How much time do students have to think about teacher questions? An investigation of the quick succession of teacher questions and student responses in the German mathematics classroom

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Abstract: *Several studies have shown that the style of the German mathematics classroom at secondary level is mostly based on the so called "fragend-entwickelnde" teaching style which means developing the lesson content by a teacher directed sequence of teacher questions and student responses. In this article we describe a study on the time the students have for thinking about a teacher question in the public classroom interaction. Our investigation is based on a re-analysis of 22 geometry lessons from grade 8 classes which mainly deal with a challenging proving content. The results show that the average time between a teacher question and a student response is 2.5 seconds. There are no remarkable differences between different phases of the lessons like comparing homework, repetition of content or working on new content. Moreover, for 75% of the teacher questions the first student was called to answer within a three second time interval.*

ZDM-Classification: D43 E53

1. Introduction

The idea that education should be a wondrous journey and not a race is nowadays a widely accepted fact. Effective learning needs time, time for reflection and thinking, time for linking new facts to the existing individual knowledge, in particular, if the learning content is on a high cognitive level. Consequently, it is clear that the learning gain during a presentation of new facts or the participation in a discourse depends on the question whether the learner can follow and process the flow of information.

With respect to the German mathematics classroom which is mainly based on a public teacher-student discourse this question leads to the time span between a teacher question and student answer. Do teachers give enough time to the students to process the information given in the question, generate an adequate answer and extend their knowledge?

In this article we describe a study about the teacher wait time in 22 mathematics lessons in grade 8 (the students are at the age of 13 or 14). The content of these lessons is reasoning and proof in geometry, i.e. a topic which requires learner activities on a high cognitive level. Therefore, one may suppose that the characteristics of the teacher-student interaction will differ to that of other mathematics lesson with a practicing or an algorithmic content. However, our results show that we can replicate the well known results for the teacher wait time in the USA for our sample.

2. Students' competencies in geometrical reasoning and proof in grade 7 and 8

In this section we will give some information about the Germany mathematics classroom on the lower secondary level and the competencies in geometrical reasoning and proof for the students in the high attaining school track[1].

2.1 Characteristics of the German mathematics classroom on the lower secondary level

According to the TIMS-Study 1995 about 70% of the grade 8 classes in Germany comprise between 21 and 30 students and about 25 % less than 20 students (Beaton et al. 1996, p. 152). The mathematics classroom is generally organized as interplay between teacher questions and student responses. The basic idea of this typical German teaching style is to integrate the role of the teacher as an instructor with the goal of student orientation. Phases of teacher talk or student work (individually or in groups) are underemphasized.

The TIMSS 1995 video study (Stiegler et al., 1999), in which 100 German mathematics lessons in grade 8 were analyzed, characterized the German teaching style as guiding students through the development of a procedure or a concept by asking them to orally fill in relevant information (so-called "fragend-entwickelnde" (= questioning-developing) teaching style).

[1] The German "Gymnasium"; it ends with a final exam in grade 12 or 13 and allows the students to continue their education at a university.

The teacher organizes the lesson so that most of the mathematical work during the lessons is done as a whole class. The teacher does not lecture much to the students; instead, she guides students through the development of the procedure by asking students to orally fill in the relevant information. (…) If the problem is a relatively new one, the teacher generally works the problem at the board, eliciting ideas and procedures from the class as work on the problem progresses. (…) The teacher keeps the student and class moving forward by asking questions about next steps and about why such steps are appropriate. After two or three similar problems have been worked in this way, the teacher summarizes the activity by pointing to the principle or property that guides the deployment of the procedure in these new situations. For the remaining minutes of the class period, she assigns several problems in which students practice the procedure in similar situations (Stiegler et al., 1999, pp. 133-134).

Though there is a teacher-student discourse in the German mathematics classroom, student activities and responses on a high cognitive level are observed comparative rarely. Already 20 years ago Voigt (1984) showed in an in-depth case study about the German teaching style that students generally do not have to solve complex problems in the mathematics classroom, because the problems are solved step by step by answering simple and closed teacher-guided questions. The required student responses are mostly only on an elementary level and the complex problem is transformed into a series of closed simple question. (cf. also Klieme, Schümer, & Knoll, 2001).

As already mentioned in the introduction the individual learning processes will take place in this special kind of teacher-student interaction which together with the mathematical content constitutes the mathematics lessons in Germany. Hence, the time a teacher gives after posing his/her question will certainly influence the characteristics of the interaction in the classroom and the individual learning processes.

2.2 Student competencies in reasoning and proof in geometry on the lower secondary level

In this subsection we will give a short overview about student competencies in geometrical reasoning and proof in Germany. Since these competencies in reasoning and proof are related to a pure mathematical context, we can assume that they are essentially influenced by the mathematics instruction and its particular conditions.

As international comparative studies like TIMSS and PISA indicate the German students on the lower secondary level perform well with routine items or argumentative items which can be performed by a one-step-argumentation. However, problems occur, if an item requires the generation of a solution strategy and the combination of arguments as it is the case for proofs (cf. Blum & Neubrand, 1998, Deutsches PISA-Konsortium, 2001).

Research in the area of geometrical reasoning competencies of grade 7 and 8 students in Germany has revealed a competency model with three competency levels (Reiss, Hellmich & Reiss, 2002, Heinze & Reiss, 2004). The first level describes the basic competency (applying facts and rules, e.g. for calculations), the second level encompass argumentative competency for one-step-argumentations and the third level argumentative competency which allows to solve tasks for which several arguments must be combined based on a solution strategy. Findings of different empirical investigations show that low-achieving students were hardly able to solve any items on level III whereas high-achieving students performed well on level I and level II items and satisfactorily on level III tasks (Reiss, Hellmich & Reiss, 2002, Heinze & Reiss, 2004). Overall this competency model and its correspondence to the different achievement groups were confirmed in several studies by empirical data of more than 2000 students.

The fact that reasoning and proof is a demanding task for students all over the world is supported by empirical evidence from other studies like in the United Kingdom (Healy & Hoyles, 1998, Küchemann & Hoyles, 2003), Taiwan (Lin, 2005) or Korea (Kwak, 2005). For the German results there are some indicators that mathematics instruction plays an essential role in the development of student proof competencies. Heinze and Reiss (2004) found significant differences on the class level. A multilevel analysis revealed that the mean test score on the classroom level in grade 7 explains about 27% of the variance of the individual student competency in grade 8. A content based analysis of German mathematics lesson on reasoning and proof indicates that there is only an incomplete representation of the proving process (as a problem solving process) in the mathematics classroom (Heinze, 2004).

3. The role of wait time in the classroom discourse

The investigation of the wait time in the teacher-student interaction was one line of research within the extensive classroom research studies in the United States in the 1970s and 1980s. One of the initial studies was conducted by May Budd Rowe (Rowe, 1969), which was followed by the investigations of several hundreds audio-taped science lessons (Rowe, 1974). Within the following 15 years the results of Rowe were replicated in a large number of studies in different countries and different subject areas (see Tobin, 1987 or Rowe, 1986 for a review). Moreover, beyond the descriptive studies several intervention studies were conducted to explore the changes of the teacher-student interaction when the wait time was extended. Stahl (1990) supplemented the factor “wait time” (after a teacher question or a student response) and defined the factor “think time”. Think time comprises eight categories which, on the one hand, encompass the classical wait time and, on the other, is also related to other kinds of pauses during the classroom

The investigation of wait time can be considered as a typical approach within the classical process-product paradigm in classroom research. However, meanwhile research methods have developed and there are more elaborated approaches like the supply and usage model for classroom processes or the process-mediation-product paradigm (cf. Helmke, 2003; Brophy, 1999). Within these new paradigms behavioral variables loose their importance and aspects like the quality of learning opportunities or cognitive activation are emphasized. However, we think that aspects of the teacher behavior like wait time are still important variables which should not be ignored.

In the following we will give a summary of the descriptive results about the wait time research (3.1) and of the effects of an extended wait time (3.2).

3.1 Descriptive results

In Rowe (1969) two types of wait time were defined: the wait time I after a teacher utterance and the wait time II after a student utterance. In most of the cases the wait time I is the time between a teacher question and a student answer, whereas wait time II is the time between a student answer and the following teacher feedback or teacher question. There are some other definitions of wait time in the literature (see Tobin, 1987), however, we will refer to the initially defined variables by Rowe and, particularly, to wait time I.

The studies of Rowe revealed that the average wait time I and II in science lessons in the USA was less than 3 seconds. In particular, the time between a teacher question and calling a student to respond tended in many lessons to one second or less. As described in Tobin (1987) different studies replicated this short wait time. Moreover, several investigations considered the wait time I as a dependent variable. It turns out that the wait time was longer after more challenging teacher questions (Boeck & Hillenmeyer, 1973; Arnold, Atwood & Roger, 1974). In the study of Boeck and Hillenmeyer (1973) nearly all high cognitive level questions were followed by a wait time of three seconds or longer. Arnold, Atwood and Roger (1974) got an average duration of the wait time I of 4.6 seconds after so called analysis questions.

In a study with 32 grade 8 students Jones (1980) measured the time the students need to think before they respond to a question during an individual interview. It turns out that the average time after convergent questions was 2.8 seconds and after divergent questions was 6.9 seconds. Jones concluded from these results that it is necessary to provide a certain time span for students to think about a question.

3.2 Effects of extended wait time

In addition to the presented descriptive studies there are various kinds of intervention studies in which effects of a manipulated wait time (average duration beyond three seconds) were analyzed in comparison to normal wait time.

A consistently reported result is the decrease of the number of teacher questions (e.g. Tobin, 1986 for mathematics lessons in grade 6/7). Moreover, it was observed that there is a change in the quality of the teacher questions: the cognitive level increases and fewer memory level questions or leading questions are posed (Swift & Gooding, 1983; Gooding, Swift, & Swift, 1983; DeTure & Miller, 1985). As Tobin (1986) observed in mathematics lessons the extended wait time is associated with a tendency for teachers to ask for more input from students (probing questions).

However, not only changes in the teachers' behavior were observed, but also changes in student variables. Rowe (1974) described that the students are more involved in classroom discourse: their responses are longer and on a higher cognitive level and the number of student utterances increases (e.g., Swift & Gooding, 1983; Tobin, 1986; Mansfield, 1996). As Tobin (1987) reported in his review there are several studies which indicate that an extended wait time is associated with an increase of the students' achievement. Here the experimental study of Tobin (1986) with 20 mathematics classes of grade 6/7 is of particular interest, since he was able to show that the extended wait time classes achieve higher class mean scores in a probabilistic reasoning test. However, as the findings of Riley (1986) for elementary students indicate the effects of an extended wait time depends also on the cognitive level of the question. His results suggest that there should be a shorter wait time after low cognitive level questions. The question whether an extended wait time in general is appropriate for all high cognitive level questions needs further research. For example, Duell (1994) reports results for university students which contradict the assumption that extended waiting time has positive influence to the achievement: She did not find the expected effect on the achievement when the wait time is extended from 1-3 second to 3-6 seconds.

Summarizing the results we can assume that there will be positive effects when the wait time after teacher questions (and student responses) is extended. However, is it clear that this has to be done in an appropriate manner, since e.g. a longer wait time after simple questions (reproduction of facts etc.) does not make sense. Moreover, it seems that an extended wait time after high cognitive level questions is a necessary condition for higher cognitive achievement. The study by Duell (1994) indicates that this condition is not sufficient.

4. Research questions and design of the study

4.1 Research questions

The empirical results in the previous section indicate that the wait time is of particular interest for the analysis of school lessons and the quality of instruction. Most of the studies cited before are related to science lessons, until now there is only a small amount of research in this area concerned with mathematics instruction. Moreover, for the situation in Germany we are not aware of systematical empirical investigations of the wait time in mathematics lessons[2].

The study presented in this article is guided by the following research questions:

1. How much time do students have after a teacher question?
2. Does this wait time depend on the type of the teacher question?
3. Does this wait time depend on the content-specific phase of the lesson?

Our research is based on the following assumptions:

When a teacher poses a question, then the students start to think about it to create a response. When the teacher calls the first student and asks to respond the question, the other students will interrupt their thinking, because they have to listen to their classmate. Since the interaction between students and their teacher constitutes a certain classroom culture (in the sense of social and socio-mathematical norms which can be regarded as part of a lesson script), the students in a class know in advanced how much time they get usually from their teacher (eventually depending on the type of questions and the content specific phase of the lesson). Hence, they are aware about the conditions or restrictions when starting to think about a teacher question and this will influence their behavior and their individual learning processes.

4.2 Sample and Design

The sample of the presented study consists of 22 videotaped mathematics lessons with grade 8 students. The lessons are from eight different classes of four schools (German Gymnasium, i.e. high attaining students). In each class we videotaped between two and four consecutive lessons with two cameras: one camera focussed the teacher and a second camera the whole class. Afterwards the two camera perspectives were

[2] Some features of the teacher wait time were recently investigated in a video study on English lessons in Germany, see http://www.dipf.de/desi/index.htm for first results.

combined to one tape. The subject of all lessons was reasoning and proof in geometry, in particular, in congruence geometry. The participating teachers were asked to provide their typical instruction. Asked by questionnaires after the lessons the vast majority of the students confirmed that the videotaped instruction was the instruction they were used to.

Since the wait time was not the initial goal for the video study, the research presented in this contribution has to be considered as a re-analysis. Previous investigations of the videotaped lessons were concerned with different aspects of mathematics instruction:

Firstly, basic elements of the lessons like instructional phases, participation of students in the discourse etc. were investigated (Heinze & Kraft, 2004). Figure 1 gives an overview how much time the teachers spend on different activities in the lessons. The findings in Figure 1 indicate that the teaching style in the lesson from our video study fits to that described in the TIMSS video study (cf. Section 2.1): Most of the time (about 65%) is spent for developing the content by teacher questions and student answers. About 16% of the time was used for individual work; however, this student work mainly consists of making geometrical drawings. Though it seems that the observed teaching style is student oriented, we have to state that the portion of students which in a lesson do not at all participate in the teacher-student discourse averages about 40% (Heinze & Kraft, 2004).

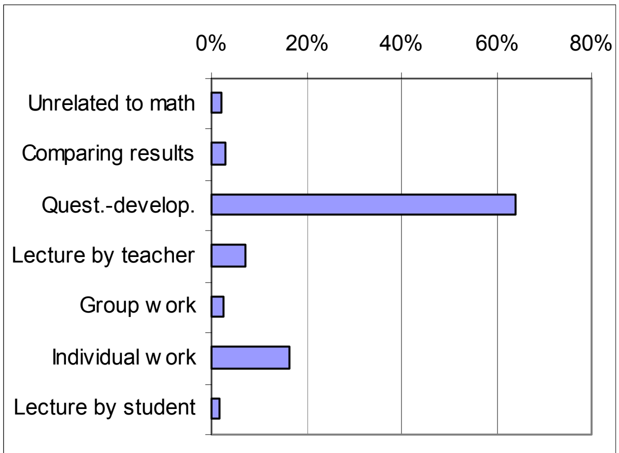


Figure 1: How much time spend teacher on various activities in mathematics lessons.

In a second step the proving processes were analyzed. We found that the stages in the proving process in which the problem situation is explored, additional information is collected and a proof idea is generated, were underemphasized by the teachers (cf. Heinze, 2004). In general, the teacher developed the proof step by step at the blackboard by asking questions to the students.

For the investigation of the wait time we firstly separated the lesson in phases which are slightly different from that in Figure 1. These phases are

(1) sharing the homework,
(2) the repetition of content during classwork,
(3) individual or group work,
(4) the development of new content during classwork.

As it was already described before, the teaching style in most of the classwork phases is the German questioning-developing style.

In addition to the separation of the lessons in phases we considered the teacher questions and assigned each question to one of seven categories. This assignment was done by the second author for all 22 lessons and checked by the first author for three randomly chosen lessons. There were only negligible differences.

The categories are:

(1) reproductive questions,
(2) convergent questions,
(3) divergent questions,
(4) evaluative questions,
(5) questions regarding the classroom management,
(6) rhetorical questions,
(7) other.

Reproductive questions are questions which ask for the reproduction of content that is already known by the students. Convergent and divergent questions are questions which require a thinking process by the students. In the case of convergent questions the teacher asks for one special response that in many cases can be obtained by a one-step thinking process ("What do we know about this angle?"). Divergent questions are open questions, which allow different responses ("How can we proceed?). Evaluative questions ask for reasoning activities ("Is this correct or not? And why?"). Questions regarding the classroom management are posed by the teacher for verifying whether the students can follow or whether the students finished a particular job in the lesson (like making drawings etc.). Rhetorical questions are responded by the teacher itself. These questions do not ask for a response.

4.3 Method of wait time measuring

The 22 videotaped lessons are the basis for the preparation of the wait time data. We restrict our analysis to parts of the lesson in which classwork took place; thus phases of group work and individual work are ignored. For each teacher question we determine the time at three specific moments:

1. when the teacher posed the question,
2. when the teacher stopped to talk,
3. when the (first) student starts to response.

In many cases Time 1 and Time 2 are identical. However, the determination of Time 1 and Time 2 is necessary, because in about 15% of the cases the teacher poses a question and then continues to give some further explanations to the question. Sometimes this supplementary teacher information takes a lot of time, because the teacher starts to make a drawing at the black board etc. We decide to ignore these cases if the time span between Time 1 and Time 2 (or Time 3) was longer than 20 seconds, because these few exceptional cases will particularly distort the mean values for the wait time. The 20 seconds bound is a more or less arbitrary bound, however, it turned out that for our sample 20 seconds was a natural threshold.

If a student response does not satisfy the teacher and he/she repeats the question and calls another student, then this second question is marked as a "repeated question". If the teacher calls a second student without repeating the question, the second response was ignored in our study.

In a few cases the teacher calls a student before he or she poses a question (e.g. "Peter, do you know why these angles are congruent?"). Here it happens that Time 1 and Time 3 are nearly identical and the question is addressed only to one student. We ignored these cases for our data collection, because they are exceptional. Other cases which are not included in the results of this article are questions of the types (5)-(7) (see Section 4.2). These questions generally are not content-specific questions.

For the time measuring we used a helpful computer program which was created by the second author. Since the videotapes were prepared as mpg-files we were able to use the computer keyboard for the time measuring: just by typing a key the exact time was saved in a spreadsheet. Experiments indicated that the accuracy of measurement was about 0.1 seconds.

5. Results

In this section firstly we will present the descriptive results for the wait time. Then in Section 5.2 and 5.3 we analyze the wait time as a dependent variable with respect to different types of questions and different phases of the lessons.

5.1 Wait time

We determined 1546 teacher questions in the classwork phases of the 22 lessons. There were 216 management questions (Type 5), 2 rhetorical questions (Type 6) and 154 questions of other type (Type 7). From the remaining 1174 questions 87 were omitted because of the 20 second bound.

For the determination of Time 1, Time 2 and Time 3 additional problems occurred, because in some cases it was not possible to determine the Time 1 or Time 3 value. Moreover, we omitted the cases in which the teacher called a student before he/she posed the question. Altogether, there are 945 remaining teacher questions, 141 of these questions are repeated questions.

Based on the values for Time 1-3 we determined three kinds of wait time (see Figure 2).

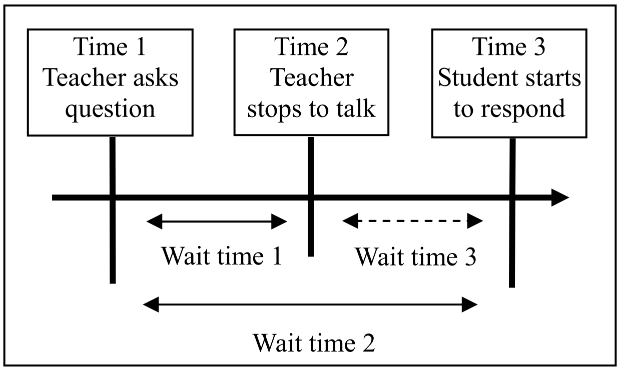


Figure 2: Definition of wait time.

Wait time 2 was determined for all 945 questions, Wait time 1 only for 307 questions, because in the remaining cases Time 1 and Time 2 were identical.

In addition to Wait time 1 and Wait time 2 we determined for all questions a Wait time 3:

Wait time 3 = Wait time 2 – Wait time 1,

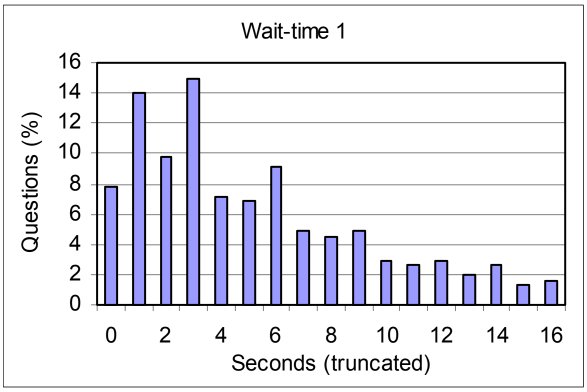
where Wait time 1 = 0, if Time 1 = Time 2.

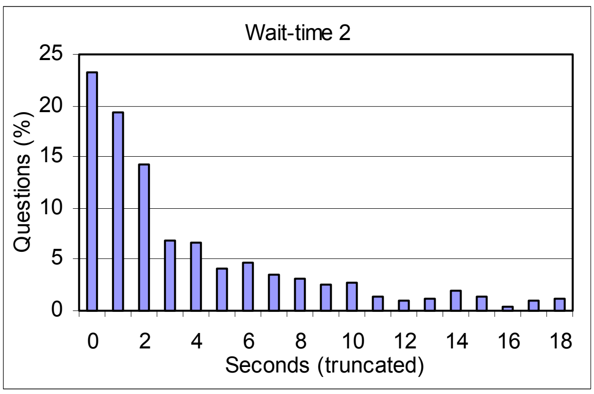
In Table 1 we give the mean values for all types of wait time. The average time between a teacher question and the student response is 4.2 seconds. However, if we take into account that the teachers in 307 case gave additional information after posing the question (average time 5.5 seconds) we obtain a mean value of 2.5 seconds for Wait time 3 (the time span between the last teacher utterance and the beginning of the student response). For all three cases the differences between first questions and repeated questions are not significant.

Mean	Wait time 1	Wait time 2	Wait time 3
(first) questions	5,6 s (N = 263)	4,2 s (N = 804)	2,4 s (N = 804)
repeated questions	5,5 s (N = 44)	4,5 s (N = 141)	2,8 s (N = 141)
all questions	5,6 s (N = 307)	4,2 s (N = 945)	2,5 s (N = 945)

Table 1: Mean values for Wait time 1-3.

More insight in the wait time situation during the mathematics lessons gives the frequency of questions with a certain wait time. For this purpose we truncate all values for the wait time to integers and give the percent of questions for each truncated value in a bar chart in Figure 3 (note that 0 represents the interval [0,1) and so on).





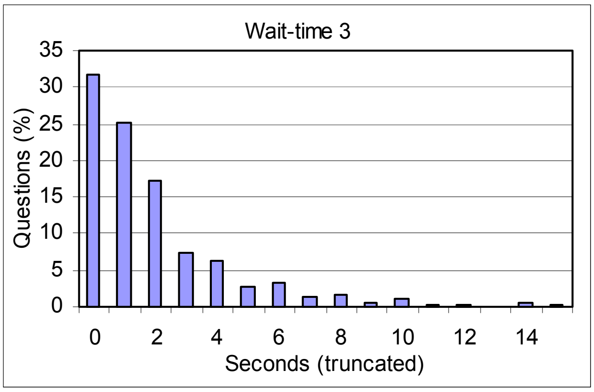


Figure 3 a-c: Distributions of Wait times 1-3.

As one can observe in Figure 3c there is a threshold for the time between the last teacher utterance and the student response (Wait time 3). In about 75% of the cases this wait time is less than three seconds. If we consider Wait time 2 (Figure 3b), which includes the 307 cases with additional teacher information after a question (Wait time 1), we also can identify the three second threshold, but here only in 57% of the cases the Wait time 2 is shorter than three seconds.

5.2 Wait time for different types of questions

As we described in Section 3 there are several empirical findings that the wait time depends on certain properties of the teacher question. The 945 questions of our data basis belong to four categories; they are distributed as presented in Table 2.

Types of questions (N = 945)	Frequency	Percentage
reproductive questions	370	39,2
convergent questions	517	54,7
divergent questions	20	2,1
evaluative questions	38	4,0

Table 2: Frequency and percentages of question types.

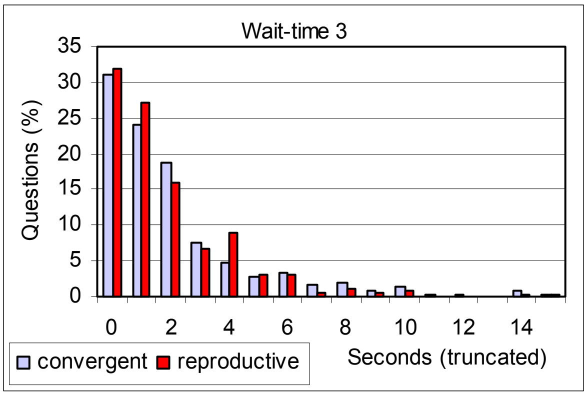
If we analyze the relation between the type of a question and the wait time, then we get the following results in Table 3 (following page).

Mean	Wait time 1	Wait time 2	Wait time 3
reproductive questions	5,4 s	3,9 s	2,3 s
convergent questions	5,9 s	4,6 s	2,6 s
divergent questions	3,1 s	3,8 s	2,6 s
evaluative questions	6 s	3,6 s	2,1 s

Table 3: Mean values for Wait time 1-3.

In each column of Table 3 the differences of the mean values are not significant.

Figure 4 gives some additional information about the distribution of convergent and reproductive questions (percentages) resp. divergent and evaluative questions (absolute values[3]). One can see that there is no noticeable difference between the Wait time 3 for the different question types. Again we observe a three seconds threshold (for the convergent and reproductive questions).



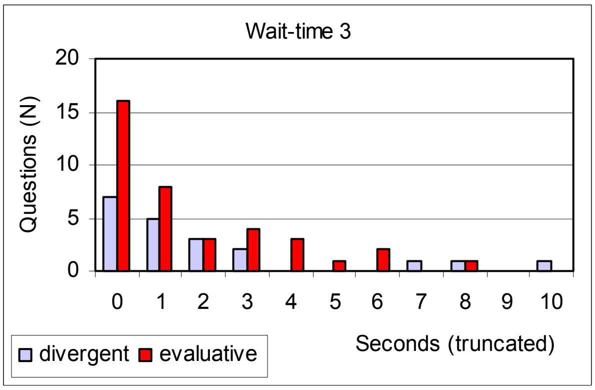


Figure 4 a-b: Wait time 3 for different types of questions.

[3] It makes no sense to consider percentages, because we have only 20 and 38 cases.

5.3 Wait time for different phases of the lessons

The last topic we will address in this article is the relation between wait time and different lesson phases. Table 4 gives an overview how many questions occur in each lesson phase.

N = 945	Frequency	Percentage
homework	202	21,4
repetition	94	9,9
new content	649	68,7

Table 4: Question per lesson phase.

The mean values for the Wait times 1-3 for each lesson phase is presented in Table 5.

Mean	Wait time 1	Wait time 2	Wait time 3
homework	6,4 s	4,1 s	2,3 s
repetition	4,5 s	3,9 s	2,2 s
new content	5,7 s	4,3 s	2,6 s

Table 5: Mean values for Wait time 1-3.

There is only one significant difference between the mean values of Wait time 1 for homework phases and repetition phases ($t = 2.155$, $df = 94$, $p = 0.034$). For Wait time 3 we have no significant differences between the mean values of the different lesson phases. This is in line with the distribution of questions with a certain Wait time 3 divided by different lesson phases (Figure 5). Except for the fact that there are some differences between repetition questions on the one hand and homework resp. new content questions on the other hand for the intervals [0,1) and [1,2), we can observe that the students have similar time for processing teacher question in all three lesson phases.

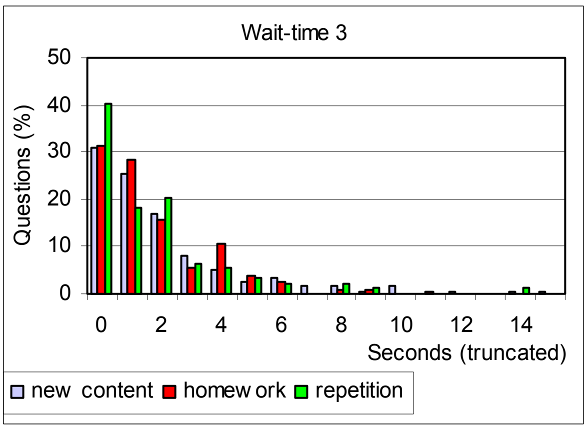


Figure 5: Wait time 3 for different lesson phases.

6. Discussion

Summarizing the presented results for the wait time in our sample we can more or less observe a similar situation as in the described studies of Section 3. If we consider Wait time 3, the time span between the moment when the teacher stops to talk and a student starts to response, then we get an overall mean value of 2.5 seconds (see Table 1). Figure 3c shows that in 75 % of the cases Wait time 3 is shorter than three seconds. Hence, we are able to replicate the findings of the three second threshold for our German sample. These results become more interesting if we take into account that the analyzed lessons are concerned with a challenging content related to reasoning and proof. Here a further study with lessons related to content like practicing might give more insight whether the wait time depends on the cognitive level of the content or not.

As Table 1 and Figure 3a show, in one third of the cases the teacher utterances do not end with a question but with additional information to a previous question. Here we may assume that the situation is more relaxed for the students, because this additional time for the students (Wait time 1) averages at 5.6 seconds. However, we have to mention that in many cases this additional information is necessary for the students to understand the question. Consequently, the extra time does not improve the situation in all cases.

Our findings for the wait time for different question types and different lesson phases indicate that there is no interdependency between the characteristics of a question or lesson phase and the wait time (cf. Sections 5.2 and 5.3). These results are not consistent with the findings of other studies (Boeck & Hillenmeyer, 1973; Arnold, Atwood & Roger, 1974, see Section 3.1). We assume that the teachers of our sample are not aware of the wait time after their questions. They do not use the wait time as an instrument in their teaching which may improve the quality of instruction and the learning opportunities for the students.

It seems that the teacher-student interaction and, thus, the wait time are dominated by routine actions of the teachers. The fact that even the content of the lessons does not influence these routines seems to be surprising. However, it is not surprising if we take into account that the German teaching style in the mathematics classroom is based on a certain script of the questioning-developing teaching style. Variations of this teaching style are comparatively rare.

If we add the small amount of time the students have to think about teacher questions to other empirical findings for these geometry lessons on reasoning and proof (cf. Section 4.2), then we may have another piece of a mosaic to explain the comparatively poor competencies of the involved students. However, our study reveals no empirical evidence that the wait time has a strong influence on the learning outcome of the students. Moreover, it is unclear, if a longer wait time is generally helpful for all students. It is possible that there are differential effects for low achieving and high achieving students.

Our study is based on a sample of 22 mathematics lesson from grade 8 in high attaining schools. Though it took a huge amount of working time to collect and prepare the wait time data, the sample is not representative. Since some characteristics of our lessons are in line with the results of the TIMSS video study for German mathematics classroom (see Section 4.2) we can assume that our sample reflects ordinary German mathematics lesson. Nevertheless, further research is necessary and our results need to be replicated.

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