

A Modified Entropy Generation Number for Heat Exchangers

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This paper demonstrates the difference between the entropy generation number method proposed by Bejan and the method of entropy generation per unit amount of heat transferred in analyzing the thermodynamic performance of heat exchangers, points out the reason for leading to the above difference. A modified entropy generation number for evaluating the irreversibility of heat exchangers is proposed which is in consistent with the entropy generation per unit amount of heat transferred in entropy generation analysis. The entropy generated by friction is also investigated. Results show that when the entropy generated by friction in heat exchangers is taken into account, there is a minimum total entropy generation number while the NTU and the ratio of heat capacity rates vary. The existence of this minimum is the prerequisite of heat exchanger optimization.

Keywords: entropy generation, heat exchanger.

INTRODUCTION

Heat exchangers are widely used in various industrial branches. In the interests of effective use of energy, heat exchanger analysis from the viewpoint of the second law of thermodynamics has attracted some researchers' attention. Of course, it would be nice to have a reversible heat exchanger, but since heat transfer process is inherently irreversible, the only thing we can do in engineering applications is to reduce the irreversibility of the heat transfer process. Bejan (1977) is the first one introducing the concept of "entropy generation" to the design of heat exchangers. This concept provides a common conceptual basis for thermal optimization of heat exchangers and the entropy generation number method of heat exchanger optimization proposed by him was the subject of several recent studies by Bejan (1978, 1980, 1982, 1988, 1994), Baclie and Selulic (1978), Sarangi and Achowdhurg (1982), Huang (1984), Sekulic and Balic (1984), de Costa and Saboya (1986), Sekulic and Nerman (1986), Sekulic (1985-1986, 1986), Lanzhou Petroleum Machinery Institute (1985), Sekulic (1990), Guo (1992) and Demirel (1995).

COMMENTS ON BEJAN'S ENTROPY **GENERATION** NUMBER

In order to evaluate the irreversibility loss in heat exchangers, Bejan (1982) redefined the entropy generation number as

$$
Ns = S_{gen}/(mc_p)_{\min} \tag{1}
$$

and he indicated that the smaller the entropy generation number, the better the performance of the heat exchanger would be.

In order to clear up the physical interpretation of Bejan's entropy generation number multiplying both the numerator and denominator by ΔT_{min} (the temperature rise or drop of the fluid with the smaller heat capacity rate) yields

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$$
Ns = \frac{\dot{S}_{gen} \Delta T_c}{(\dot{m}c_p)\Delta T_c} = \frac{\dot{S}_{gen} \Delta T_c}{Q} \tag{1a}
$$

Obviously, the physical interpretation of *Ns* is the dimensionless overall entropy generation rate, or the entropy generated per unit amount of heat transferred in the heat exchanger multiplied by the temperature change of the fluid with smaller heat capacity rate. The overall entropy generation rate can not be used to evaluate the performance of a heat exchanger. Although the entropy generated per unit amount of heat transferred in the heat exchanger is the criteria, multiplying the ΔT_{min} , which becomes larger when the area of the heat transfer surface of the heat exchanger increases, makes the *Ns* dimensionless, might lead to some ambiguities as illustrated in the following exampies.

Example 1: Entropy generation due to heat transfer in condensers or evaporators.

Take a condenser as an illustrative example. In a condenser the fluid with larger heat capacity rate is the hot fluid, i.e. the condensing vapor, its temperature keeps constant through the process. Then the entropy generation rate of the whole condenser can be expressed as:

$$
\dot{S}_{gen} = -(m h_{fg})_h + (m c_p) \ln(T_{co}/T_{ho}) \tag{2}
$$

where entropy changes associated with the frictional pressure drops have not been included which will be described shortly. The outlet temperature of the cold fluid is

$$
T_{co} = T_{ci} + \Delta T_0 (1 - e^{-NTU}) \tag{3}
$$

in which $\Delta T_0 = T_{hi} - T_{ci}$ is the maximum possible temperature difference. Combining Eqs.(1), (2) and (3), noticing $(\dot{m}h_{fa})_h/(\dot{m}c_p)_c = \Delta T_c$, and setting $\theta =$ T_{in}/T_{ci} , one obtains the entropy generation number of the condenser

$$
Ns = \ln[1 + (T_{hi}/T_{ci} - 1)(1 - e^{-NTu})]
$$

$$
-(1 - T_{ci}/T_{hi})(1 - e^{-NTu})
$$
(4)

Curve 1 in Fig.1 shows the relationship between *Ns* and *NTU.*

On the other hand, the problem might be analyzed directly on the basis of unit amount of heat transferred as follows. The total amount of heat transferred in the condenser

$$
Q = (\dot{m}c_p)_c \Delta T_c \tag{5}
$$

Then, from Eqs.(2) and (5), the entropy generation per unit amount of heat transferred in the condenser is

$$
\frac{\dot{S}_{gen}}{Q} = -\frac{1}{\Delta T_0 (1 - e^{-NTU})} \cdot \ln \left[1 + (T_h/T_{ci} - 1)(1 - e^{-NTU}) \right] - \frac{1}{T_h} \quad (6)
$$

and shown diagrammatically in Fig.1 (curve 2).

Similar results were obtained for evaporator. It is surprising to see that variations of *Ns* are conceptually opposite to those of S_{gen}/Q . As a matter of fact, in a condenser or evaporator, the mean temperature difference between the hot and cold streams becomes smaller as the area of heat transfer surface increases, thus the irreversibility of the heat exchanger as a whole should decrease. Therefore, it seems that Eq.(6) is more reasonable for evaluating the irreversibility of heat exchangers. The reason why the *Ns* increases with increasing *NTU* is the multiplication of ΔT_{min} , since ΔT_{min} increases as the area of heat transfer surface increases, so the larger the *NTU,'the* higher the *Ns* might be.

Fig.1 Effects of *NTU* on Ns , Ns' and \dot{S}_{gen}/Q for condenser; $(1 \ Ns; 2 \ Ns', 3 \ \dot{S}_{gen}/Q)$

Example 2: Entropy generation due to heat transfer in heat exchangers without phase change.

Take the whole heat exchanger as the control volume. The entropy generation rate only due to heat transfer can be expressed as

$$
\dot{S}_{gen} = (\dot{m}c_p)_h \ln \frac{T_{ho}}{T_{hi}} + (\dot{m}c_p)_c \ln \frac{T_{co}}{T_{ci}} \tag{7}
$$

The outlet temperatures of the hot and cold fluids in a heat exchanger can be calculated according to the equations given by Xu et al. (1995).

$$
T_{ho} = T_{hi} + WE\Delta T_0 \tag{8}
$$

$$
T_{co} = T_{ci} + E\Delta T_0 \tag{9}
$$

where the constant E for counterflow heat exchanger

$$
E = \begin{cases} \frac{1 - e^{(1 - W)NTU}}{W - e^{(1 - W)NTU}}, & W \neq 1 \\ \frac{NTU}{1 + NTU}, & W = 1 \end{cases}
$$
(10)

--(I+W)NTU

for parallel-flow heat exchanger

$$
E = \frac{1 - e^{-(1 + W)ATU}}{1 + W}
$$
 (11)

in which $W = (\dot{m}c_p)_c/(\dot{m}c_p)_h, NTU = UA/(\dot{m}c_p)_c$.

Substituting Eqs. $(7)-(9)$ into Eq. (1) , one obtains the entropy generation number

$$
Ns = \frac{1}{W} \ln \left[1 - WE \left(1 - \frac{1}{\theta} \right) \right]
$$

+ $\ln[1 + E(\theta - 1)]$ (12)

The curves 1 and 2 in Fig.2 show the relationships between *Ns* and *NTU* for counterflow and parallelflow heat exchangers.

Fig.2 Dependence of Ns and \dot{S}_{gen}/Q on *NTU* for heat exchangers $(1,3$ counterflow Ns , S_{gen}/Q ; 2,4 parallel-flow $Ns; \dot{S}_{gen}/Q$)

Now, we analyze directly the entropy generation per unit amount of heat transferred in the heat exchanger. The total amount of heat transferred in the heat exchanger is

$$
Q = (\dot{m}c_p)_h \Delta T_h = (\dot{m}c_p)_c \Delta T_c \tag{13}
$$

From Eqs. (7) , (8) , (9) and (13) , the entropy generation rate per unit amount of heat transfer

(8)
$$
\frac{S_{gen}}{Q} = \frac{1}{\Delta T_0 W E} \ln \left[1 - W E \left(1 - \frac{1}{\theta} \right) \right] + \frac{1}{\Delta T_0 E} \ln \left[1 + E(\theta - 1) \right]
$$
(14)

The calculated results of Eqs.(13) and (14) for counterflow and parallel- flow heat exchangers are shown by curves 3 and 4 in Fig.2.

Again, comparing the curves 1, 3 and 2, 4 in Fig.2, we found that the physical interpretation of the entropy generation number proposed by Bejan was somewhat ambiguous. In a parallel-flow heat exchanger the temperature difference between the hot and cold streams becomes smaller and smaller. Then

MODIFIED **NUMBER ENTROPY GENERATION**

In order to clarify the above-mentioned ambiguity of the original entropy generation number suggested by Bejan as defined by Eq.(1), we recommend that the entropy generation number might be modified as

$$
Ns' = \Delta T_0 \dot{S}_{gen}/Q \tag{15}
$$

Since the maximum possible temperature difference ΔT_0 is always given as a definite value in designing heat exchangers, the physical interpretation of the modified entropy generation number is very clear. It is a dimensionless criterion which is directly proportional to the entropy generated per unit amount of heat transferred in the heat exchanger and thus elimihates the ambiguity of the original entropy generation number.

Taking the pressure drop into account, one may obtain the entropy generation in a single phase heat exchanger as follows

$$
\dot{S}_{gen} = (\dot{m}c_p)_h \ln \frac{T_{ho}}{T_{hi}} + (\dot{m}c_p)_c \ln \frac{T_{co}}{T_{ci}}
$$

$$
-\dot{m}_h \int_{P_{h,A=0}}^{P_{h,A=A}} \left(\frac{\partial v}{\partial T}\right)_{P,h} dp_h
$$

$$
-\dot{m}_c \int_{P_{c,A=0}}^{P_{c,A=A}} \left(\frac{\partial v}{\partial T}\right)_{P,c} dp_c \qquad (16)
$$

Then substitute Eqs. (13) and (16) into Eq. (15) , the modified entropy generation number then can be calculated as

$$
Ns' = \frac{\Delta T_0 \dot{S}_{gen}}{Q} = \frac{\Delta T_0}{\Delta T_h} \ln \frac{T_{ho}}{T_{hi}} + \frac{\Delta T_0}{\Delta T_c} \ln \frac{T_{co}}{T_{ci}}
$$

$$
-\frac{\Delta T_0}{c_{ph} \Delta T_h} \int_{P_{h,A=0}}^{P_{h,A=A}} \left(\frac{\partial v}{\partial T}\right)_{P,h} dp_h
$$

$$
-\frac{\Delta T_0}{c_{pc} \Delta T_c} \int_{P_{c,A=0}}^{P_{c,A=A}} \left(\frac{\partial v}{\partial T}\right)_{P,c} dp_c \qquad (17)
$$

The sum of the first two terms on the right side of $Eq.(17)$ is the entropy generated by heat transfer and denoted as $Ns'_{\Delta T}$. Evidently, the first term and

second term represent the entropy generation by the hot fluid and cold fluid respectively. The sum of the last two terms is the entropy generated by friction and denoted as $Ns'_{\Delta P}$ which includes the effects of hot fluid and cold fluid.

Entropy Generated by Heat Transfer

The modified entropy generation number taking only the effect of heat transfer into account can be calculated by substitution of Eq. (14) into Eq. (15)

$$
Ns'_{\Delta T} = \frac{1}{WE} \ln \left[1 - WE \left(1 - \frac{1}{\theta} \right) \right]
$$

$$
+ \frac{1}{E} \ln [1 + E(\theta - 1)] \tag{18}
$$

The dependence of $Ns'_{\Delta T}$ on *NTU* is illustrated by the curve 1 in Fig.3 and the curve 1 in Fig.4.

Fig.3 Relationships between *Ns'* and *NTU* for counterflow heat exchanger $(1 \ Ns'_{\Delta T}; 2 \ Ns'_{\Delta p}; 3 \ Ns')$

Fig.4 Relationships between *Ns'* **and** *NTU* **for** parallel-flow heat exchanger $(1 \ Ns'_{\Delta T}; 2 \ Ns'_{\Delta p}; 3 \ Ns')$

For condensers and evaporators, the effects of *NTU* on $Ns'_{\Delta T}$ can be expressed as

$$
Ns' = \frac{1}{(1 - e^{-NTU})} \ln[1 + (\theta - 1)
$$

$$
\cdot (1 - e^{-NTU})] - (1 - \frac{1}{2}) \tag{19}
$$

$$
Ns' = \frac{1}{(1 - e^{-NTU})} \ln\left[1 - \left(1 - \frac{1}{\theta}\right) \right]
$$

$$
\cdot \left(1 - e^{-NTU}\right)\right] + \left(\theta - 1\right) \tag{20}
$$

The curve 3 in Fig.1 shows this relationship for condenser. Obviously, it is in consistent with curve 2.

Entropy Generated by Friction

When liquids are used as the working media *Ni* (1985) has recommended $\left(\frac{\partial v}{\partial T}\right)_P \approx \frac{v}{T} = \frac{1}{\rho T}$, and pointed out that the difference of the simplification is less than 1% for water. It is reasonable to assume that the density of the liquid remains unchanged, i.e. $\rho = const$ and $dp/d(NTU) = const.$ Using Eqs.(8) and (9) and integrating them, we may obtain the entropy generation (due to friction only) numbers: for counterflow heat exchangers $W \neq 1$

$$
Ns'_{\Delta P_h} = \frac{(-dp_h/dNTU)\theta}{WE\rho_h c_{ph}T_{hi}[\theta - W(\theta - 1)]}
$$

$$
\cdot [(\theta - 1)\ln\frac{1 - WE + WE/\theta}{1 - WE}
$$

$$
+\frac{1}{W - 1}\ln\left|\frac{1 - E}{1 - WE}\right|]
$$
(21)

$$
Ns'_{\Delta P_c} = \frac{-dp_c/dNTU}{E\rho_c c_{pc}T_{ci}(\theta + W - 1)}
$$

$$
\cdot \left[\frac{\theta - 1}{\theta}\ln\frac{1 + E\theta - E}{1 - E}\right]
$$

$$
+\frac{W}{W - 1}\ln\left|\frac{1 - E}{1 - WE}\right|\right]
$$
(22)

for counterflow heat exchangers $W = 1$

$$
Ns'_{\Delta P_h} = \frac{(-dp_h/dNTU)\theta}{E\rho_h c_{ph} T_{hi}}
$$

$$
\cdot \left[\frac{\theta E}{1 - E} + (\theta - 1) \ln \frac{(1 - E)\theta}{\theta - E\theta + E} \right] (23)
$$

$$
Ns'_{\Delta P_e} = \frac{(-dp_c/dNTU)}{E\rho_c c_{pc} T_{ci} \theta^2}
$$

$$
\left[\frac{\theta E}{1-E}+(\theta-1)\ln\frac{(1-E)}{1+E\theta-E}\right] (24)
$$

for paraUel-flow heat exchangers

$$
Ns'_{\Delta P_h} = \frac{(-dp_h/dNTU)\theta}{WE\rho_h c_{ph}T_{hi}(\theta - 1)}
$$

$$
\cdot \ln \frac{1 - WE + WE/\theta}{1 - WE - E}
$$
(25)

$$
Ns'_{\Delta P_c} = \frac{(-dp_c/dNTU)}{E\rho_c c_{pc} T_{ci}(\theta - 1)}
$$

$$
\cdot \ln \frac{1 + WE - E}{1 - WE - E}
$$
(26)

For gaseous working media, using the equation of state for perfect gas, the modified entropy generation numbers are

$$
N s'_{\Delta P_h} = -\frac{R}{c_{ph} W E} \ln \frac{p_{ho}}{p_{hi}} \tag{27}
$$

$$
Ns'_{\Delta P_c} = -\frac{R}{c_{pc}E} \ln \frac{p_{co}}{p_{ci}} \tag{28}
$$

The pressure drop or *dp/d(NTU)* in the above equations can be obtained by using correlations given in literatures. Take a shell and tube heat exchanger (Fig.5) as an example. Suppose the shell side fluid is hot and the tube side fluid is cold, then the frictional loss on the tube side will be

$$
\Delta p_c = \left(f\frac{l}{d} + \sum \xi\right) \frac{\rho V_c^2}{2} \tag{29}
$$

According to Zhukauskas (1982), the frictional loss on the shell side may be expressed as

$$
\Delta p_h = \zeta d_o C_z C_k C_\beta \frac{\rho_h V_h^2}{2} \tag{30}
$$

Substituting $l = zL$, $A = zn\pi L d$, $\dot{m}_c = \rho_c V_c n\pi d^2/4$, the frictional losses become:

$$
\frac{\Delta p_c}{p_{ci}} = \left(f + \frac{d}{L} \sum \xi\right) \frac{NTU}{8\beta Eu_c St_c} = F_c N T U \qquad (31)
$$

$$
\frac{\Delta p_h}{p_{hi}} = \zeta dC_z C_k C_\beta \frac{NTU}{8\beta L Eu_h St_c} = F_h NTU \qquad (32)
$$

where the Euler number $Eu = p_i/(\rho V^2)$, Stanton number $St = \lambda/(\rho V c_p)$. Substituting Eqs.(31) and (32) into Eqs.(27) and (28), the entropy generation number due to friction for gaseous media can be obtained and are graphically illustrated with curve 2 in Fig.3 and Fig.4. For liquid media, from Eqs.(31) and (32), the derivatives can be obtained

$$
\frac{dp_c}{dNTU} = p_{ci}\left(f + \frac{d}{L}\sum\xi\right)\frac{NTU}{8\beta Eu_cSt_c} = p_{ci}F_c \quad (33)
$$

$$
\frac{dp_h}{dNTU} = p_{hi} \zeta dC_z C_k C_\beta \frac{NTU}{8\beta L Eu_h St_c} = p_{hi} F_h \quad (34)
$$

The corresponding entropy generation numbers may be calculated by Eqs.(33), (34) with Eqs.(21)-(24) or Eqs.(25), (26). Then, it is straightforward to calculate the entropy generation number due to friction for a gas-liquid or liquid-liquid heat exchanger.

The sum of entropy generation number due to heat transfer and that due to friction equals to the total entropy generation number of a heat exchanger. The relationship between the total entropy generation number and the *NTU* is shown by curve 3 in Fig.3 and Fig.4, Fig.6 shows the effects of *NTU* and the ratio of heat capacity rates of the two fluids on *Ns ~.* It is worthwhile to note that there exists a minimum total entropy generation number while *NTU* and the ratio of heat capacity rates vary. The existence of this minimum value is the prerequisite of heat exchanger optimization.

(a) counter-low heat exchanger

CONCLUSION

1. The original entropy generation number suggested by Bejan (1982) is somewhat ambiguous in evaluating the thermodynamic performance of a heat exchanger.

2. A modified entropy generation number is suggested by the present authors and the ambiguity of the original one can be eliminated by using the modified one.

3. The entropy generated by friction is noticeable while the temperature difference between the hot and cold fluids is relatively small.

4. A minimum entropy generation dose exists which is the basis of heat exchanger optimization.

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