Learning by Developing Knowledge Networks

A semiotic approach within a dialectical framework

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Abstract: A central challenge for research on how we should prepare students to manage crossing boundaries between different knowledge settings in life long learning processes is to identify those forms of knowledge that are particularly relevant here. In this paper, we develop by philosophical means the concept of a *dialectical system* as a general framework to describe the development of knowledge networks that mark the starting point for learning processes, and we use semiotics to discuss (a) the epistemological thesis that any cognitive access to our world of objects is mediated by signs and (b) *diagrammatic reasoning* and *abduction* as those forms of practical knowledge that are crucial for the development of knowledge networks. The richness of this theoretical approach becomes evident by applying it to an example of learning in a biological research context. At the same time, we take a new look at the role of mathematical knowledge in this process.

Kurzreferat: *Lernen als Entwicklung von Wissensnetzen. Ein semiotischer Ansatz in einem dialektischen Rahmen*. Die Frage, wie wir Schülerinnen und Schüler darauf vorbereiten können, die Grenzen zwischen verschiedenen Wissenssystemen in Prozessen lebenslangen Lernens zu meistern, macht es nötig, solche Wissensformen zu identifizieren, die hierzu besonders relevant sind. Im vorliegenden Text entwickeln wir dazu mit philosophischen Mitteln das Konzept eines dialektischen Systems. Dieser Ansatz dient als Rahmen, um die Entwicklung von Wissensnetzen beschreiben, die den Ausgangspunkt für Lernprozesse bilden. Dabei verwenden wir die Peircesche Semiotik, um (a) die erkenntnistheoretische These zu diskutieren, dass jeder kognitive Zugang zu den Dingen unserer Welt durch Zeichen vermittelt ist, und (b) um diagrammatisches Schließen und Abduktion als diejenigen Formen praktischen Wissens zu beschreiben, die entscheidend für eine Entwicklung von Wissensnetzen sind. Die Reichhaltigkeit dieses theoretischen Ansatzes wird in der Anwendung auf einen Lernprozess im Rahmen biologischer Forschung sichtbar gemacht. Gleichzeitig wird die Rolle des mathematischen Wissens in diesem Prozess beleuchtet.

ZDM Classification: C30, D20, D30, D50, M10, M20, M60

1. Introduction

This study is part of a research project about forms of knowledge that are of particular importance for crossing and navigating the boundaries between formal education and the workplace. Its focus is thus on an essential condition for the possibility of lifelong learning. To understand the nature of knowledge involved during boundary crossing, we need a more differentiated concept of knowledge than provided by the philosophical tradition (Hoffmann; Roth 2004), which defines knowledge usually in the following form. Person X knows the proposition *p* if and only if (a) X believes that *p* is true, (b) *p* is indeed true, and (c) X can formulate reasons, or can explain, why *p* is true. While this traditional approach reduces knowledge to what can be expressed in a proposition—“knowing that” (Ryle 1949)—other forms of knowledge are of much greater relevance for learning processes. These other forms include not only Ryle’s “knowing how” (practical and strategic knowledge) but also knowledge about oneself (self-estimation, self-confidence, etc.), about others, about the respective situations, and about available tools (Roth, in press).

We are now investigating the idea that learning is possible by developing networks of different knowledge forms. In the present study, we articulate this approach by developing two theoretical tools that are foundational for a more systematic elaboration of learning by developing knowledge networks: (a) the concept of a “dialectical system” and (b) a “semiotic model” that focuses on signs as means for getting cognitive access to a world of objects, and to ourselves.

Here, we do not use the term dialectic in the metaphysical sense (e.g., Hegel), which identifies ontological processes (the development of “being”) with processes of thinking. Rather, we use the term in the sense of dialectic mediation: a process of theorizing in which a set of opposing elements becomes related to one another by a further element (Hoffmann, in preparation). Such a further mediating element can be, for example, the activity of a scientist who puts the following two opposing methods into a relation of mutual dependency: (a) the method of generating general concepts out of empirical observations (e.g. the natural kinds of mammals, birds, etc.) and (b) the method of dividing those general concepts again for a classification. Going beyond this initial definition of dialectic mediation, we define our concept of a dialectic system (DS) here as a set of four elements, one of them a relation $R(a_1, ..., a_n)$ between a set of at least two further elements ($n \geq 2$):

(1) $DS\{R(a_1, ..., a_n); LM; Agt(Mot, Abil); Dev\}$

These four elements are defined as follows: *R* signifies a relation between opposing elements a_1 to a_n. “Opposing” has to be understood here in a rather broad sense ranging from contradictions between propositions to certain tensions between a set of propositions a_1 to a_n that are, with regard to their consistency within a given belief system, difficult to maintain at the same time. Or R might be a breakdown of a part of our belief system as a result of new experiences. *LM* is a level of mediation. The idea is that any problems within R can only be overcome at a level beyond R. There must be a ‘room’, so to speak, to develop a relation. The level of mediation provides a range of possibilities for thinking and acting (cf. Holzkamp 1983). These possibilities are defined by the concepts, theories, techniques, instruments, or sign systems (see below) as developed within a culture when what has to be learnt is already known within a culture, and they are not defined when new knowledge must be developed to cope with the problems within R. *Agt* is an (individual or collective) agent, defined here by two characteristics: (a) an emotionally based motive or motivation (Mot) to overcome the difficulties described by R and (b) given abilities (Abil). The crucial point is that the abilities mediate the availability of specific cultural possibilities. For example, an illiterate person cannot use certain cultural means for solving his or her problems. Based on these considerations, the idea of *expansive learning* can be developed:

"Expansive motivated learning means ... learning for the sake of an enlargement of control/quality of life as accessible by penetrating a subject matter. Thus, expansive motivated learning actions ... means overcoming my isolation towards a realization of generalized societal action possibilities—as accessible by a learning based opening up of subject matters—in my subjective experience." (Holzkamp 1993, p. 1, our translation)[1]

A connection between culture (society) and agent becomes possible in the concrete realization of general possibilities available in a culture (society) on the basis of my own particular abilities in a specific situation.

The last of our four elements that define a dialectical system is *Dev*, that is any development or change that takes place in one of the three other elements R, LM, or Agt. Thus, a new formulation of, a new arrangement of, or an addition to the elements a_1 to a_n might resolve the experienced difficulties within the relation R; or the impossibility of solving the problems may lead to a frustration of agents and negatively affect their motivation; or the agent is able to develop new tools and ideas that enlarge the range of possibilities at the level of mediation, that is within a culture.

This framework of a dialectic system has been developed to put together all those elements that are necessary for describing a certain form of learning, that is, learning based on the experience of personally relevant problems. There are, however, other forms of learning. Based on the idea that the function of learning is to open up new action- and thinking-possibilities for an agent, learning is possible either in the form of a dialectical development or in the adoption of tools that are available to us in our sociomaterial environment. Small children, for example, learn their mother tongue not so much by genuine creative acts but rather by taking up words which are perceived as most powerful means to enlarge their action possibilities within those interaction frames which are already well-known for them (Bruner 1983). But in this case, too, it is obvious that we need a description of what happens to an agent with motivations and abilities, a social level of mediation, and a description of what exactly develops. We therefore define a learning system in a more general way as:

(2) LS{LM; Agt(Mot, Abil); Dev}

From this point of view, dialectical systems constitute a subclass of learning systems. In both forms of learning, however, the central problem is the same: How is it possible to enlarge, or to improve action and thinking possibilities?

To study this question in greater detail we propose a semiotic model that is embedded, on the one hand, in the dialectical framework described above and, on the other hand, in a more general epistemology. In his *Critique of Pure Reason*, Kant made it quite clear that there is no direct way of grasping the world that surrounds us: consciousness never has direct access to reality. Any cognition is mediated in different ways, be it by (a) concepts used to identify what we see, (b) forms of perception that determine the sequential and spatial order of what we see, or (c) theories that determine our focus of attention, our expectations, and so forth.

From this general epistemological perspective, the most important problem of any learning system is this: In which way can we *perceive* the general possibilities provided by our culture or society if we do not have direct access to them? In a more general sense, this problem is the problem of interpretation. Because, the problem is not to see the tools or means of a culture, the problem is to see their possibilities for my own acting and thinking. Thus, a student might see the sign π at the blackboard, but he might be unable to see that, and in which way, it is a cultural tool for calculating circumference and area of a circle. From this perspective, the problem of how to interpret a sign can be taken as an alternative formulation of the epistemological problem of how to access the objects that constitute our world. However, the problem of interpreting signs is also fundamental for the possibility of learning, for to be motivated to learn "I must have the capability, faced with a certain learning problem, to directly experience or anticipate, the internal *relations between learning-induced lifeworld expansion, increase of control, and an improved quality of life*" (Holzkamp 1993, p. 190, our translation, original emphases). But how can we perceive or anticipate this internal relation between available tools and their mediation of our personal action possibilities? The fundamental problem is that to recognize this internal relation, we already have to presuppose certain knowledge that is supposed to be the end result of the learning process (e.g., Roth; McRobbie; Lucas; Boutonné 1997).

We argue as follows: In a first step, we show how, based on Peirce's triadic model of a sign relation, a semiotic model can be developed that allows a discussion of the problem of interpretation in a very general way. We highlight that the possibility of interpreting signs depends on prior knowledge, and then—in a second step—more precisely: on a network of different knowledge forms that are relevant in a situation. In a third step, we propose—based on the notion of dialectic system—a dialectical model of learning that explains the development of a knowledge network by an ongoing process of mutual tuning of its nodes. Essential preconditions for this process are (a) a representation of this knowledge network in a way that it can become an object of reflection (diagrammatic reasoning) and (b) the capacity to generate new hypotheses in relation to a range of possible interpretations of those representations (abduction). In a fourth step, we test the adequacy of this model by applying it to a concrete case of a learning process among research biologists.

[1] Holzkamp (1993: 190, footnote) insists that there is no relation to the concept of "learning by expanding" as developed by Engeström (1987), though others showed that the two forms of expansive learning can be related (e.g., Roth; Tobin 2002).

2. The Semiotic Approach

Charles S. Peirce's (1839–1914) essential innovation in the history of semiotics—the theory of signs, or representations—becomes visible if we relate it to the distinction of "meaning" and "sense" as it is known since Frege (1892): For Peirce, the meaning of a sign is not definable by merely listing references to possible objects and the sense is not definable only by its relation to other signs. Rather, both meaning and sense, or extension and intension depend first of all on *interpretation*. This at least is our epistemological interpretation of the triadic relation in which a sign, to be a sign, must be embedded according to Peirce (Figure 1)[2]:

> "I define a *Sign* as anything which on the one hand is so determined by an Object and on the other hand so determines an idea in a person's mind, that this latter determination, which I term the *Interpretant* of the sign, is thereby mediately determined by that Object. A sign, therefore, has a triadic relation to its Object and to its Interpretant." (Peirce, CP 8.343; original emphases)

In contrast to Saussure's *sémiologie* that consists—based on an analysis of language—only of a dyadic relation between signifier and signified, the essential characteristic of Peirce's semiotics is the role of the interpretant. In later writings, he defined his interpretant as "the proper significant outcome of a sign", or as its "effect" (CP 5.473, 5.475). Thus, the interpretant can be a reaction to a sign or its effect in acting, feeling, and thinking, or its meaning. Every interpretant therefore both translates the content of the sign *and* increases our understanding of it (Eco 1984). According to Peirce's later differentiation of possible interpretants (cf. Hoffmann, in press-b), this meaning again can either be located within an individual or a collective. What might be called the objective meaning of a sign is, for Peirce, the "final logical interpretant", that is, the meaning that ideally comes to be in the long run in the scientific community. Peirce also coins the concept of a "cominterpretant" to signify an interpretant as it might evolve, perhaps only momentarily, as shared meaning between dialogue partners in concrete interaction (cf. Sáenz-Ludlow, in press).

It is important not to confuse the Peircean interpretant with an interpreter. The interpretant might be a spontaneous reaction within a person's mind or it might be the normal reaction "produced on the mind by the Sign after sufficient development of thought" (Peirce, CP 8.343), but it also can be any arbitrarily created meaning within a certain group of persons or the shared standard reaction to a certain sign within a group which may be defined in a general way by certain societal or cultural characteristics.

Peirce often highlights that the sign and its object "determine" the interpretant. He says, for example, that the triadic "relation must ... consist in a power of the representamen to determine some interpretant to being a representamen of the same object" (CP 1.541).

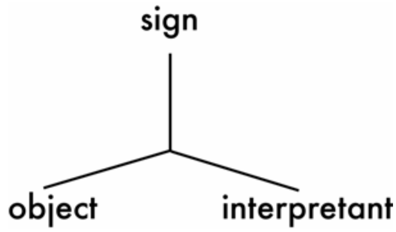

Figure 1. Peirce's triadic sign relation

Considering determination in this way seems to reduce the interpretant to an automatic response to a sole input, which is not very plausible. By hinting at the fact that a normal interpretant will be generated "after sufficient development of thought", Peirce makes clear that any interpretant is also determined by the respective circumstances of sign interpretation (i.e., cognitive capacities), the knowledge of an interpreting person, the possibilities of a culture, etc.

The fact that we indeed need a considerable amount of background knowledge for interpreting signs becomes evident when we consider the following: If we are right in saying that both meaning and sense of a sign depend first of all on *interpretation*, then what the sign represents, its object, can only be described by looking at what is represented in the interpretant. Whereas the distinction between object and interpretant is necessary to mark *what* is represented *for whom* or to signify the intended reference of a sign in contrast to understanding this reference, there is no distinction between object and interpretant with regard to the respective *content* of this "what". The content of the respective objects exists only in so far as it is represented in a certain interpretant.

One therefore has to be careful in applying the triadic sign relation to the interpretation of communication, cognition, or learning: What is visible from the outside is only the sign. The sign—a word, statement, gesture, or diagram—is the only thing we observe. The interpretant may be visible in so far as it has a physical reaction to a sign. The interpreted meaning of a sign, however, its understanding as generated in one's mind, and the question of what a sign signifies for a certain person or group of persons, is never directly observable. Only if this "internal" interpretant is transformed in a further sign can we formulate hypotheses about the respective content of the object/interpretant. The only thing that can be said from the very beginning is that the content hypothetically ascribed to the interpretant must be precisely the same ascribed to the object. An object can be grasped only in so far as it is represented by an interpretant.

From an epistemological perspective, this means that interpreting signs is crucial for cognitively accessing the objects of our world; "a sign ... enables one ... to know" (Peirce, MS 599: 31). However, if the content conveyed by a sign about its object depends on the interpretation of this sign, the decisive question is, "In which way are interpretants generated?"

A student who already knows the function of the sign π in geometry will interpret this sign, when written on the blackboard, in a different way than a student who knows this sign only as a Greek letter. Thus, it is different background knowledge that leads to different

[2] Our representation of Peirce's triadic sign relation—an alternative to the usual triangle —is justified based on our reading of Peirce, NEM IV 307 ff., CP 1.347, and SEM II 137.

interpretants of signs. From this perspective the relation between sign, object, and interpretant (Figure 1) is itself mediated by knowledge.

3. Knowledge, Forms of Knowledge, and Learning

With regard to the question of how knowledge mediates the interpretation of signs, we argue elsewhere for two theses and develop two terminological suggestions that we now use (Hoffmann; Roth 2004): Our theses indicate that (a) a wide range of different forms of knowledge that establish a kind of network is needed for explaining the possibilities of interpretation and problem solving and (b) interpreting signs can be modeled as a reflexive process in which considerations regarding the sign's meaning for a person, and considerations regarding the object the sign signifies according to her or his interpretation, are developed in mutual interdependence. Thus, we would say, based on earlier results, that "interpretation is a reflexive and constitutive process in which particular readings of signs and potential content are mutually adjusted until they are consistent" (Roth; Bowen 2003, p. 439; see also Roth; Bowen 1999).

Our terminological suggestions concern general definitions of knowledge and learning, and a distinction between different forms of knowledge whose main point has been a distinction between two modes in which these knowledge forms can exist. When a certain knowledge form is explicitly in focus, because it is problematic in some way, or has to be learned, we call it *focal knowledge*; when it is only implicitly relevant—as a precondition for understanding focal knowledge—we call it, following Peirce, *collateral knowledge*, that is, knowledge that "runs side by side" with focal knowledge. Peirce gave the example of understanding the proposition "Napoleon was a lethargic creature". To be surprised by the idea of seeing the great conqueror as "lethargic", one must already know who Napoleon was and what he accomplished. If we do not know about Napoleon, we can only guess it is the name of a lethargic person. We are surprised only when our collateral or prior knowledge about Napoleon contradicts the proposition.

Collateral knowledge is not only important for the understanding of sentences. It is obvious that we have to know what a circle is according to geometry before we can learn the meaning of π. More interesting is the consideration that in any single situation, a complex network of different knowledge forms has to be presupposed. These forms include embodied, practical, symbolic, propositional, argumentative, iconic, indexical, situational, situated, and knowledge about ourselves—each with its focal and collateral type (Hoffmann; Roth 2004).

If we take, for example, a scientist who tries to interpret an unfamiliar graph, we can say with regard to the collateral knowledge necessary in this situation that she must have practical knowledge to develop a strategy about what to do; she needs iconic knowledge to identify, for instance, the specific properties of a coordinate system, or to see structure in her data; she requires symbolic knowledge concerning the symbols and concepts used in the diagram; and she has to trust in her ability to cope with problems like those in question. To us, collateral knowledge is of greatest interest for explaining the possibility of learning.

In order to encompass all the forms of knowledge distinguished in the list above, we formulated a general definition of knowledge as the ability to perform habits. The concept of habit can be defined as signifying a general form of acting as it is triggered by a certain kind of situations. A habit is thus no particular event like an action. It is a specific, but general form to act. The advantage of this definition of knowledge is, on the one hand, that habits are indirectly observable by looking at the ways in which people act in certain situations and, on the other hand, that all the listed knowledge forms can be described in terms of more or less given, or missing abilities. For example, knowing that $2 + 2 = 4$ (propositional knowledge) means to be able to calculate this task correctly, having the embodied knowledge to distinguish colors means being able to do so, and so on. Based on this definition of knowledge we can define learning as any change of a habit.

This definition of learning is similar to Holzkamp's (1993) notion of "expansive learning". Changing habits implies to enlarge, or to improve, one's possibilities to act, think, and control what happens. Both definitions can be discussed as pragmatic approaches in the sense of Peirce, since they focus on the outcome and on the effect of learning (cf. Hoffmann, in press-a). The difference between the two notions might be that improving and enlarging possibilities implies a certain direction of learning, while habit change can also mean a decrease of action possibilities.

4. A Dialectical Model of Learning

Based on these preliminaries, a more abstract model of habit change in expansive learning can be developed. In the next section, we provide a concrete analysis as an exemplary application of this model. In this way, the richness of the proposed model becomes visible.

Taking the dialectic system as a framework, we now focus on the question of how an agent can develop an interpretation of a sign that is new for him, that is, that transcends the possibilities of his prior knowledge. Let us assume that our agent experiences a certain tension between what he knows and what he should know. A student in geometry, for example, observes others in the classroom handling the sign π in a way that sounds quite mathematical whereas he has no clue. Maybe, he is motivated in this way to draw on his abilities to look for a level of mediation at which he could integrate the culturally shared knowledge about π into his own knowledge network, thereby developing his abilities.

The collateral knowledge that is available to him for this task, thus forming a network of already given abilities, may be the following: (a) He knows that π is a symbol signifying an object which has to be defined mathematically; (b) he knows that the situation is a class about geometry, more precisely, about circles; (c) he knows from earlier classes about how to define and how to construct a circle; (d) he has more or less a vague idea about the concepts of radius, diameter, circumference, ratio, and square; (e) he has the ability to transform his vague ideas about the problem in focus into a more determinate question; (f) and he may have the self-confidence in doing so—in spite of the risk that his question may sound stupid to others. There are of course several more abilities relevant in this situation: being able to listen, observing, focusing attention, identifying definitions in contrast to arguments, or distinguishing a figure drawn on the blackboard from its general mathematical properties. Which forms of knowledge are relevant in a certain situation depends on how this situation is interpreted respectively within a certain research context. That means, within the analysis we are about to perform, we must uncover all the levels and forms of knowledge necessary to explain convincingly that a particular learning process is being enacted in this concrete case—this will require the analytic technique of zooming, whereby events of different spatial and temporal range are brought into focus (Roth 2001).

There are certainly many possible learning trajectories, but if we look for the most important conditions for performing them, there are two that we will highlight here: Firstly, it is crucial to find a *representation* of what we already know and what might be relevant for the problem in question or for the interpretation of a sign. The point is that in order to realize cultural possibilities that depend on a network of prior knowledge, we have to *work* with this knowledge, that is, we have to reflect on it. Doing this, however, presupposes a representation of this knowledge and only in this way can it become an object of our reflection. To describe this process of learning by representing knowledge, we use a semiotic concept: Peirce's concept of "diagrammatic reasoning" (Hoffmann; Roth 2004). Diagrammatic reasoning is a three-step activity that contains *constructing* representations, *experimenting* with them, and *observing* the results.[3] The idea is that by representing a problem in a diagram, ephemeral and fleeting thoughts become concretized in a fixed and visible format, so that self-control of thought and experimenting with our own knowledge and cognitive means becomes possible. "The diagram becomes the something (non-ego) that stands up against our consciousness"; "reasoning unfolds when we inhibit the active side of our consciousness and allow things to act on us" (Hull 1994, 282, 287).

Secondly, another condition for learning is the practical (strategic) knowledge of how to find out more information in a problematic situation. We need to know how to ask people, what to read, and how to connect all this information to what we already know. Of particular importance here is the ability to perform hypotheses. It is this ability that Peirce first discussed under the heading of "abductive reasoning". Abduction, for him, "is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea" (CP 5.171; cf. Hoffmann 1999; Roth 2003, chap. 3).

[3] Peirce, NEM IV 47 f. Cf. Stjernfelt 2000, Hoffmann 2003. Attempts to apply this concept to problems of mathematics education are formulated by Dörfler, in print, and by Bakker; Hoffmann, in print.

Figure 2. Samantha has just captured a lizard in the wild. Clearly visible is one of the front leg, the length of which, together with leg length, was thought to be correlated with spring speed, itself thought to be a correlate of survival rates.

The essential point of our dialectic model is that within a process of connecting all the elements relevant in a certain situation, they may, in turn, become objects of modification. It is a complex, reflexive process in which many considerations are related in order to solve a problem: Meanings of signs, potential operations which are permitted by certain signs, but not by others, all this has to be developed, or to be questioned, on a level of mediation until we achieve something which seems to be a new consistent network of knowledge.

5. An Exemplary Analysis

5.1 Ethnographic background

The present case study draws on data collected by the second author (WMR) over a three-year period among ecologists, which involved serving as a research assistant in an ecological field research camp in a mountainous area of British Columbia. There, he assisted in particular one research group by hunting and capturing lizards, skinks, rubber boas, and garter snakes and conducting measurements in the field and field laboratory. The main informant was a doctoral student (pseudonym Samantha) in her fourth of six years doing independent research; her work was partially funded by different organizations interested in the topic of her research. The database is extensive, consisting of observations recorded in fieldnotes, photographs, audiotaped conversations in and about fieldwork, videotapes of data collection in the field and field laboratory work, and formal interviews conducted during the winter months, which Samantha spent on her home campus. The database further includes a complete set of Samantha's laboratory notes from 1996–97, her dissertation, and the articles and reports published to date based on this work. There are also videotapes of poster sessions at local and national conferences, videotaped talks about her work in university seminars, and all slides and notes used for these diverse presentations. Among her peers (graduate students and professors), Samantha stood out in her ability to understand mathematical representations and do statistics. Her undergraduate background was in mathematical biology, and she repeatedly taught a fourth-year undergraduate course in statistics. She extensively used multivariate statistics and was known in the department as a "statistical wizard".

Figure 3. In captivity, Samantha generated a variety of measurements from each animal. Here, Samantha "races" a lizard by chasing them down a 1.5-meter long, narrow box and into a black bag ("cover"). The researcher uses a stopwatch for measuring total time taken for the given distance.

The purpose of Samantha's research was (a) to describe the natural history of a particular lizard species (e.g. body size, habitat preferences, movement patterns); (b) to determine basic life history traits (e.g. life span, survivorship, and litter size); and (c) to identify the fecundity and survival costs of reproduction. Samantha conducted her research at the northern-most boundary of the area where the particular lizard (Figure 2) was believed to occur. Although southern relatives of the species had been researched by others on occasion before, very little was known about this species. Samantha drew on research on other reptilians for ideas about how to capture life history information, but also thought that there were particular adaptations that her subspecies must have undergone to be able to live so far north. Finding out how to represent the lizard and its environment was central to her work. Her task, therefore, was one of bringing order to this lizard species and the lizards' lifeworld without knowing beforehand what that order might be. She therefore was in a situation very similar to our model student facing the sign π about which he has no clue; in contrast to the student, everybody else in the ecology community is in the same situation. Samantha's problem consisted in having to create new cultural knowledge, which involved becoming intimately familiar with the phenomenal world and structuring it in common (e.g., temperature) or new and not so common ways (sprint speed, body measures); these structural aspects of the setting, that is new iconic knowledge, then became starting point for her statistical analyses. The common ways of structuring are constituted by existing cultural knowledge, whereas her statistical competencies were constituted primarily by given practical knowledge. The new ways of structuring involve processes of structuring the environment in such a way that it reveals signs and representations that she can subsequently interpret. Her work therefore may even involve a form of radical invention of code, as Samantha delved "directly into the as yet unshaped perceptual continuum" (Eco 1976, p. 254) that subsequently would constitute the novelty in her discipline.

The collection of data began with the capture of an animal. After capture, each animal was first entered a

Figure 4. An alternative method for measuring a lizard's body length.

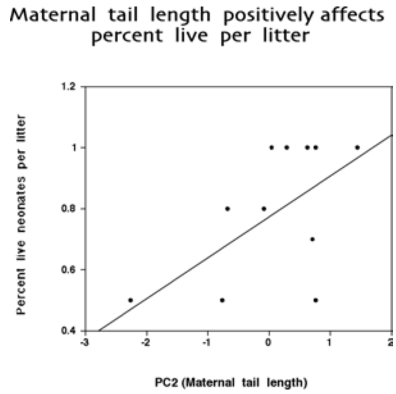

Figure 5. One of the graphs constructed after one season in the field shows a positive correlation between maternal lizard tail length and the survival rates of their offspring (these lizards are live-bearing [viviparous]).

holding facility (sock) and brought to the field laboratory at the end of the day. Samantha took measurements from all animals such as sprint speed (Figure 3) and a variety of body measures, including front and back leg length, snout-vent length, and tail length. The leg lengths were measured directly by holding a ruler to the "shoulder" of the animal, stretching the leg along the ruler, and reading the measure; as can be inferred from Figure 2, the legs can be measured directly because the animal can be pinned between thumb and index. The body measures took a more complicated procedure, in part because lizards are able to tail autotomy, that is, let go of part or all of their tails. Samantha therefore measured the body lengths by placing the lizards in a clear plastic box and pinning them against the bottom using a piece of foam. Using a felt marker, a trace along the mid-line of the animal was produced on the box, as well as markers for snout, vent, and tail end (Figure 4).

5.2 Creating variables

Because there are no grand theories in ecology, much of the research is conducted bottom up, beginning with some core concepts and intuitions and pursuing them through fieldwork. These intuitions constitute a part of the collateral knowledge that both enables interpretation and allows signs to emerge from the setting before their interpretation can begin. In the course of fieldwork, ecologists gain substantial tacit knowledge about their species and its habitat (Roth, in press; Roth; Bowen 1999, 2001). For example, Samantha explained the emergence of some of the ideas about sprint speed during a seminar at her home institution:

"Typical scenario is like this. I flip a rock. I see a lizard. Lizard sees me. There's a brief moment when everybody is frozen in time ((laughter)) and then lizard makes a run for it and I run after it. ((A lot of laughter.)) ((Projects picture of four lizards in a plastic bin.)) There's something that became abundantly obvious early on, it's that not all lizards are created equal. There are obvious size difference in the lizards, but something I started to suspect is that there are some, there are differences in sprinting ability, 'So how could you guys get away from me'. And I started wondering, 'What is driving these differences?' There are a variety of reasons why it might be important for a lizard to get away. The first and most obvious one is to escape voracious predators such as myself or ((Picture of bear in field)) Mister Bear".

Samantha thought that sprint speed was a survival strategy, for she found animals not only underneath rocks but also basking on rocks some distance away from a cover object. That is, having seen lizards often escape her capture attempts, she thought about sprint speed. She then used an ecological theory to interpret sprint speed with respect to the lizards as she had observed them. Thus, sprint speed would provide an advantage in the case of a predator approaching. Sprint speed also should be of advantage when it comes to feeding, for the faster an animal can sprint, the easier it is to catch its live prey. In support of this hypothesis, she not only found many crickets and grasshoppers in the area, but also the fact that her captive females actually fed on them. Samantha also thought that sprint speed would come in handy during mating. During mating, males have to capture and hold the female (by the head) or the latter would run away. Faster males therefore would be able to mate more frequently than slower males, some of which may not mate at all. Again, sprint speed of male lizards would be a survival strategy for the species. That is, linking sprint speed to existing knowledge allowed Samantha to generate hypotheses, which she could then link into and support by means of other knowledge elements. These field data subsequently became signs that she further elaborated through interpretation. Her prior knowledge network was the necessary fundament of all this.

5.3 Constructing lizard traits

After one field season, Samantha returned to her home institution where she 'cleaned up' the computerized database and began initial statistical analyses. Although she only had a small number of animals captured, especially of the kind that she was most interested in, gravid females ($N = 12$), she began statistical analyses in part to test existing hypotheses (often based on, as she said, "gut feeling") and to generate new hypotheses. After having conducted a series of statistical tests, Samantha felt that most of her intuitions have been born out. That is, what previously existed more or less as indexical, situational, and situated knowledge now was expressed in the form of formal propositions and relations between sign complexes. For example, being interested in reproduction and reproductive costs, she had found that the tail lengths of female lizards correlated positively with the percentage of offspring born alive (Figure 5).

Figure 6. Typical box in which Samantha kept the lizards one animal per box because of their territorial nature. In comparison to the wild, there was much less reason to sprint distances.

She found this correlation sensible given that the lizard tail contains a lot of fat, and therefore is a measure of the amount of internal energy available to the animal. Longer tails therefore correspond to more energy available as reproductive energy, which would explain the positive correlation with the percentage of live-born offspring. That is, her explicit knowledge made sense because she already had the collateral knowledge that allowed knowledge to become explicit. Her interpretation of such sign complexes as the graphs she generated is rooted in forms of knowledge that typically resists explicit formulation.

The one aspect, however, that did not bear out after the first season concerned the intuitions related to sprint speed, for example, the differences between males and females, especially between males and non-gravid females on the one hand, and the gravid females on the other hand. It seemed intuitive that gravid females move more slowly because of the added weight related to pregnancy and the encumbrances of an enlarged belly area. This constituted a contradiction similar to the case of hearing that Napoleon was lethargic. This contradiction motivated Samantha to seek a deeper understanding. Here we have a situation where a person is stimulated to seek further signs that assist in the interpretation of the problem, which is possible only when the person has self-confidence and knowledge about herself that she has the ability to find out.

The intuited differences and correlations with size of different body parts (leg length, tail length) did not turn out, to Samantha's great surprise. She was starting to wonder, attempting to come up with possible reasons, factors that had influenced and therefore contaminated her data. That is, facing the contradiction, she sought for a development of her initial knowledge network, turning it into a dialectic system that would lead to an interpretation that made sense on intuitive grounds. Samantha then had a hunch, which emerged from remembering the events on one particular day, when she and a helper had caught many animals and did not immediately get to collect the data in the laboratory.

"I mean, you know you have, it's probably only a few cases where you know how an animal responded when you first caught it and within a day of capture and then you see its behavior. So particularly, the particular problem was for the gravid females, because when [a helper] came out. We got on a run, where we were just searching and bringing animals back. And we were doing really well, catching gravid females. And so

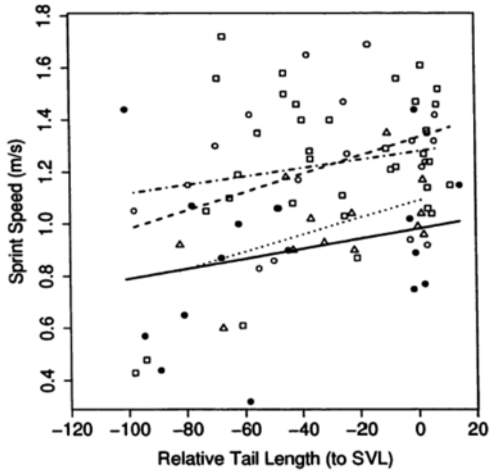

Figure 7. In Fig. 3.5 of her thesis, Samantha correlated the lizards' relative tail lengths and their sprint speed, after days in captivity had been controlled for. If tail length is used as a measure of internal energy, the positive correlations also make intuitive sense.

I just didn't take time off searching because we seem to be in a real. Because we seemed to be getting animals and, and so they were held for a bit. So those guys were held so I have this sort of sense of what they're like when I first captured them and then...".

When Samantha did not have time, she waited with taking measurements. She took her time especially with the gravid females, which she kept in boxes filled with rocks that simulated natural environments (Figure 6). She was beginning to sense that captivity might have to do with a change in sprint speed.

"I discovered after I did the data, when I did the data analysis last year I just, I had a feeling that the time that I held them in captivity was having an effect so I included it in the analysis and in fact it was. So, when I did the analysis I statistically controlled for it but I think the better thing to do would be to actually control for it in the methodology. So everybody gets raced immediately like the next day after capture or the same day basically. I think that would be a better way to control for it than to control for it 'cause there is, there is individual variation in how long they respond to being held in captivity so just using numbers of days I don't think it's good. So, my hope is that when I'll control for all the things that I think are there will be there but… Yeah, I'm not sure."

Originally Samantha wanted to bring females into the lab and then race them every two weeks of their pregnancy in order to study changes in sprint speed with pregnancy. But after she had discovered the change in sprint speed, she knew she had to change her approach. That is, based on the special case, she hypothesized a general rule, "lizards get slower with days in captivity". She then used her database including all concrete cases to verify the hypothesis and found it supported. This process is what we have mentioned above as abduction, a synthetic inference "where we find some very curious circumstances, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition" (Peirce CP 2.624).

Samantha also began to think that the time of the year would make differences in sprint speed. Again, this idea was based on a hunch having spent entire seasons (April–September) in the field, attempting to catch lizards during different climatic situations and also at different stage in their annual cycles. These hunches are, so to speak, grounded in different expression channels (cf. Eco 1976: 176), here thermal, kinesic (e.g., slopes of lizard capture areas), and positional gradients in the course of the year. For example, females caught late in the year, for example, in August, would be very "chubby" because of their advanced pregnancy, which should have an effect on their spring speed. The effect was not easily controlled, as she could never know how many animals she would be bringing back to the laboratory. When she had a number of animals, she could race them immediately the same or next day. But if she was getting one animal for three days in a row, she could not make valid comparisons.

In her dissertation, Samantha ultimately reported her results in symbolic form that has little resemblance with anything that she had done and produced during her field season. This very different system of representation was, in part, the outcome of a radical shift from focusing on particulars and individual cases (horizontal rows in Samantha's spreadsheet) to focusing on generalizations across individuals (vertical columns in the spreadsheet). With respect to sprint speed and tail length for different types of animals, Samantha reported:

"Sprint Speed
The mean sprint speed was 1.13 m/s ($SE = 0.03$, $n = 84$). Sprint speed increased with relative tail length ($F_{1,79} = 8.54$, $P = 0.01$; Fig. 3.5). Juveniles were not significantly faster than adult gravid females ($t = 0.59$, $P = 0.56$; Fig. 3.5), but both adult males and adult non-gravid females were significantly faster than adult gravid females ($t = 3.65$, $P = 0.001$; $t = 3.32$, $P < 0.001$; Fig. 3.5) ".

For the reader, "Fig. 3.5" (here Figure 7) was provided as an additional, related sign in support of the interpretive process. In fact, the text and statistics, on the one hand, and the graph, on the other, are mutual interpretants the combination of which serves to constrain any reading (Bastide 1990).

Here, then, we have a process of double mapping (e.g., Eco 1976), in which a perceptual model (thermal, kinesic, positional gradients) was mapped onto a semantic model (intuitions about the lizard lifeworld), and onto an expression, the formal statistics and graphical representation. It is a process of invention as code making, which begins when "relevant elements are picked up in an unshaped perceptual field and organized in order to build a percept" (Eco 1976, p. 250). In subsequent processes, percepts are mapped onto semantic representations, which constitute simplifications. The semantic representations are then coded arbitrarily into an independent set of expressive units—the intuitions about sprint speed and tail length are dissimilar to the graph in which the correlations are expressed (Figure 7).

5.4 Discussion

In this case study, we describe learning as the development of a knowledge network that marks the starting point of Samantha's research project. Existing forms of knowledge—propositional knowledge as provided by prior research, intuitions that were generated by experiences in the field, iconic knowledge about possible structures of data, practical knowledge about research strategies, and so on—form a network in which new knowledge has to be integrated.

Samantha's research was motivated by the goal of formulating a precise representation of her initial vague intuitions with regard to the relation between differences of sprint speed and various groups of lizards. These intuitions were part of a consistent belief system, but, at the end, they could not be confirmed by the experiments of her research. Thus, tensions arose between different elements of her knowledge network, motivating a search for a level of mediation at which these tensions could be overcome by a development of the prior knowledge network.

Within the framework of this developing dialectic system, signs, and the interpretations of these signs, were crucial. To formulate her problem more precisely, Samantha had, first of all, to make explicit those parts of her knowledge network that might be relevant. That is, she focused on certain elements of her knowledge, and represented them in signs. From a semiotic perspective, any sign can be interpreted in different ways, that is, the multitude of interpretants is virtually infinite (Eco 1984). What Samantha did in this situation can be described as a processes of abductive reasoning in which she tried to hypothesize a new general rule that permits an interpretation of her representations in a way that eventually could be a basis for a new consistent knowledge network.

Knowledge networks can be described as "rhizomes" (Deleuze; Guattari 1976), in which every point can and sometimes must be connected to every other point. A rhizome may be broken off at some point and reconnected at another. This metaphor allows us to understand Samantha's accomplishments, whereby knowledge forms initially unconnected to her research questions were connected in new and innovative ways, especially as a consequence of the contradictions or tensions within the relation R between her different knowledge elements (a_1, … a_n). But crucial for working in a reflective way with ones own rhizomic knowledge network is that we have to *focus* on it, that is, we must represent it in signs that we can interpret.

Samantha's motivation and abilities (Agt[Mot, Abil]) were essential for starting the dialectical process. She supposed general rules and new interpretations of her signs from nothing more than intuitions—based on considerable collateral knowledge—and then went on to confirm (or disconfirm) these rules and interpretations by testing them in the domain of cases. Elsewhere, we analyzed the relation between the tremendous hardship and physical discipline involved in ecological fieldwork and the ecological knowledge developed by researchers (Roth; Bowen 2001). Samantha's mental discipline was linked to her physical discipline and the associated self-confidence and knowledge that she could complete the project over its three-year span. Our case study therefore shows that the dialectical and learning systems specified in our initial definitions (1) and (2), respectively, requires a level of mediation LM between motivation and abilities, on the one hand, and the set of opposing elements R, on the other hand, that transcend the cognitive realm. It is for this and similar reason that some scholars have begun to talk about the embodied nature of knowing (e.g., Varela; Thompson; Rosch 1991). Therefore, the rhizomic network of salient relations with respect to the mathematical knowledge of an individual ultimately "fades into the indeterminate background of mostly unarticulated and inarticulate bodily experiences in the social and material world" (Roth 2004, p. 87).

6. Coda

An important aspect of our research is to mediate philosophical thinking with our practical interest to improve our understanding of live long learning processes. Empirical studies that try to identify forms of knowledge most relevant for crossing the boundaries between formal education and workplace need a theoretical language that philosophical considerations can provide. For this purpose, we develop in this paper the concept of a dialectical system to describe learning—learning that is based on the experience of contradictions, or tensions between different parts of given knowledge networks—in a most general way. And we show in which way theoretical tools developed in the context of semiotics can be used to describe the epistemological problem how we can have access to our world of objects (namely mediated by signs and representations), and how a development of knowledge networks is possible. For this, we stressed the relevance of (a) representing given knowledge in *diagrams,* and (b) generating new hypotheses and new interpretations of those representations by *abduction*. Semiotics (the theory of signs, or representations) not only provides an adequate framework for understanding knowledge development, it also offers specific tools (like diagrammatic reasoning and abduction) to understand these processes.

To apply our results within the concrete practice of education, we suggest the development of specific learning environments in which diagrammatic and abductive reasoning play a central role. Both these abilities seem to be of eminent importance if we are looking for knowledge crucial for moving between the highly differentiated knowledge settings that humans have to manage daily.

Looking back to the role of mathematics in the processes of knowledge development, one can distinguish two different aspects: On the one hand, our example from biology shows clearly that mathematics—in this case: statistics—offers a repertoire of representation systems that play an essential role for diagrammatic reasoning and abductive hypotheses generation in other sciences. On the other hand, we have to realize that learning mathematics—as our running example of understanding π makes evident—presupposes a lot of knowledge that is clearly not based in formal mathematics: an intuitive understanding what a circle is, the ability to relate class discussions to prior experiences, the practical skills of solving problems, the ability and self confidence to ask questions, and so on.

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