Modeling of Local Strains in Ductile Fracture

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The concept of dividing microvoid coalescence (MVC) ductile fracture into three constituent processes, nucleation, growth, and coalescence, is discussed, with emphasis on needs for additional analytical and experimental work. Statistical and stochastic aspects of the problem are presented. Recent work on modeling of local strains during ductile fracture, and particularly as components of fracture toughness, is summarized and discussed in light of current knowledge of ductile fracture. Such local strain modeling is especially attractive because it permits micromechanisms of fracture to be explicitly included in the fracture model.

I. INTRODUCTION

THE phenomenon of ductile fracture encompasses a wide range of both types and processes of material behavior. It is of major technological importance because it is the preferred fracture mode in structural materials. It does not by any means always correspond to high values of fracture toughness, yet the implication of high (local) fracture strain is attractive to the materials engineer and comforting to the structural designer. Ductile fracture is also a complex and challenging problem in fundamental materials science, with a number of unsolved problems accompanying appreciable understanding of the phenomenology, as has recently been reviewed.[1] The present paper addresses ductile fracture which occurs by microvoid coalescence, particularly including recent work aimed at modeling the microscale and microstructural aspects of the process.[2,3]

As a general description of the microvoid coalescence (MVC) process, Figure 1 identifies the role of particles as nuclei, subsequent growth of holes by processes which can be regarded as macroscopically plastic, and finally the growing together or coalescence of these microvoids.[4] It is equally important to recognize that data exist to indicate that microvoid nucleation can occur in the absence of particles,[5,6,7] and that the growth phase can be terminated in several ways other than by void impingement,[1] as discussed below. Nevertheless, the dominant observations, over a wide range of materials, do tend to resemble Figure 1. The principal topics of this paper are the needs for a fuller description, both experimentally and analytically, of ductile fracture; second, statistical and stochastic aspects of the problem; and third, the issues in extending the understanding of ductile fracture, by means of local strain models, to quantitative estimation of fracture toughness.

II. DUCTILE FRACTURE: ELEMENTARY PROCESSES

The elements of ductile fracture are described here to clarify some of the issues which arise in modeling, issues which are discussed in somewhat greater detail in another review.[1] The idea of Figure 1, that ductile fracture can be regarded as occurring with a sequence of three "elementary processes", which are the nucleation, growth, and coalescence of microvoids, has been treated at some length in a series of previous reviews.[8-13] This division of the fracture sequence into elementary processes is convenient for discussion and for some analytical aspects, but it must be kept in mind, as is shown below, that actual fractures usually comprise multiple or overlapping processes over a wide range of strain, such as continued nucleation of new voids during growth of the previously-nucleated voids. Accordingly, most existing models for the elementary processes are necessarily considerable oversimplifications of reality. In particular, models for the elementary processes ordinarily cannot be used to describe the entire fracture sequence, even as an approximation.

A. *Microvoid Nucleation*

In most real materials, as mentioned in the Introduction, microvoids are largely nucleated at inclusions or other particles, as has been known since the pioneering work of Tipper.[14] Similarly, it has long been established that these microvoids then grow and coalesce to comprise the fracture event.[14,15] Subsequent work has elaborated this knowledge to a considerable extent.[10,13,16] A common mode of void nucleation is by decohesion of the particle from the matrix, and most evidence supports the view that interfacial strength is a dominant factor in such nucleation. A number of earlier workers had suggested that the critical parameter was the energy of the new free surfaces formed by nucleation of a microvoid. It now seems clear that this is a necessary, not a sufficient, condition for nucleation, and is often satisfied at or near yield.[10,13] Pivotal work on this topic was accom-

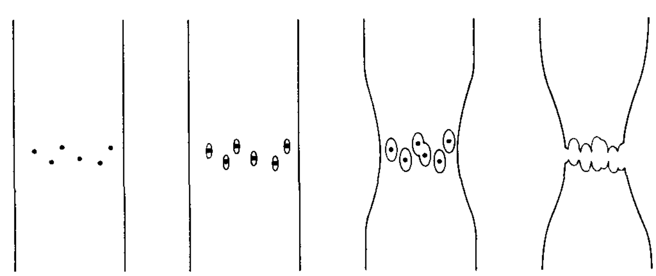

Fig. 1 — Schematic representation of ductile fracture process, left to right, from pre-existing particles or inclusions at left, to hole nucleation at particle poles with tensile stress, to transverse growth when necking (or other source of transverse stress) is present, to final fracture with half-voids or dimples on the two fracture surfaces (right).

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plished by Argon and co-workers,[17,18,19] who combined an extended Bridgman analysis with contributions from local stresses due to plastic flow. In addition, a method of using this relation for evaluation of the interfacial strength was devised by Argon and Im,[19] and has been used in a number of subsequent investigations.[20,21,22] It has been claimed by Fisher and Gurland,[23,24] however, that more detailed calculations show that the required elastic energy is not necessarily supplied near yield, which would restore the importance of an energy criterion in initiation.

There are a number of strong indications of the role of stress (and indirectly, of the role of an approximately constant interfacial strength for a given microvoid nucleus), as has been reviewed,[13,16] particularly the observations of Cox,[25] which have been discussed elsewhere.[1,13] There are, however, indications of a number of other factors which, though secondary and perhaps indirect compared to the interfacial strength, may still play a major role in special circumstances or when acting in concert.[10–13,16,18,19,26] These include particle size, primarily an experimental observation[10,13] but with some analytical[18] and explicit experimental[27] understanding; particle shape, which along with volume fraction is known to affect nucleation;[28] particle strength, in cases for which, as first shown clearly by Gurland and Plateau,[29] void initiation occurs by particle cracking instead of particle-matrix decohesion; particle location and distribution; state of stress, strain, or both;[1,30] interfacial structure and composition, as strikingly shown in recent work of Hippsley and Druce;[31] and particle-matrix bond strength, presumably related to structure and composition.[32]

B. *Void Growth*

The process of void growth often appears to be the province of mechanics analysts. The primary work here is that of McClintock,[33,34] which together with the refinements of Rice and Tracey[35,36] and Rice and Johnson,[37] makes clear the major parameters in this process. The first to introduce both matrix hardening and interactions among voids into these calculations was Needleman,[38] but the resulting equations are sufficiently complex that it is unclear how to apply them to experimental data. Moreover, the equations break down before coalescence is reached.[38] Although McClintock's treatment is simplified in a number of ways, subsequent work suggests that it appears to include the major parametric relations correctly. In particular, there is a very large effect of transverse (or hydrostatic) stress on void growth, as is frequently observed. It seems often to be overlooked, however, that this treatment substantially overstates the observed fracture ductility. McClintock, in a rarely-cited paper,[34] made it clear that he was aware of this, and Gurland[39] has presented a graphical comparison between the McClintock calculation and the well-known data of Edelson and Baldwin,[40] as shown in Figure 2. The reason for the overestimate (Figure 2) may involve termination of stable void growth by strain localization,[34] or for several other possible reasons, as discussed below.

Additional factors in void growth appear to be evident in some experimental treatments, but as yet have not had a detailed analytical treatment. These include the state of strain (independent of the state of stress);[10,13] nucleation strain, a topic of recent interest in the literature;[3,26,41] nucleation density; the matrix flow relation, or constitutive relation, for the hole-containing matrix (discussed below); and criteria for the termination of stable void growth, such as strain localization. Experimental work on void growth in recent years has been sparse, perhaps because the mechanics appears extensive if not complete, but additional work to address some of the issues listed above, on both the experimental and analytical fronts, is especially needed.

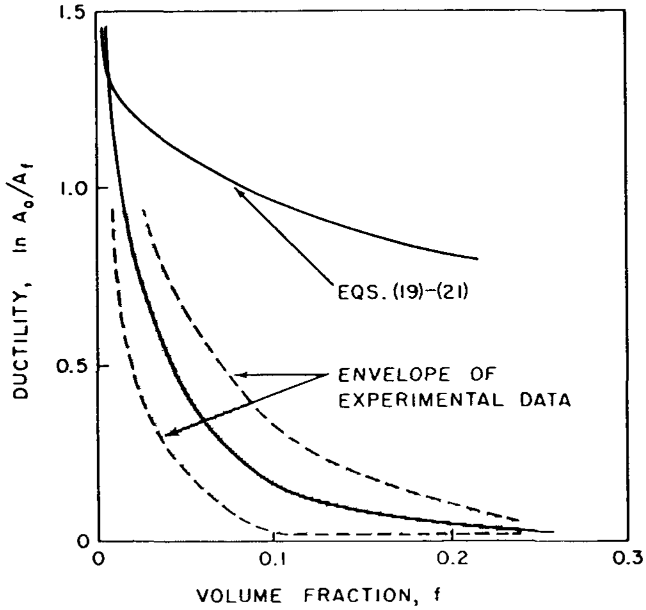


Fig. 2 — Comparison of McClintock's Eqs. [19] to [21], from Ref. 33, with the experimental data of Edelson and Baldwin.[40] Figure from Gurland.[39]

C. *Void Coalescence*

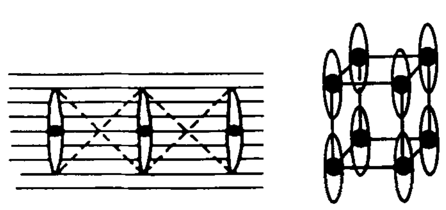
Coalescence is the third and final part of the MVC process, and in many older treatments it is neglected, or merely regarded as the moment of fracture. However, it is now recognized as a distinct stage, during which strong void-void interactions can occur, in which void growth as well as strain localization may be accelerated significantly, and which may take place in an extensively voided or porous matrix. This stage remains less well studied than the others, with no clear mechanics basis for description. Moreover, it is a difficult experimental subject. However, it is becoming clear that appropriately-stiff load trains for mechanical testing, or use of strain control (with extensometer strain measurement) in a closed-loop machine, can overcome these problems.

It is in any case already clear that void coalescence is a physically complex process.[10,13] It can take place by direct impingement of voids, that is, by voids growing until they touch, as envisioned in Figure 1 and elsewhere;[33–36] it can also occur by void sheet formation, as first recognized by Rogers.[9,42] Void sheets in turn can occur by profuse voiding in an existing slip band or strain localization. They can also occur if the growing void density can trigger localization as the voided matrix becomes less resistant to such an instability. There are even experimental indications[43] that both sequences can occur in a single specimen. But in either case, the coalescence process can arise in groups as well as between pairs of voids, and either detailed observation or description of the attendant phenomena are challenging and accordingly rare.

From the mechanics perspective, there are two important issues. First, it is essential to identify the appropriate matrix flow relation. Whether or not localization is already proceeding, the void density will often be high enough that the flow relation must be that for hole-containing material, sometimes called "Gurson" material after the originator of a widely-used constitutive relation for such a circumstance,[44] though that mechanics is still emerging.[45–48] There has been recent concern that the Gurson constitutive relation may not correctly reproduce the experimentally-observed stiffness of hole-containing materials,[49] potentially an important point. The second issue relates to the onset of localization. Localization by shear intensification, or possibly by adiabatic shear, has been recognized for some time.[50] It appears experimentally that the profuse voiding usually precedes, rather than follows, strain localization (as concluded in Cox and Low's work,[51] for example), but additional observations, as well as better mechanics criteria, would be welcome. Other aspects of the termination of stable void growth, which may apply to the discrepancy shown in Figure 2, are discussed below.

The literature now contains extensive analysis of the localization event. One simple, microscale criterion for the onset of localization, proposed a number of years ago, is that of Brown and Embury,[52] shown in Figure 3(a). The concept of Figure 3(a) is appealing, that when the void length is equal to the spacing, and 45 deg lines can be drawn between adjacent void tips, the material will be unstable against shear between the voids, *i.e.*, to intervoid localization, thus terminating the fracture process. One problem with Figure 3(a), as pointed out by Garber,[53] is that the material in three dimensions cannot experience instability under the same two-dimensional condition, as shown in Figure 3(b). A column of material remains among the voids which are at spacings equal to their length.

In addition to this conceptual problem, there have been several observations reported which contradict this criterion,[54] by showing more closely-spaced voids (or, equivalently, longer voids) than the proportions shown in Figure 3(a). An example, taken from the thesis of Park,[55] is shown in Figure 4, which shows the presumed stages of a process of combining individual microvoids into larger voids, or macrovoids. This research[55] demonstrates two important points. First, coalescence is not a single event in a real material, but occurs repeatedly and (in many cases) over a considerable range of strain. Second, microvoids in Figure 4 are quite long relative to their spacing, another example of appearing to violate the Brown-Embury criterion of Figure 3(a), possibly for the reason sketched in Fig-



(*a*) (*b*)

Fig. 3 — Termination of stable void growth by shear, according to the criterion of Brown and Embury. (*a*) Two-dimensional criterion[52] for termination, as void length becomes equal to spacing, permitting a 45 deg shear line to be drawn between voids. (*b*) Sketch of three-dimensional equivalent of (a), with "column" of material in center of cell. Figure from Garber.[53]

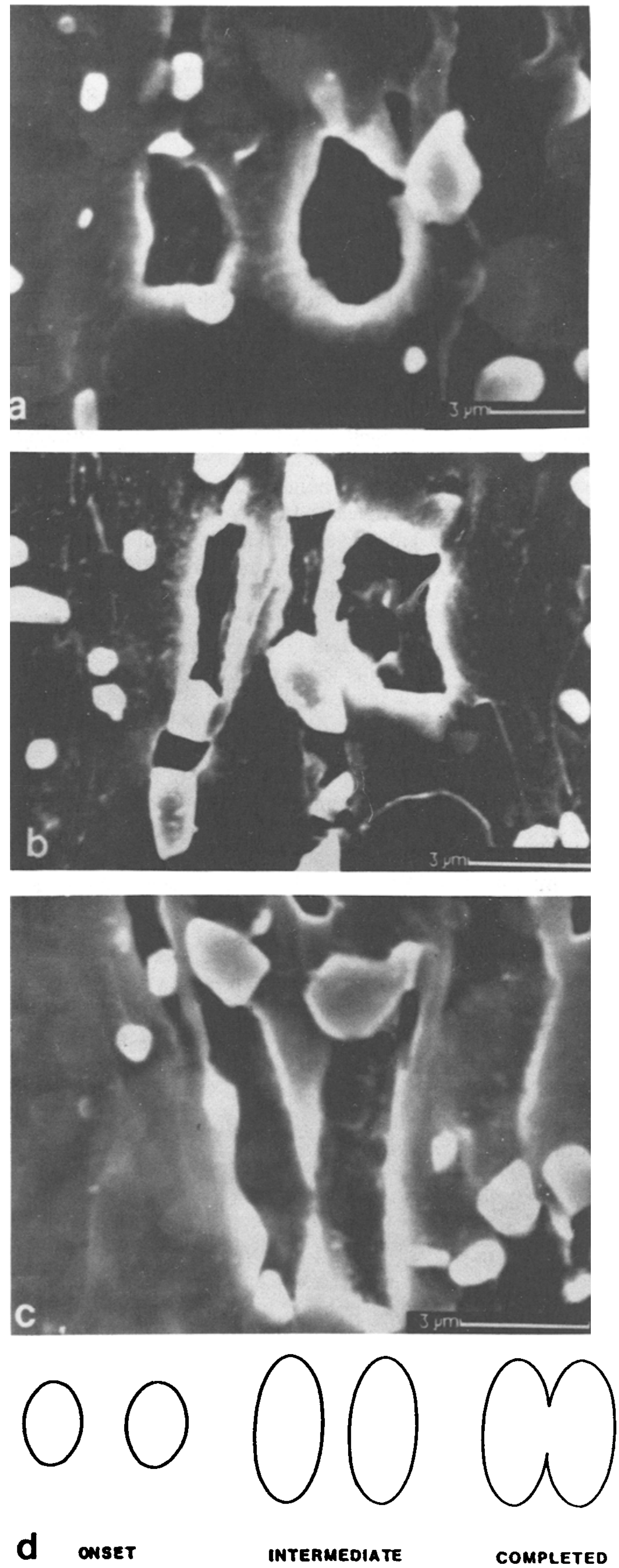


Fig. 4 — Scanning electron micrographs showing presumed stages of transverse microvoid coalescence. (*a*) Onset, (*b*) intermediate stage, (*c*) completion, and (*d*) schematic depiction of overall coalescence process. Figure from Park.[55]

ure 3(b). Additional work is clearly needed, on the topics of localization criteria, of general analytical descriptions of coalescence which can supplement preliminary efforts,[41,56–58] and of more extensive experimental observations of the coalescence process.

D. *The Strain to Fracture*

The foregoing description of the three processes proposed as components of the typical ductile fracture sequence is necessarily brief. The integration of these processes is illustrated by the topic of fracture strains, that is, the strain for all three processes together. In any such discussion, the most important data are those of Edelson and Baldwin,[40] on a variety of powder metallurgy materials with controlled volume fractions of various particles in copper matrices. A scatter band for those data is included in Figure 2. Although the original authors stated that their experimental materials exhibited a wide range of bonding strength between intermetallic or oxide particles and the copper matrix, in retrospect it is clear that all these particles were relatively weakly bonded. Indeed, all the data are consistent with the data for voids, prepared by incomplete densification of copper powder metallurgy compacts. It is also noteworthy that all the particles were at least as large as 5 μm diameter, so no information about truly small particles is contained in these data. There are indications[5,59,60] that particles smaller than 1 μm diameter have a smaller effect on ductility than suggested by Figure 2. In nickel containing 2 vol pct thoria, for example, Figure 2 predicts a fracture strain of about unity, while published fracture strain data[61] range from 1.4 to over 2. Nevertheless, the striking and marked dependence of fracture strain on volume fraction of particles remains as a profound result of Edelson and Baldwin's work, and a continuing benchmark against which other work is measured.

The point was made above, and should be reiterated here, that there is no simple basis on which data like those of Edelson and Baldwin,[40] or those of Gladman *et al.*[62] on sulfides in steels, comprising fracture strains, can be realistically compared to analytical expressions for the constituent processes of ductile fracture. Few if any instances of fracture occur solely as nucleation events; fractures which proceed from a virtually zero nucleation strain usually are not terminated by pure void impingement, and the coalescence process can occur only in voided or porous material. Accordingly, it seems unrealistic to evaluate any expression for one of the constituent processes by comparing it as in Figure 2, unless very special experimental conditions have been devised (and verified). On the other hand, efforts have been made to combine criteria, or to evaluate them separately, for the constituent processes.[26,41] These efforts deserve expansion and further development, although numerical approaches involving large numbers of free parameters[63] cannot be expected to provide either detailed physical understanding or predictive capability.

In principle, combined or integrated models should be feasible, but uncertainty about generalizing many of the factors mentioned above makes such a procedure difficult. Even to insert a single value for nucleation strain, for example, can serve to represent data more effectively, as in Figure 5 from LeRoy *et al.*[41] Yet many other studies, including a number cited above, make clear that nucleation would not be expected to occur at a single strain, even for a unimodal size distribution of a single particle type, making Figure 5 at best an approximation to physical reality. (This point was also recognized by LeRoy *et al.*[41] in presenting Figure 5.) One way of approaching this particular issue is to recognize the progressive nature of nucleation, as schematically suggested in Figure 6(a), from Embury,[64] in which a void nucleation "front" is imagined to pass through the particle size distribution, with the rate of movement of the front with plastic strain not explicitly specified. It should be pointed out that as that size distribution changes, the consequences of an increment of motion of the front can be quite different, as shown in the remainder of Figure 6, but details are difficult to specify without knowing either explicit movement rates or specific distributions.

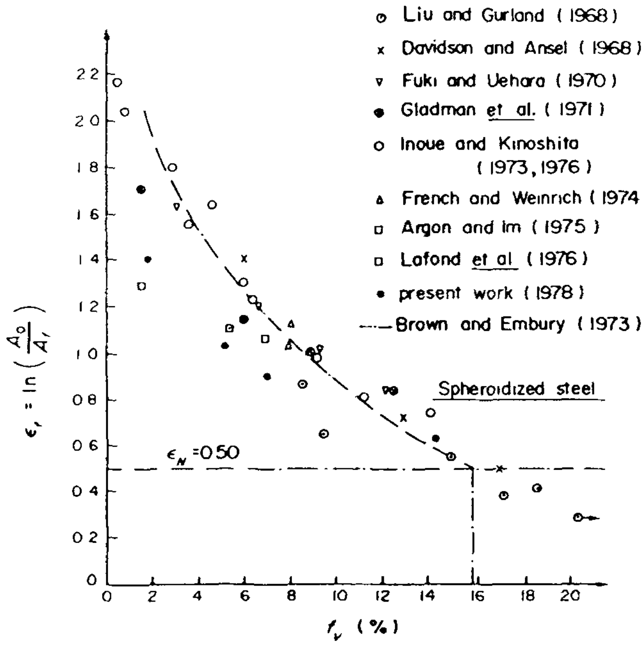


Fig. 5—Relationship between fracture strain and volume and fraction of carbides in spheroidized plain carbon steels, with nucleation strain of 0.50 indicated. Literature references given in original paper.[41]

One approach to collecting the behavior of the constituent processes into a unified model is to consider the behavior as the voids begin to coalesce on a macro scale into a crack or crack-like feature. In a tensile specimen, with relatively modest gradients of stress and strain even in a deep neck, this process can still be treated as a plastic one, as Figure 7 shows. But when a fracture mechanics perspective is taken, for behavior at the tip of a sharp crack, the problem can become a difficult exercise, compared to the tensile case, in states of stress and strain, as well as in questions of appropriate constitutive relations for material in the crack tip plastic zone, and in questions about the role of strain gradients and restricted plastic volumes in the plastic zone. However, given what is known about the role of stress and strain states in nucleation and growth, this is, at least qualitatively, a straightforward problem.

It would nevertheless be useful to have a careful, quantitative, and extensive experimental comparison of ductile fracture behavior for various specimen geometries, to verify that these expectations are reasonably accurate. The questions of constitutive relations and of the volume and strain gradients of the plastic zone (and of the appropriate values

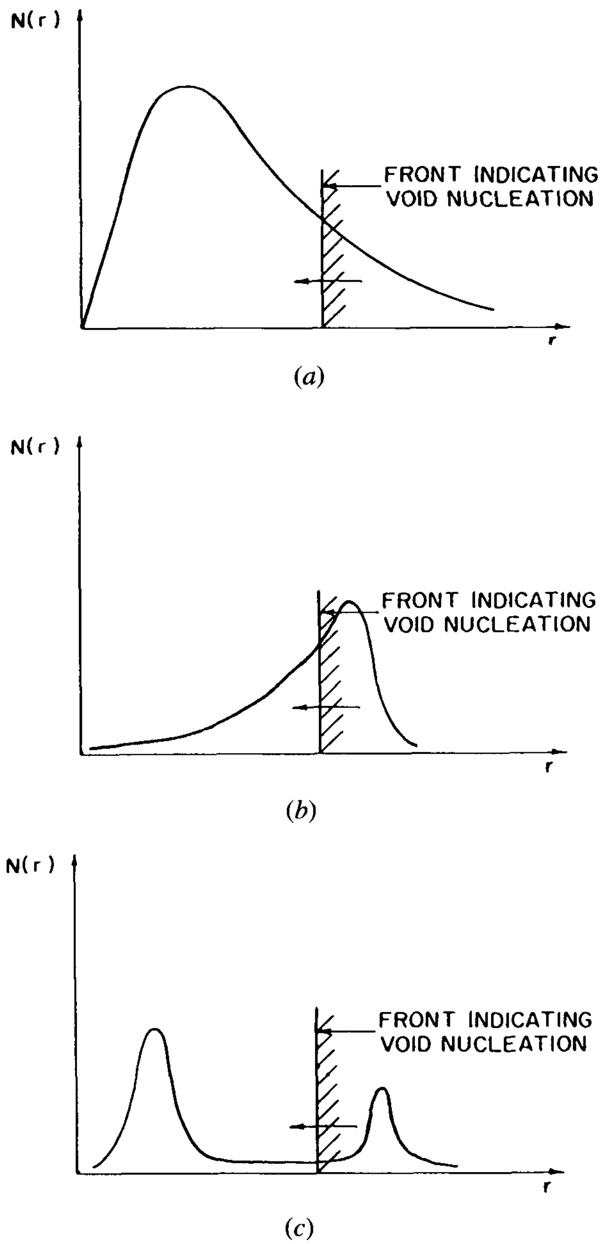


Fig. 6—Schematic depiction of nucleation process in ductile fracture, indicating progress of nucleation "front" through particle size distribution. (*a*) Suggestion of Embury.[64] (*b*) Alternative size distribution with opposite skewness, indicating implication of unit motion of nucleation front. (*c*) Motion effects in bimodal particle size distribution.

of quantities like local fracture strain) are much more open in a fundamental sense and thus potentially much more troublesome. It is beginning to be clear, however, that certain opportunities exist to make improved measurements (or estimates) of these local quantities. Those opportunities are discussed following a summary of some stochastic aspects of the ductile fracture process.

III. STATISTICAL AND STOCHASTIC ASPECTS

A significant barrier to both physical understanding and also modeling procedures is the existence of statistical and stochastic aspects of the ductile fracture process. Although these aspects are not the primary topic of this paper, it is

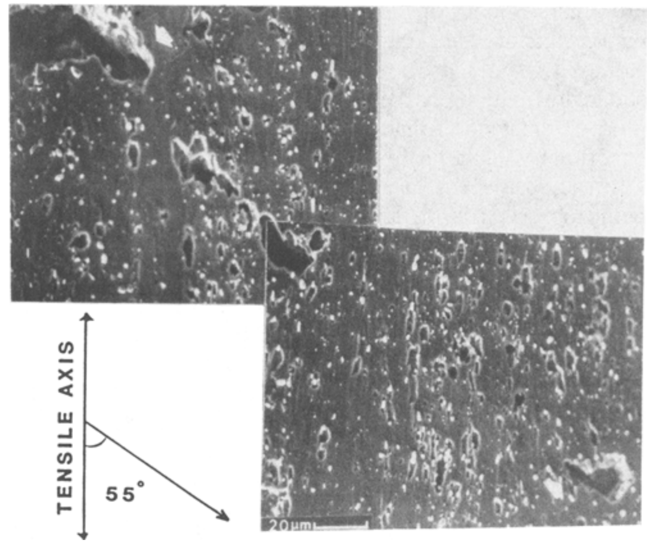


Fig. 7—Scanning electron micrograph of voids beginning to coalesce with main ductile crack (upper left) in tensile specimen of spheroidized plain-carbon steel. Average angle of high-density void regions to tensile axis is about 55 deg. Figure from Park.[55]

appropriate to mention them for completeness in describing needs for additional work. One of these aspects is the fact that all the microstructural parameters discussed above, such as particle size, shape (or aspect ratio), spacing, interfacial strength, and so forth, are in reality not single-valued parameters but exist in real materials as distributions of values. Moreover, these distributions in general are unknown, and typically are reported only as mean values, sometimes not even measured properly in quantitative metallographic terms. There do exist, however, some experimental determinations of such distributions, such as Tierlinck *et al.*'s apparent observation[65] of a log-normal size distribution for carbides in steel. They also discussed the effects on fracture of an oppositely-skewed size distribution. The problems of assessing experimental distributions are addressed further below, but it is encouraging that some such determinations are emerging.

Another aspect of this problem is that microvoid nuclei, generally particles, are not uniformly distributed in materials but instead are more nearly randomly distributed. The existence of a random or approximately random distribution means that the spatial distribution is significantly nonuniform, so that parameters such as the average particle spacing may not be descriptive of the physical behavior taking place between particles which are more closely spaced than the average. Such particles in turn can be the location of both the earliest nucleation and also the most rapid void growth in the material, and can readily dominate the fracture process. Comparison of experimental results for such materials, with models based on uniform void distributions, can clearly be subject to systematic and in some cases considerable error.

An analytical approach to this problem is emerging, based on dividing volume into cells surrounding random (or nonrandom) points, which are called Voronoi polyhedra or Dirichlet cells.[66–69] It is still difficult to conduct a good statistical test of whether observed particle spacing distributions conform to particular theoretical distributions, but approximate conclusions can be drawn.[68,69] Many of the sparse data in this area are unilluminated with statistical analysis. Robust tests of the form of a distribution, such as a chi-square test, are rarely if ever applied, and in any case require more extensive data than are generally available. Moreover, experimental data of this kind frequently contain sufficient intrinsic scatter that a cogent decision on the distribution being observed is difficult. An alternate means of testing distributions[70] is to plot the data in rank order on various probability papers (normal, log normal, Weibull, *etc.*) and examine the goodness of fit. For other distributions, similar procedures are available.[70,71] There has been a certain amount of loose talk in the literature about particle spacings obeying Poisson distributions, for example, but to the author's knowledge, there is no case in which a rigorous statistical test has been applied to support such a description of experimental observations. The Poisson distribution has a few pitfalls for the unwary in any case, for example its variance being equal to its mean, contrary to the assumptions of normal distribution procedures involved in t statistics and variance and regression analysis. Another potentially sensitive issue is the degree of skewness of any particular observed distribution, which can complicate analysis. On the experimental side, there do exist extensive quantitative metallographic procedures for spatial distributions.[72,73]

What may be the effects of nonuniform particle spacings on ductile fracture? The intuitive conclusion might be that the nonuniform distribution, having some particles much more closely spaced than the average, would exhibit a lower fracture strain than that of a uniform distribution. Such a conclusion has some support in the literature.[74,75] However, this point may depend sensitively on the work-hardening capacity of the matrix. The intuitive conclusion may be appropriate for low hardening situations, but in the presence of high work-hardening, the closely-spaced particles cause early formation of zones of intense strain, which can then shield nearby regions. The nearby regions, in turn, have higher than average spacing. The result could then be higher fracture strains with the random, nonuniform distribution. This latter point, of higher strain with a nonuniform void distribution, was in fact the conclusion of a series of computer simulations by Melander,[76,77,78] although only a general comparison with experiment seems to have been performed.

These issues of the spatial distribution become of particular importance in the coalescence stage of ductile fracture, since any nonuniform distribution implies local void impingement which is a function of location (without considering strain localization or other means of terminating stable void growth). It is well known that extensive connectivity of voids can occur well before the impingement of the "average" void spacing,[79] an example of which is shown in Figure 8. A more general description of the development of Figure 8 is the calculation described in Figure 9, which shows the actual volume occupied by randomly located holes which touch, at a particular volume fraction of holes.[79] Other treatments are possible with the methods of percolation theory, or with a Dirichlet tesselation with adjustments for void growth. Additional work in this area seems badly needed, in order for experimental coalescence and fracture strain observations to be compared to a realistic analytical criterion, on the basis of actual particle distributions.

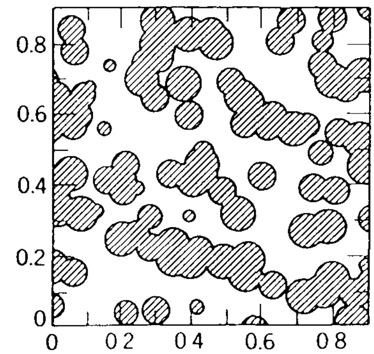


Fig. 8 — Cross-section through a material containing a random distribution of spheres in three dimensions, with sphere volume fraction 0.5. Each sphere on average overlaps or touches 5.5 other spheres, and only 0.4 pct of the spheres touch no others. From Haller.[79]

Especially for spatial distributions, much remains to be learned in both analysis and experiment, although many of the tools for this work are in place. Careful description of distribution type, width or variance, and skewness, may well play a role in ductile fracture models, and deserves further investigation.

IV. LOCAL FRACTURE STRAINS

There is a long history of interest in this topic, which will not be reviewed here in light of several extant treatments.[1,8,13,80-82] Much of this work has focused on "process zones", following the work of Krafft,[83] although such an approach seems unable[1] to address issues of microscale plasticity or of microstructural effects. A more promising approach, for several reasons, is the "characteristic distance" concept of Ritchie, Knott, and Rice, or RKR.[84] Here the assumption is made that the fracture events on a local scale within the plastic zone must occur at some critical stress which is present at a critical distance ahead of the crack tip. That distance is the characteristic distance. An illustration of the idea is shown in Figure 10(a), which includes the notation of Ritchie and Thompson's review.[2] Figure 10(a) is for cleavage fracture nucleated by cracking of grain boundary carbides, and the RKR result was that the characteristic distance was about two grain diameters. That distance was presumed to arise as a consequence of the need for a suitably sized and oriented carbide to be within range of the crack tip. By using concepts developed by McClintock[33] and MacKenzie, Hancock, and Brown,[85] the same RKR concept can be extended to ductile fracture,[2,86] as shown in Figure 10(b). Thus the same approach to fracture using a characteristic distance can be taken for both brittle and ductile fractures. Analysis of "micromechanisms" of fracture, that is, mechanisms which operate on the scale of the materials' microstructure,[2,80] can proceed with additional information about that scale.

Any equation developed in this way will have the same conceptual form, with the toughness expression (for generality in terms of the J integral[2]) including a distance and a fracture strain:

$$J_{Ic} \sim \sigma_0 \bar{\varepsilon}_f l_o^*$$

where σ_0 is the local flow stress, $\bar{\varepsilon}_f$ is the equivalent local fracture strain, and l_o^* is the characteristic distance. The problem with this approach is that the local fracture strain

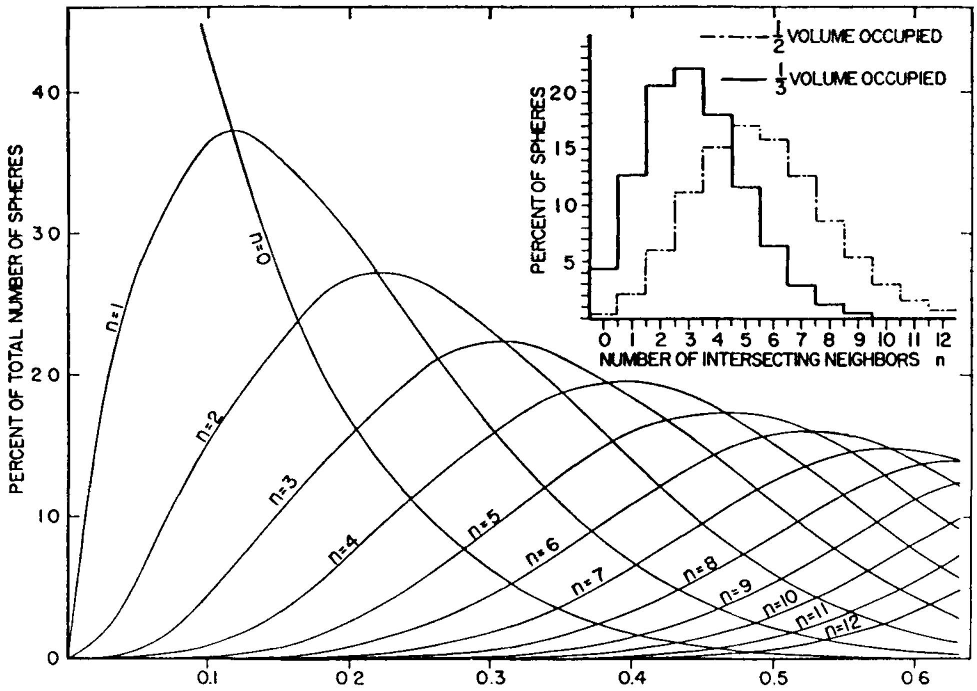


Fig. 9—Relation between the frequency (pct) of spheres with a given number n of intersecting neighbors, and the volume fraction (occupied volume) of these spheres, for Poisson-distributed spheres in three dimensions. To expand on Fig. 8, for example, at a volume fraction of 0.33, the mean number of touching or overlapping neighbors is 3.2. From Haller.[79]

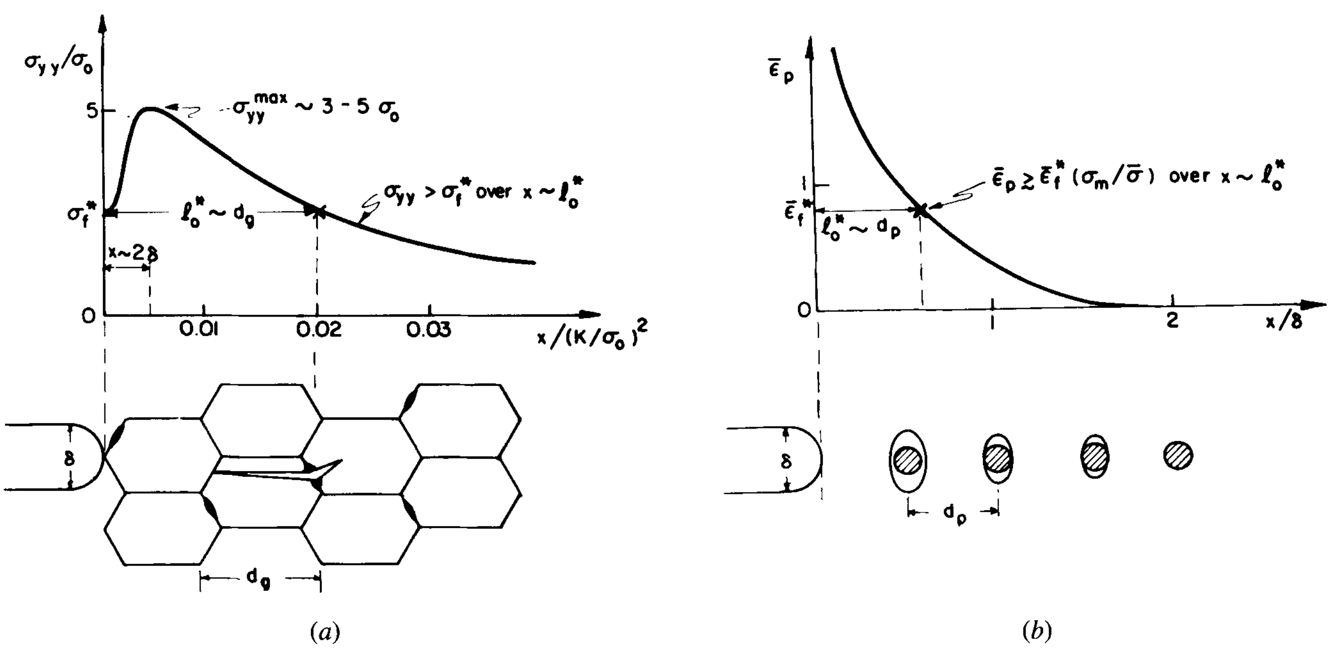


Fig. 10—Schematic depiction of microscopic fracture criteria at crack tips (*a*) Original RKR proposal for cleavage under stress control, as discussed in text. (*b*) Stress-modified strain-control model for microvoid coalescence. From Ritchie and Thompson.[2]

need not be, and generally will not be, the same as the conventionally-measured fracture strain in smooth-bar tension or plane-strain tension. One approach to this problem is presented in the following section.

Before considering recent models for local strains, it is worth mentioning that the fracture events at a crack tip may not occur in the same way as in a geometrically simpler specimen such as a tensile specimen. For example, under conditions of little or no triaxiality and ample shear stresses, a particular material may fracture by strain-controlled MVC processes, while under the triaxial stresses, plane strain, and limited shear stress conditions at a crack tip, fracture may occur by a different (usually more brittle) mode, such as cleavage, which also is typically a stress-controlled fracture mode. Discussion of this point in detail is beyond the scope of this paper (it has been addressed elsewhere[30]). It is an important point to keep in mind, however, particularly when tensile-test information is used to model process zone events. It should be evident that such a procedure is risky unless detailed information is available about crack tip fracture processes.

Modeling of Local Strains in Ductile Fracture

Development of local strain models has been reviewed elsewhere[1–3,82] and only will be summarized here. The essential concept is that the local plasticity around void-nucleating particles should preserve a record of the post-strain initiation, as the microvoid lengthens in the loading direction. Thompson and Ashby[87] analyzed this problem with the use of the fracture surface "microroughness" parameter M suggested earlier,[26] following considerations of quantitative metallography (remembering that the fracture surface in general is neither planar nor necessarily a random section). When toughness is experimentally measured, the characteristic distance is then obtained as the "free" parameter in the toughness expression. Additional work by Garrison and Thompson[3,82,87–89] has extended this idea.

Void growth as a measure of local strain accumulation is based on the calculations of McMeeking,[90,91] who used Rice and Tracey's results[35] to construct a relationship between the crack tip opening displacement δ and the distance of a void-nucleating particle from the crack tip, X_0. The dependence of void dimensions on the ratio of δ/X_0 is shown in Figure 11. This figure essentially is a prediction of observations on voids, whether in sectioned specimens or on the fracture surface, in connection with a measure of toughness, δ, and the particle spacing, which may be taken as X_0. Of particular interest is the linear relationships displayed, for both R_y and R_x. In general, R_x is more convenient to measure on fracture surfaces.

To make experimental comparisons, Garrison[89] then averaged R_x and R_z, the void dimensions in the projected plane of fracture, as R_v, the void radius. Then, following Rice and Johnson[37] and treating X_0 as the nearest-neighbor distance among particles in three dimensions, one would expect the critical crack tip opening displacement δ_{Ic} to scale as the product of X_0 and (R_v/R_i), where R_i is the initial void size, identified with the particle radius $d/2$. δ_{Ic} in turn is related to J_{Ic} as $\delta_{Ic} = d_n\ (J_{Ic}/\sigma_0)$, where d_n is a function of yield strain, work-hardening exponent n, and whether plane stress or plane strain conditions obtain, while σ_0 is the flow stress, *e.g.*, the average of yield and ultimate strengths. Measurements on a variety of tough steels, in various microstructural conditions, have been made and are shown in Figure 12. Included are the McMeeking calculations from Figure 11 for comparison. It is evident that these steels display amounts of void growth R_v/R_i and toughnesses δ_{Ic} well in excess of the range of values calculated by McMeeking, but appear to retain a linear relation in this plot. More work is certainly of interest on this topic, and is continuing.

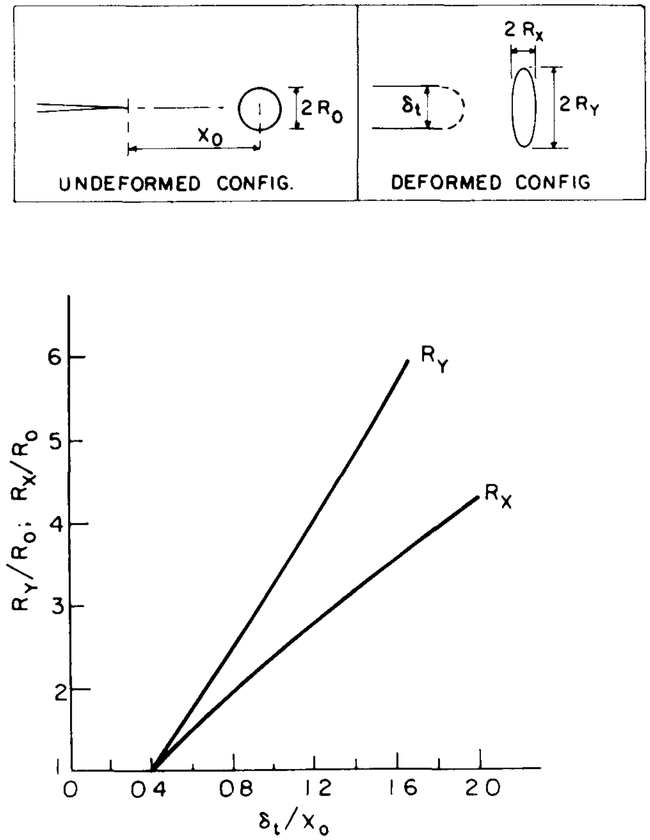


Fig 11 — Relation between void growth and δ_t/X_0, from McMeeking's calculations. Figure after Garrison.[89]

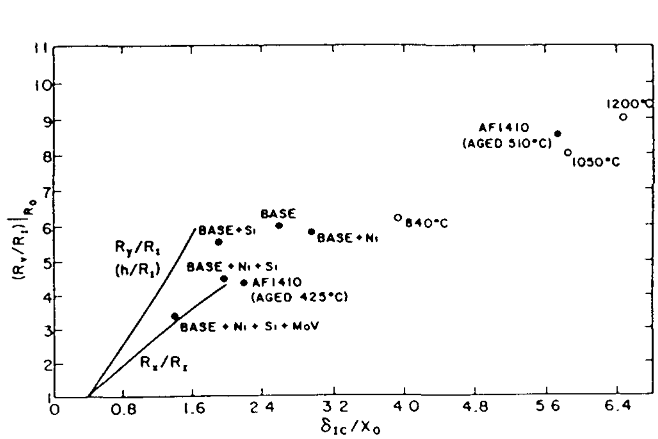


Fig. 12 — The calculated critical crack tip opening displacement, δ_{Ic}, as a function of the spacing and void growth parameters, expressed as described in the text Data are for 4340-type steels ("BASE") with alloying modifications, the 9Ni-4Co steel HP 9-4-20 (open circles), and the 14Ni-10Co steel AF1410 (unpublished data of Garrison). Also included are the McMeeking results from Fig. 11.

The criticism is sometimes voiced that this fractographic approach to local strains integrates the entire fracture process and therefore is not very informative about details, and moreover that it neglects the fact that different portions of the specimen may fracture differently. The first point is quite true, although for the kind of toughness relation in which only a strain appears, it seems clear that the fractographic strain is an appropriate, if not the appropriate, strain. It should be obvious that to obtain information about the constituent processes of ductile fracture, other experimental techniques are necessary, many of which are described above or elsewhere.[1–19,65,84–89]

Regarding the second point about positional dependence, this is also correct, has been recognized explicitly from the outset,[26,87] and continues to be a feature of experimental work. For example, local void shapes and sizes have been found to vary with distance from the end of the pre-crack, and from centerline to side surface, in compact tension specimens, as would be expected from the original suggestions.[26,87,88,92] If anything, this positional dependence appears to be a strength of the approach, since one can in principle determine local strains over a wide variety of stress and strain states in a single specimen, as well as examine local microstructural dependences of the fracture processes. But until additional data are accumulated, more detailed descriptions of these aspects must be deferred.

It may be appropriate here to mention the fractal approach[93] to description of fracture surfaces, which has had much recent attention. As has been pointed out elsewhere,[94] the problem with fractal analysis is that even changes in fracture surface appearance which a fractographer would call major, such as a transition from ductile MVC to brittle intergranular fracture, appear to change the fractal dimension only very slightly,[95] and moreover do not specifically identify the change other than as a change in local roughness. The appearance of even a few intergranular facets on an otherwise MVC fracture, for example, would be a vivid signal of an impending change in fracture mode, and unless this is evident in fractal analysis, few fractographers will be sympathetic to such an approach. It is encouraging to see efforts underway to modify the fractal concept to reflect fractographic needs,[95] but it is still unclear whether fractal analysis either provides information not readily available otherwise, or can realistically represent significant fracture surface changes.

V. CONCLUDING REMARKS

The purpose of this paper has been to present a summary of what is known, as well as what seems less well known, about the process of ductile fracture. It seems clear that both analytical and experimental work is badly needed in several topic areas within the constituent processes of nucleation, growth, and coalescence of microvoids. Statistical and stochastic aspects have been relatively neglected. Analyses which explicitly address the problems of distributions, rather than single values, of parameters like size, shape, and distribution are very desirable, but will likely be quite complex. Modeling of local strains at crack tips is an active and interesting topic of current research, although much remains to be done in this area as well.

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