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A Numerical Analysis of the Forced Convection Condensation of Saturated Vapor Flowing Axially Outside a Horizontal Tube

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Physical and mathematical models are developed to describe the forced convection condensation heat transfer of saturated vapor flowing axially outside a horizontal tube. The numerical solution of the models indicates the effects of vapor velocity on the liquid film thickness. The result verifies the enhancement of condensation heat transfer caused by such flow.

Keywords: Condensation, Axial flow, Numerical solution, External flow.

INTRODUCTION

Laminar film condensation of saturated vapor on a vertical surface has been studied widely. In 1916, Nusselt[1] first developed a very simple and effective correlation to describe natural convection condensation of pure saturated vapor on a vertical wall. Then he derived a theoretical solution for natural convection condensation of saturated vapor on a horizontal tube surface. In 1959, Sparrow[2] applied boundary layer theory to the problem. Since then, advances in the treatment of the boundary layer have allowed the solution of the condensation problem with increasing accuracy[3-10]. However, most work is concerned with natural or forced convection condensation of vapor across horizontal tubes. Little reasearch has been done on condensation due to forced convection axial flow outside of a horizontal tube. Recently, the enhancement of condensation heat transfer due to coiled wires on a tube was studied by Wang et al.[11,12]. In these papers, the surface tension of the condensate caused by the coiled wire was considered to be an axial force. The surface tension exists only between the wires. No study has been reported concerning forced convection condensation heat transfer with axial shearing stress on the condensation film. This kind of condensation phenomenon exist in heat exchangers, especially in rod-baffle condensers. This paper gives a theoretical study and numerical computation of the condensation phenomemon. The results show that this pattern of vapor flow with shearing stress enhances the heat transfer of condensation.

PHYSICAL AND MATHEMATICAL MODELS

1. Assumptions

(1) Thermophysical properties of liquid film and vapor film are constant;

(2) There are no noncondensable gases;

(3) The flows in the liquid and vapor films are laminar and the liquid film is very thin;

(4) The axial speed of the vapor flow is far greater than that of the liquid film; the circumferential speed of the vapor flow is neglected;

(5) The inertia terms in the motion equations and the convection terms in the condensate film energy equation are neglected;

From these assumptions, the liquid flow is taken as two-dimensional in the (φ, z) plane and the vapor flow is taken as two-dimensional in the (y,z) plane. As a result, the original three-dimensional problem is simplified to a combination of two-dimensional problems.

Nomenclature

C_p	specific heat [J/kg· K]
D	diameter of tube [m]
Fr	Froude number [-]
g	gravitational acceleration [m/s^2]
J_a	Jakob number [-]
Nu	Nusselt number [-]
Pr	Prandtl number [-]
q	heat flux [W/m^2]
t	temperature [K]
r	latent heat of vaporization [J/kg]
u, w	liquid velocity in $\varphi-$ and $z-$ directions respectively [m/s]
W, V	vapor velocity in $z-$ and $y-$ directions respectively [m/s]
x, y, z	rectangular coordinates [m]

Greek Symbols

δ	thickness of condensate film [m]
Δ	thickness of vapor boundary layer [m]
λ	thermal conductivity [W/m· K]
μ	dynamic viscosity [kg/m · s]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
φ	angle in cylindrical coordinate [-]

Subscripts

f	infinite
i	interface of liquid and vapor
l	liquid phase
s	saturated
v	vapor phase
w	wall
-	dimensionless

2. Governing Equations

The physical model and coordinate system used

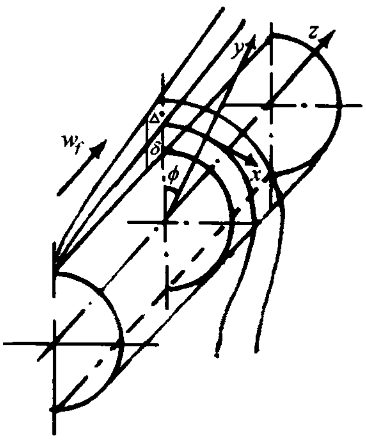

Fig.1 Physical model and coordinates

for the forced convection condensation heat transfer of saturated vapor flowing axially outside a horizontal tube is shown in Fig.1. The following mathematical model can be derived according to the above-mentioned physical model.

The motion equations of the model are:
for the liquid film,

$$\mu_l \frac{\partial^2 u}{\partial y^2} + \rho_l g \sin(\varphi) = 0 \quad (1)$$

$$\mu_l \frac{\partial^2 w}{\partial y^2} = 0 \quad (2)$$

$$\frac{\partial^2 t}{\partial y^2} = 0 \quad (3)$$

for the vapor boundary layer,

$$W\frac{\partial W}{\partial z} + V\frac{\partial W}{\partial y} = \nu_v \frac{\partial^2 W}{\partial y^2} \quad (4)$$

$$\frac{\partial W}{\partial z} + \frac{\partial V}{\partial y} = 0 \quad (5)$$

The energy conservation equation at the liquid vapor interface is:

$$\frac{2}{D}\frac{\partial}{\partial \varphi}\left(\int_0^\delta u dy\right) + \frac{\partial}{\partial z}\left(\int_0^\delta w dy\right) = \frac{\lambda_l(t_S - t_w)}{(\rho_l r \delta)} \quad (6)$$

with the boundary conditions:

$$y = 0 \qquad u = w = 0 \qquad t = t_w \quad (7)$$

while for $y = \delta$ or $\xi = 0$ (liquid-vapor interface)

$$w_i = W_i \qquad t = t_S \quad (8)$$

$$\mu_l \frac{\partial w}{\partial y} = \mu_v \frac{\partial W}{\partial y} \quad (9)$$

$$-\rho_v V_i = \frac{q_i}{r} \quad (10)$$

where $\xi = y - \delta$

$$y = \delta + \Delta \text{ or } \xi = \Delta \qquad W = W_f \qquad \frac{\partial W}{\partial y} = 0 \quad (11)$$

Directly integrating Equation (1) twice and using the boundary conditions in Equations (7) and (8) gives:

$$u = \frac{\rho_l g \sin\varphi}{\mu_l}\left(\delta y - \frac{y^2}{2}\right) \quad (12)$$

Directly integrating Equation (2) twice and using the boundary conditions in Equations (7) and (8) gives:

$$w = \frac{w_i y}{\delta} \tag{13}$$

Directly integrating Equation (3) twice and using the boundary conditions in Equations (7) and (8) gives:

$$t = (t_S - t_w)\frac{y}{\delta} + t_w \tag{14}$$

Considering heat conduction in the liquid film:

$$q_i = -\lambda_l \left(\frac{\partial t}{\partial y}\right)_{y=0} = -\lambda_l \frac{(t_S - t_w)}{\delta} \tag{15}$$

Substituting Equations (12) and (13) into Equation (6) gives:

$$\frac{2}{3}\frac{\rho_l g}{D\mu_l}\left(\delta^3 \cos\varphi + \sin\varphi \frac{\partial \delta^3}{\partial \varphi}\right)$$

$$+\frac{1}{2}\frac{\partial}{\partial z}(w_i \delta) = \frac{J_a \nu_l}{Pr_l \delta} \tag{16}$$

Equation (16) is put into dimensionless form using the following dimensionless numbers:

$$Fr = \frac{{W_f}^2}{Dg} \tag{16a}$$

$$J_a = \frac{C_{Pl}(t_S - t_w)}{r} \tag{16b}$$

$$Pr = \frac{\nu_l}{a_l} \tag{16c}$$

$$H = \frac{J_a}{Pr_l} \tag{16d}$$

$$\overline{z} = \frac{zg}{{W_f}^2} \tag{16e}$$

$$\overline{\delta} = \delta\sqrt{\frac{g}{W_f \nu_l}} \tag{16f}$$

$$\overline{w_i} = \frac{w_i}{W_f} \tag{16g}$$

Then Equation (16) becomes:

$$Fr\left(\overline{\delta}^4 \cos\varphi + \frac{3}{4}\sin\varphi \frac{\partial \overline{\delta}^4}{\partial \varphi}\right)$$

$$+\overline{\delta}\frac{3}{4}\frac{\partial(\overline{w_i}\overline{\delta})}{\partial \overline{z}} = 1.5H \tag{17}$$

Integrating Equation (4) for ξ from 0 to Δ gives:

$$\int_0^\Delta W\frac{\partial W}{\partial z}d\xi + \int_0^\Delta V\frac{\partial W}{\partial \xi}d\xi = \int_0^\Delta \nu_v \frac{\partial^2 W}{\partial \xi^2}d\xi \tag{18}$$

since

$$\frac{\partial W}{\partial z} + \frac{\partial V}{\partial \xi} = 0$$

and

$$\frac{\partial(WV)}{\partial \xi} = W\frac{\partial V}{\partial \xi} + V\frac{\partial W}{\partial \xi} = -W\frac{\partial W}{\partial z} + V\frac{\partial W}{\partial \xi}$$

$$V\frac{\partial W}{\partial \xi} = \frac{\partial(WV)}{\partial \xi} + W\frac{\partial W}{\partial z} \tag{19}$$

Substituting Equation (19) into Equation (18) gives:

$$\int_0^\Delta \frac{\partial(WW)}{\partial z}d\xi + \int_0^\Delta \frac{\partial(WV)}{\partial \xi}d\xi$$

$$= \int_0^\Delta \nu_v \frac{\partial^2 W}{\partial \xi^2}d\xi \tag{20}$$

Integrating Equation (20) gives:

$$\frac{\partial}{\partial z}\left(\int_0^\Delta (WW)d\xi\right) + W_f V_f - W_i V_i$$

$$= -\nu_v \left(\frac{\partial W}{\partial \xi}\right)_{\xi=0} \tag{21}$$

Integrating Equation (5) for ξ from 0 to Δ gives:

$$\int_0^\Delta \frac{\partial W}{\partial z}d\xi + \int_0^\Delta \frac{\partial W}{\partial \xi}d\xi = 0$$

therefore (using Equation (21)),

$$V_f = V_i - \int_0^\Delta \frac{\partial W}{\partial z}d\xi \tag{22}$$

Substituting Equation (21), then gives

$$\frac{\partial}{\partial z}\left(\int_0^\Delta (WW - WW_f)d\xi\right)$$

$$+(W_f - W_i)V_i = -\nu_v\left(\frac{\partial W}{\partial \xi}\right)_{\xi=0} \tag{23}$$

From Equation (10)

$$V_i = -\frac{q_i}{r\rho_v} \qquad q_i = -\frac{\lambda_L(t_s - t_w)}{\delta}$$

so that:

$$V_i = \frac{\lambda_l(t_S - t_w)}{(\rho_v \delta r)} \tag{24}$$

Substituting Equation (24) into Equation (23) gives:

$$\frac{\partial}{\partial z}\left(\int_0^\Delta (WW - WW_f)d\xi\right)$$

$$+(W_f - W_i)\left(\frac{J_a}{Pr_l}\frac{\rho_l}{\rho_v}\frac{\nu_l}{\delta}\right) = -\nu_v\left(\frac{\partial W}{\partial \xi}\right)_{\xi=0} \tag{25}$$

Assuming a parabolic velocity profile in the vapor boundary layer:

$$W = a + b\xi + c\xi^2 \tag{26}$$

and using the conditions:

$$\xi = 0 \quad W = w_i$$

$$\xi = \Delta \quad W = W_f \quad \frac{\partial W}{\partial y} = 0$$

the velocity distibution within the vapor boundary layer is given by:

$$W = w_i + (W_f - w_i)\left[2\frac{\xi}{\Delta} - (\frac{\xi}{\Delta})^2\right] \tag{27}$$

Substituting Equations (13) and (27) into Equation (9) gives:

$$\frac{\Delta}{\delta} = \frac{2(W_f - w_i)}{w_i}\frac{\mu_v}{\mu_l} \tag{28}$$

Subtituting Equations (27) and (28) into Equation (25) then gives:

$$\frac{\partial}{\partial z}\left\{\int_0^\Delta\left\{\left[w_i + (W_f - w_i)\left[2\frac{\xi}{\Delta} - \left(\frac{\xi}{\Delta}\right)^2\right]\right]^2 - \left(w_i + (W_f - w_i)\left[2\frac{\xi}{\Delta} - \left(\frac{\xi}{\Delta}\right)^2\right]\right)W_f\right\}d\xi\right\} + (W_f - w_i)\frac{J_a\rho_l\nu_l}{Pr_l\rho_v\delta} = -w_i\frac{\mu_l}{\rho_v\delta} \tag{29}$$

Integrating Equation (29) gives:

$$\frac{\partial}{\partial z}\left[\delta\frac{(W_f - w_i)^2(3w_i + 2W_f)}{w_i}\right] + \frac{15R^2}{2\delta}\left[(W_f - w_i)\frac{J_a\nu_l}{Pr_l} - w_i\frac{\mu_l}{\rho_l}\right] = 0 \tag{30}$$

where

$$R^2 = \frac{\mu_l\rho_l}{\mu_v\rho_v}$$

Substituting Equations (16a)-(16g) into Equation (30) gives the dimensionless equation

$$\frac{(1 - \overline{w_i})^2(3\overline{w_i} + 2)}{2\overline{w_i}}\frac{d\overline{\delta}^2}{d\overline{z}} + \overline{\delta}^2\frac{d}{d\overline{z}}\left[\frac{(1 - \overline{w_i})^2(3\overline{w_i} + 2)}{\overline{w_i}}\right] + \frac{15R^2}{2}[H(1 - \overline{w_i}) - \overline{w_i}] = 0 \tag{31}$$

NUMERICAL SOLUTION FOR TWO LIMITING CASES

1. No Surface Shear on the Liquid Film

For natural convection condensation of saturated vapor on a tube:

$$W_f = 0 \quad and \quad w_i = 0$$

so that Equation (16) becomes:

$$\frac{2\rho_l g}{3D\mu_l}\left(\delta^3\cos\varphi + \sin\varphi\frac{\partial\delta^3}{\partial\varphi}\right) = \frac{J_a\nu_l}{Pr_l\delta} \tag{32}$$

Equation (32) describes natural convection condensation of saturated vapor on a horizontal tube. This is Nusselt's equation[1], which must be solved numerically.

2. No Gravitational Force in the Liquid Film

The phenomenon, similar to forced convection condensation of vapor flowing on a horizontal plate, is defined by:

$$g = 0$$

Therefore, Equation (16) becomes:

$$\overline{\delta}\frac{d(\overline{w_i}\overline{\delta})}{d\overline{z}} = 2H \tag{33}$$

while equation (30) becomes:

$$\frac{d}{d\overline{z}}\left[\frac{(1 - \overline{w_i})^2(3\overline{w_i} + 2)\overline{\delta}}{\overline{w_i}}\right] + \frac{15R^2}{2\overline{\delta}}[H(1 - \overline{w_i}) - \overline{w_i}] = 0 \tag{34}$$

Assuming the following functions for $\overline{w_i}$ and $\overline{\delta}^2$ which satisfy Equations (33) and (34),

$$\overline{w_i} = A_o \tag{35}$$

$$\overline{\delta}^2 = B_1\overline{z} \tag{36}$$

and substituting Equations (35) and (36) into Equation (33) gives:

$$A_oB_1 = 4H \quad or \quad B_1 = \left(\frac{4H}{A_o}\right) \tag{37}$$

Substituting Equations (35) and (36) into Equation (34) and using Equation (37) gives:

$$\frac{4(1 - A_o)^2(2 + 3A_o)}{R^2} - \frac{15A_o^3}{H} + 15A_o^2(1 - A_o) = 0 \tag{38}$$

From Equation (36):

$$\bar{\delta}^2 = \left(\frac{4H}{A_o}\right)\bar{z} \tag{39}$$

Substituting Equations (16e) and (16f) into Equation (39) gives:

$$\frac{Nu_z}{\sqrt{Re_z}} = \frac{\sqrt{A_o}}{2\sqrt{H}} \tag{40}$$

In Equation (40), $Nu_z = \alpha_z z/\lambda_l$ and $Re_z = W_f z/\nu_l$. Equation (40) is the relationship between the local Nusselt number and local Reynolds number for forced convection condensation on a horizonal plate. In Equation (40), A_o is obtained from Equation (38).

3. Including the Effect of Both Forces on the Liquid Film

When both surface shear and gravitational forces act on the liquid film, no terms in Equations (17) and (31) can be neglected. When $\varphi = 0$, Equation (17) becomes:

$$Fr\bar{\delta}^4 + \frac{3\bar{\delta}}{4}\frac{d}{d\bar{z}}(\overline{w_i}\bar{z}) = 1.5H \tag{41}$$

Equations (31) and (41) form a set of first order differential equations that can be used to obtained the characteristic solution at $\varphi = 0$. The variations of $\bar{\delta}$ and $\overline{w_i}$ as functions of $\bar{z}$ at $\varphi = 0$ was determined by numerically solving Equation (31) and Equation (41). Since it is assumed that the speed of the liquid film flowing circumferentially does not force the vapor to move in the circumferential direction, then the axial flow of the vapor forces the liquid film to move only axially. Therefore, the derived speed at the liquid-vapor interface is a function of only the axial coordinate. Therefore, the derived speed at the liquid-vapor interface is not only applicable to the location, $\varphi = 0$, but also applies at any angle φ. If the variations of $\bar{\delta}$ and $\overline{w_i}$ as functions of $\bar{z}$ at $\varphi = 0$, and the relationship between $\overline{w_i}$ and $\bar{z}$ are substituted into Equation (17), then the variation of $\bar{\delta}$ for all $\bar{z}$ and φ can be found using the Runge-Kutta-Gill method.

NUMERICAL RESULTS AND ANALYSIS

1. Relationship between Liquid Film Thickness, $\bar{\delta}$, with φ and $\bar{z}$

Fig.2 and Fig.3 show the variation of the liquid film thickness $\bar{\delta}$, with φ and $\bar{z}$. Fig.4 shows the liquid film thickness variation as a function of the Fr number. When the Fr number increases, the thickness decreases indicating that a large vapor flow velocity produces a strong

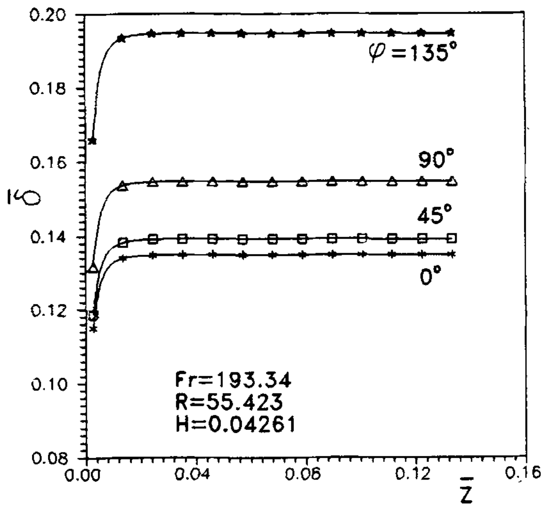

Fig.2 Axial liquid film thickness variation

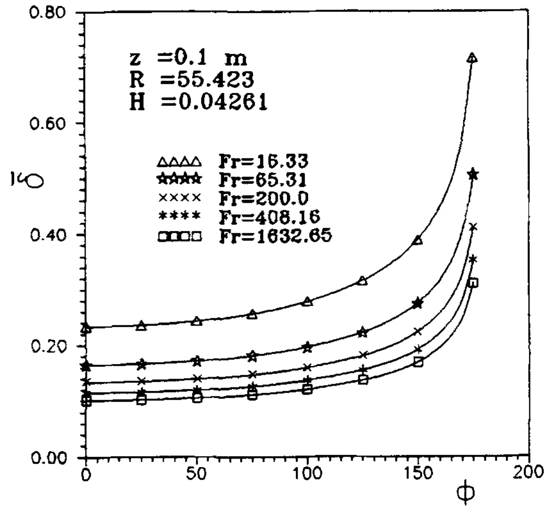

Fig.3 Circumferential liquid film thickness variation

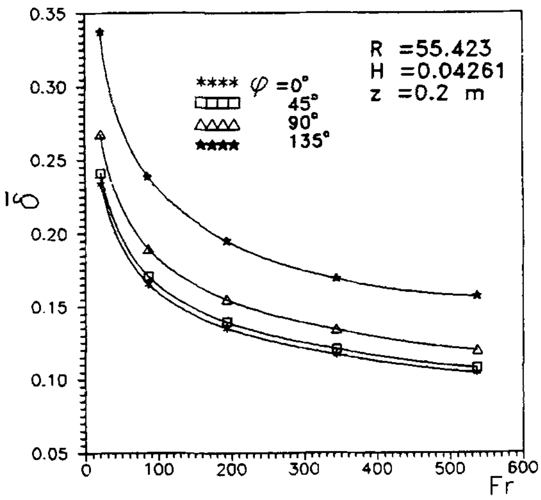

Fig.4 Liquid film thickness variation with Fr number

shearing stress at the liquid-vapor interface which, as a result, makes the liquid film thin. The figures indicate clearly that the thickness of the liquid film increases with the increase of either φ or z. To show the enhancement of heat transfer with the flow pattern, a comparison between the thickness of the film on the top of a tube with free convection condensation and with forced convection condensation is shown in Fig.5. It is clear that when the Fr number is large, the liquid film is very thin near the entrance due to the high vapor velocity. Further down the tube (larger z), the vapor velocity decreases rapidly due to condensation and the liquid film thichness increases until it reaches a value typical of free convection. Therefore, there is significant enhancement of the condensation heat transfer in the enchance region, especially at larger Fr numbers.

2. The Variation of the Local Heat Transfer Coefficient α with φ and z

Taking R11 as an example, the variation of the local heat transfer coefficient, α, as a function of φ,z is shown in Fig.6 and Fig.7. As Fig.6 and Fig.7 show, the local convective heat transfer coefficient decreases with increasing z and φ. The mean heat transfer coefficient in the circumferential direction decreases with increasing z and with decreasing Fr number, as shown in Fig.8.

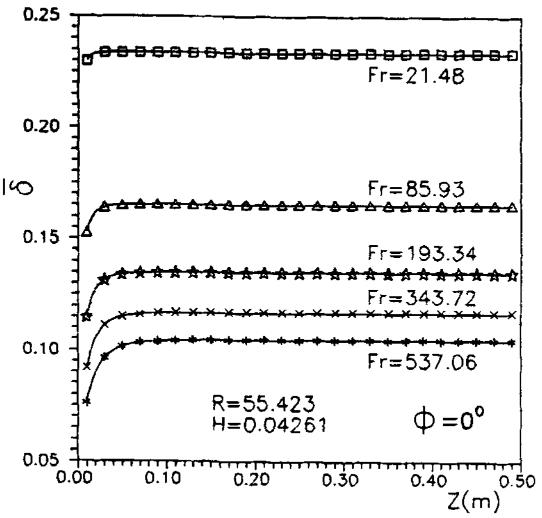

Fig.5 A comparision of liquid film thickness for forced and natural convection condensation

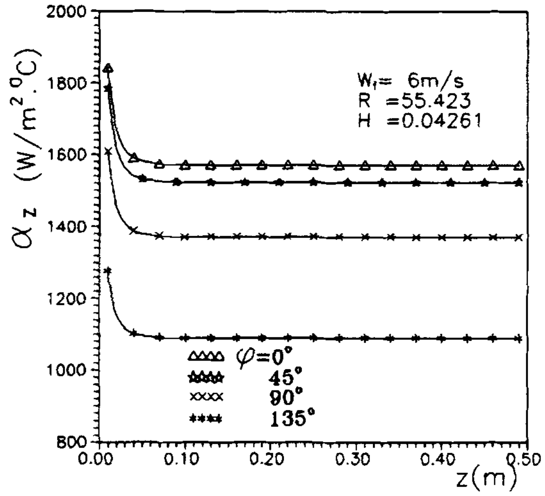

Fig.6 Axial variation of the local heat transfer coefficient

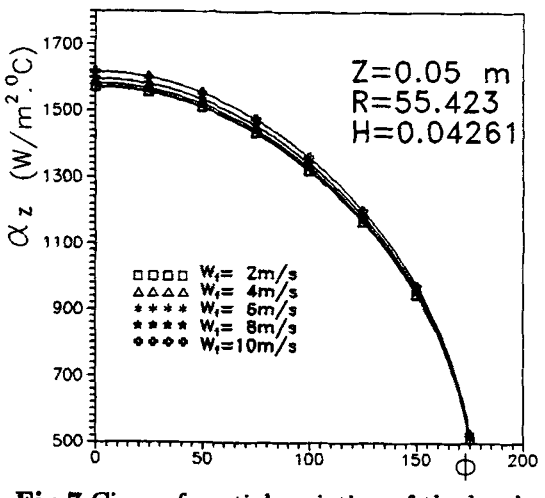

Fig.7 Circumferential variation of the local heat transfer coefficient

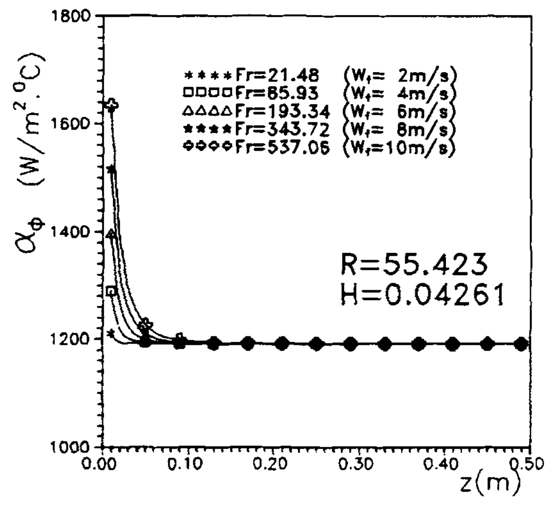

Fig.8 Mean circumferential heat transfer coefficient variation in the axial direction

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