is naïve to suppose that all "thinking" goes on inside the minds of isolated individuals. For example, at least since the age of written media, mathematicians have been off-loading formerly internal functions (Lesh & Sriraman, 2005a).

"Genetic" Mathematics Instruction: Early versions of Constructivism.

In Germany, the word "genetic" was used to exemplify an approach to mathematics instruction to prevent the danger of mathematics taught completely via procedures (Lenné, 1969). Several theorists stressed that mathematics instruction should be focussed on the "genetic" or a natural construction of mathematical objects. This can be viewed as an earlier form of constructivism. This approach to mathematics education did not gather momentum. The word "genetisch" occurs frequently in the didactics research literature until the 1990's.

The New Math

In the United States, the "new math" movement began due to events linked to World War II. The mathematical community became interested in mathematics education, stimulated by both their war time experiences as well the new importance that mathematics, science and technology had achieved in the public eye. Mathematicians along with politicians and the press became leaders of the movement to change school mathematics. However these reformers didn't always have the same results in mind. Mathematicians were interested in students learning to understand the structure of the subject whereas the public was interested in test scores. Several prominent university professors like Beberman were involved in the reform movement (Hayden, 1983). The launch of Sputnik in 1957 , witnessed numerous articles in the press claiming that schools were responsible for the American lag in technology. The other press arising after Sputnik was placing the burden of social reform on the public schools (Hiatt, 1986). This resulted in mathematicians and experts from other fields designing curriculums for schools (e.g School Mathematics Study Group or SM SG). However many of these curriculum materials were controversial since the public and the government were more interested in reform in mathematics teaching rather than having students understand the structure and content of modern mathematical ideas. Due to conflict of interests between the public and government versus the mathematicians, the "new math" movement became controversial and unpopular, and ended in an unsatisfactory note. It is important to analyze why the "new math" movement failed for there are lessons that can be learnt from such an analysis. The "new math" movement failed because the teachers couldn't implement the changes in themselves to make a change in what and how children learnt. This led to frustration which eventually led to the demise of "new math". (Miller, 1990). One must understand that the intentions of mathematicians like Beberman and Begel was to change the mindless rigidity of traditional mathematics . They did so by emphasizing the *whys* of problem solving rather than the *hows* but it seems rather ridiculous to expect teachers trained in direct instruction to change and guide younger students toward the concrete discovery of abstract mathematical principles by deduction. While Beberman was a gifted mathematician and teacher, to make every teacher into a Beberman was virtually impossible. The plan to "upgrade" teachers to enable them to teach this new curriculum failed miserably. Beberman realized that the "new math" movement failed because teachers were unprepared and addressed this at the 1966 meeting of the National Council of Teachers of Mathematics. Other experimental programs like the Madison Project were also launched during this era. The primary reason for the demise of the "new math' movement was the country's penchant for a "quick fix" (Miller, 1990). Miller comments that "had Sputnik not flown", UICSM, SMSG, the Madison Project and other experimental programs might have evolved slowly and carefully into a national curriculum. The lesson to be learnt from the failure of the "new math" movement is that reform takes time to implement. On a positive note this era brought forth the use of manipulatives to learn mathematics (for example: Cuisenaire rods for use in understanding division and fractions). The end of the "new math " movement gave rise to a "back to the basics" movement, and the general tenor now was on a return to memorization and drill and practice. However, rather than help the program, this movement caused a reduction in scores on problem solving and concepts. Americans began to hold schools accountable for a slowing economy and standardized testing became a major tool for proving schools achievement. To provide a comprehensive "basic skills" program , the NCTM, in 1977, defined 10 basic skill areas: problem solving, applying mathematics in day to day situations, alertness to reasonableness of results, estimation and approximation, appropriate computational skills, geometry, measurement, reading, interpreting and constructing tables, charts, and graphs, using mathematics to predict and computer literacy (NCTM, 1977).

Parallel to the new math movement occurring in post-Sputnik United States, an analogous reform movement took place in Germany (mostly in the West, but partly adopted by the East, see [1]). A superficial inspection seems to point to a realization of Klein's dream of teaching and learning mathematics by exposing students to its structure. This reform took on the dynamic of polarizing scientists (mathematicians) to work in and with teacher training, the resulting outcome being a lasting influence on mathematics instruction during this time period. Unlike the United States teachers were able to implement a structural approach to mathematics in the classroom. This can be attributed to the fact that during this time period there was no social upheaval in Germany, unlike the U.S where the press for social reform in the classroom (equity and individualized instruction) interfered with this approach to mathematics education. The fact that German "new math" did not survive the tide of time indicates that there was difficulty in implementing it effectively.

Didactics/ Mathematics Education as a research discipline

While the new mathematics movement was subject to a host of criticisms, one positive outcome was the founding of the Gesellschaft für Didaktik der Mathematik (German Mathematics Didactics Society) which stresses that mathematics didactics was a science whose concern was to rest the mathematical thinking and learning on a sound theoretical (and empirically verifiable foundation). This was a radical step search for mathematics education research in Germany, one that consciously attempted to move away from the view of a math educator as a part-time mathematician (recall Klein's garden). Needless to say, we could easily write an entire book if we wanted to spell out the ensuing controversy over the definition of this new research discipline in Germany (see Bigalke, 1974; Dress, 1974; Freudenthal, 1974; Griesel, 1974, Laugwitz, 1974; Leuders, 2003; Otte, 1974; Tietz, 1974 Wittmann, 1974; 1992). However the point to be taken from the founding of this society and a new scientific specialty is that the very debate we have undertaken here, i.e., to globally define theories of mathematics education has in fact many localized manifestations such as in Germany.

Although there was no analogous consolidated effort to establish a society of mathematics education researchers per se in the United States, the founding of the Journal for Research in Mathematics Education (JRME) in 1969-1970 serves as an important bench mark for comparative purposes. Lesh & Sriraman (2005b) in the previous issue of ZDM, argued that research as we mean it today only started in the 1960's and depended mainly on theory borrowing (from other fields such as developmental psychology or cognitive science) with no real stable research community nor a distinct identity. They write:

> „We really had no stable research community – with a distinct identity in terms of theory, methodologies, tools, or coherent and well-

defined collections of priority problems to be addressed. Only recently have we begun to clarify the nature of research methodologies that are distinctive to our field (see Lesh & Sriraman (2005b) for related citations of Biehler et al., 1994; Bishop et al., 2003; Kelly & Lesh, 2000; Kelly & Lesh, in press; English, et. al., 2001); and, in general, assessment instruments have not been developed to measure most of the constructs that we believe to be important." (p. 490).

Mathematical Teaching and Learning- A Socialistic and an Individualistic Process ?

One of the consequences of founding a new discipline of science was the creation of new theories to better explain the phenomenon of mathematical learning. The progress in cognitive science in tandem with interdisciplinary work with social scientists led to the creation of "partial" paradigms about how learning occurs. Bauersfeld's (1988,1995) views of mathematics and mathematical learning as a socio-cultural process within which the individual operates can be viewed as one of the major contributions to theories of mathematics education.

In the United States, by the mid 80's several researchers began to investigate how children learn mathematics. The focus of research shifted from the teacher to the student with the focus being on how students think and learn. The learning theories of Piaget and Vygotsky were used to find ways to educate students in mathematics. The NCTM commissioned the *Curriculum and Evaluation Standards for School Mathematics*. The primary goal of the Standards is to help students (1) become mathematical problem solvers, (2) learn to communicate mathematically, (3) learn to reason mathematically, (4) value mathematics, and (5) become confident in one's ability to do mathematics. (NCTM, 1989). The Standards envisions well rounded students having good reasoning abilities, readily adapting their mathematical knowledge to solve "real" world problems. One can see the evolution of the notion of what it means to be mathematically literate person from the 1940's to the 1990's by noting the shift from being a procedurally competent student to the student envisioned by the Standards. One can also see the evolution from teacher centered classrooms to learner centered classrooms. The NCTM published the *Professional Standards for Teaching Mathematics* in 1991, to serve as a guideline for teachers to implement changes in their classroom. It seems that the reform movement has finally learnt from the mistakes of the past, by addressing the needs of both students and teachers to implement reform. The Professional Standards (1991) present several recommendations for teachers, namely (1) Posing worthwhile mathematical tasks (2) the teacher's role in discourse and (3) the student's role in discourse.

The Professional Standards (1991) suggest when creating worthwhile tasks, the teacher should base her decisions on the context of the lesson and the learning styles of the students. This includes the appropriateness of the task for the type of students, the assessment of the conveyance of the task and which skills are to be developed from the task. Discourse is described as a way of thinking, talking, representing ideas, and generally the way a class is run. Teachers need to pose thought provoking questions, listen to students ideas as well as have students listen to one another. Students should be free to make conjectures and explore but also listen to other students' ideas in the class and mathematical meaning emerges in the classroom through negotiation and discourse.

The Presence of New Technology

The influx of new technology, particularly graphing calculators posed several conundrums in the 1990's to the mathematics education community in the United States. The three basic issues were (1) the problem for teachers (2) the problem of implementation (through reform) and (3) the problem for students. It is impossible to have a "discrete" discussion of these three areas because they are interconnected, as the reader will find out. We think the past may yield insights into this question. During the late 1970's in the U.S, education was viewed as a new and viable market for computer and software products. This resulted in a large influx of first-generation microcomputer technology and software into schools. Unfortunately, little or no related professional development for the teachers accompanied the technology. The training that did occur was often limited to a few hours or days and was oriented around computer literacy or experimentation with drill-and-practice or tutorial software. With this limited experience, many mathematics teachers were expected to use the computer in their mathematics classrooms and were responsible for determining how to use it. Technical support for the teacher was minimal or non-existent with no pedagogical support to assist teachers in developing and implementing the technology within their classrooms. What has been the outcome of this? Few teachers engaged in more than a cursory attempt to use technology in their classrooms. This was reflected in the 1989 I.E.A. Computers-in – Education survey (Becker, 1990, 1991) on the use of technology in high school mathematics. Of the secondary mathematics teachers that responded, 42% used the computer in class at least one day, but only 17% classified themselves as using the computer regularly throughout the year. The dominant computer activity for these teachers was the use of drill-and-practice software. Learning to write programs was also a common instructional goal within computer mathematics courses. The study also reported that in "only 10% of high school mathematics classes where computers were used did students use mathematical graphing programs or spread sheets on more than five occasions" (1990,p.2).

During the 1980's, national reports such as A Nation at Risk (NCEE, 1983), and The Underachieving Curriculum: Assessing U.S. School Mathematics From and International Perspective (McKnight et al., 1987) were published and were critical of the state of mathematics education in this country. These findings along with the recommendations in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) provided a rationale and direction for the types of changes that mathematics education should undertake. Interestingly enough, these reports consistently portrayed technology as one factor that stimulated the need for reform and recommended the integration of technology in the teaching and learning of mathematics. The act of placing technology in the classroom does not constitute reform and should not be mistaken for reform (Kilpatrick, 1989). Reform must involve teachers. To continue the purchasing and placing of educational technology in schools without any change in the professional development and support for teachers would repeat what was done in the 70's and have little or no impact on how and to what extent technology is used in the teaching of mathematics. Wilson and Kilpatrick (1983) identified the teacher as "the focal point for any change in the schools", and say that this aspect of reform has not been well-learned (p.117). This suggests that a better understanding of how teachers define and interact with technology in their school settings would be useful information that could help the design and implementation of programs for math reform.

So, the most fundamental problem for teachers in the U.S with the use of technology in a traditional curriculum was not knowing how to use it effectively to enhance mathematical learning. Looking back at what happened in the 70's, it seems ironic that teachers mostly used computers for drill-and –practice. However, it was not their fault because they did not receive the technical and pedagogical support to become aware of how to use the technology to the students benefit. This could be one way of looking at it or could it be that the teachers use of technology for drill-and-practice reflected their beliefs about what mathematics is? The research done by Magdalene Lampert indicates what is possible when teachers are provided the right support. One research project at the Educational Technology Center (ETC) at Harvard involved seven secondary geometry teachers in various school sites collaborating with researchers to "understand the process of implementing technology-enhanced guided explorations in school classrooms" (Lampert, 1993,p.143). The teachers were selected based on their willingness to participate in a project that would use the Geometric Supposer in a "guided inquiry" approach that would revise the curriculum and instruction for the entire course. The classrooms were equipped with computers and the software and a set of handouts that "pose problems to lead students to make conjectures based on the data they collected" (Lampert, 1988,p.1). It is not stated whether the teachers received any initial training on the use of Geometric Supposer .They did however participate in monthly "user group" meetings with a researcher from ETC. The reason we are citing this particular study is because of Lampert's findings. Lampert found that the mathematics teachers were confronted with considerable conflict during the course. Issues of control, ordering of content, the role of induction and deduction were areas of common concern. The dynamics of the classroom, the increased possibilities that geometry offered, student ownership of an agenda, and the existence of another "authority" in the classroom all contributed to the pressure to change their role in the classroom. The teachers found this role to take more time, energy, and required "a more demanding relationship with the students than teaching by telling" (Lampert, 1993,p.173). Although, this was a study where teachers volunteered to revise the curriculum and instruction for the entire course. We attribute the difficulties that the teachers faced during this course to conflicts in their existing belief structures, and the sudden change from a traditional setting to a non-traditional one.

There are several bodies of research on the influence of technology on students learning. Research on student achievement (Ruthven, 1990; Quesada and Maxwell, 1992 , Harvey, 1993) involving the comparison of test scores of students receiving graphing calculator based instruction to those receiving traditional instruction found significant differences in favor of experimental groups (using technology in the classroom). Students who use graphing technology placed at higher levels in a hierarchy of graphical understanding (Rich, 1991), and obtained more information from graphs (Beckmann, 1989). There are other studies that report improved problem solving by students using graphing calculators (Dick, 1992). When one looks at all the studies quoted above, it seems that graphing calculators are working wonders for students. It is important to realize that the positive results of most of these studies were obtained from the experimental groups. Could one generalize these findings and conclude that they could apply in a "traditional" classroom. If we (the teacher) believe in direct instruction and subscribe to the view that mathematics is a set of truths and rules, would the introduction of technology promote improved understanding in my students? The answer is no. Simply because my use of the technology would reflect these beliefs. What are some of the problems that students will have in such a setting? They would now use the calculator to find the right answers. In other words, they would use the graphing calculator to execute the algorithms that they normally did with pencil and paper. One cannot expect the students to change if the teachers don't change. To me, the biggest problem that students might have with the use of technology in a traditional curriculum is the "black-box" syndrome where the machine is viewed as a means of getting the right answers, instead of a problem-solving tool. The problem at the moment seems that technology like hand-held graphing utilities are introduced only in high school. One cannot expect students to suddenly change their pre-conceived notions about technology especially if technology is used incorrectly by inexperienced teachers. It makes more sense to introduce graphing utilities in the earlier grades, so that with sufficient practice, under "trained" teachers, students will learn to use technology as a problem-solving tool.

In Germany, Weigand's (1995) work posed the analogous rhetorical question as to whether mathematics instruction is undergoing yet another crisis. The advent of new technologies opened up a new realm of unimagined possibilities for the learner, as well as researchable topics for mathematics educators. The field of mathematics education in Germany oriented itself to address the issues of teaching and learning mathematics with the influx of technology. However the implications of redefining mathematics education, particularly the "*hows*" of mathematics teaching and learning in the face of new technology poses the conundrum of the need to continually re-orient the field, as technology continually evolves (see Noss / Hoyles (1995) for an ongoing global discussion).

The influence of international testing

The results of TIMMS and PISA brought the previously discussed trends to a collision with mathematics educators and teachers feeling under-appreciated in the wake of the poor results. These assessments also brought mathematicians and politicians back into the debate for framing major policies, which would affect the future of mathematics education in Germany. Mathematics education was now in the midst of new crisis because the results of these assessments painted German educational standing in a poor global light. A detailed statistically sieved inspection of the results indicated that poor scores could be related to factors other than flaws in the mathematics curriculum, and/or its teaching and learning, that is to socioeconomic and cultural variables in a changing modern German society. Thus mathematics education in Germany would now have to adapt to the forces and trends creating havoc in other regions of the globe (see Burton, 2003; Steen, 2001). Similarly the relatively poor performance of the U.S on TIMMS led to increased cross-national studies on classroom practices, comparisions of curriculum between several countries (U.S., Japan, Germany, Singapore) as well as the issue of teacher training and support structures for implementing new reform oriented curricula in the U.S. The heterogeneity of approaches within the U.S makes it impossible to adequately describe any systemic approaches which have been successful in the post-TIMMS phase. The repurcussions of the poor performance of the United States on PISA are yet to be felt and not received the same outcry or attention that TIMMS and SIMMS results did. International testing bring into focus assessment issues along with societal and political variables which are changing conceptions of mathematics education as we speak. In a sense we have come full circle because in this paper we have not defined what mathematics education or mathematics didactics really is. However, in the search through history for the answer, we have understood the epochal nuances of this interesting term. Perhaps the advent of globalization requires that we finally define it.

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Authors

Prof. Dr. Günter Törner,
Institute of Mathematics
University of Duisburg-Essen
Duisburg Campus
Lotharstr. 63 / 65
D-47048Duisburg
Germany
E-mail: toerner@math.uni-duisburg.de

Prof. Dr. Bharath Sriraman
Editor, *The Montana Mathematics Enthusiast*
Dept. of Mathematical Sciences
The University of Montana
Missoula, MT 59812
USA
E-mail: sriramanb@mso.umt.edu