Calculations of Forming Limit Diagrams for Changing Strain Paths

ALEJANDRO F. GRAF and WILLIAM F. HOSFORD

The method of calculating the shape of forming limit diagrams (FLDs) using a high-exponent yield criterion with the Marciniak and Kuczynski (M-K) analysis has been extended to include the effects of changing the strain paths and applied to aluminum alloy 2008 T4. Calculations incorporating abrupt path changes agreed with the general trends found experimentally. If the first stage of strain is under biaxial tension, the subsequent FLD shifts to the right and down with respect to the original FLD, whereas it shifts to the left and up when the first stage of strain is in uniaxial tension. Calculations introducing gradual strain-path changes, characteristic of stretching over a hemispherical dome, predict that the minimum of the FLD shifts to the right.

I. INTRODUCTION

SINCE the article by Marciniak and Kuczynski[1] (M-K), there have been many analytical predictions of forming limit diagrams (FLDs) following their approach.[2–11] The M-K analysis assumes that an initial defect in the sheet, in the form of a long groove, grows and eventually fails during stretching along a linear strain path in the surrounding material. The M-K calculations have been quite successful in predicting the left-hand side of the FLDs. However, there are discrepancies between the shapes of the experimental and calculated FLDs for the right-hand side (RHS), with most predictions showing a strong dependence on the value of the strain ratio R, which is not observed experimentally.[12] Sowerby and Duncan,[6] and later Chan,[8] explained the influence of the yield function used on the shape of the predicted FLDs. Lian *et al.*[9] showed that the shape of the FLD is affected by the ratio $p = \sigma_p/\sigma_b$, where σ_p and σ_b are the major principal stresses for plane-strain and biaxial tension, respectively. Hill's 1948 yield criterion[13] predicts that p changes quite drastically with R. Recent articles by the authors[10,11] showed good agreement of calculations using a high-exponent yield criterion[14] in the M-K analysis with experimental FLDs for different materials.

During an actual forming operation, a material element may undergo considerably large changes in strain path, and these changes can significantly alter the forming limits. An experimental study of the effects of such path changes on the FLDs of Al 2008 T4 was reported in a companion article. The ability to include path changes in FLD calculations is important, because the number of potentially significant changes is too great to be thoroughly covered by experiments and because calculations allow general trends to be explored over a large range of variables. There have been a number of calculations using the M-K analysis of the effect of strain path changes on forming limits.[2,3,7] Unfortunately, these were all done using Hill's yield criterion, and the results exhibit the same R-value dependence mentioned before.

This article discusses the incorporation of abrupt as well as gradual strain-path changes into FLD calculations using a high-exponent yield function. The calculations are applied to Al 2008 T4 alloy and compared with experimental FLDs.

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II. THEORETICAL ANALYSIS

A. *Calculation Method*

The RHS of the FLDs was calculated with the M-K analysis, modified to incorporate strain-path changes. It is assumed that there is an initial imperfection in the form of a long groove oriented perpendicular to the direction of the largest stress (Figure 1). Although an infinitely long trough is unrealistic, the results of finite element method (FEM) calculations with finite grooves are nearly indistinguishable from those assuming infinite grooves if the level of FLDo is held constant.[8] The computation method was similar to that previously used by the authors[10,11] in which a high-exponent yield criterion is incorporated into the M-K analysis. It was assumed that the material is characterized by planar isotropy and normal anisotropy. The yield criterion is

$$(\bar{R} + 1)\bar{\sigma}^a = \sigma_1^a + \sigma_2^a + \bar{R}(\sigma_1 - \sigma_2)^a \quad [1]$$

with $a = 8$ for fcc materials.[17] Strain hardening is described by

$$\bar{\sigma} = k\bar{\varepsilon}^n \quad [2]$$

The initial imperfection can be characterized by the ratio of the initial thicknesses:

$$f_0 = t_{0\,\text{groove}}/t_{0\,\text{bulk}} \quad [3]$$

It is assumed that the strains parallel to the groove are equal inside and outside the groove, and that the strain ratios outside the groove remain constant during prestraining and after the strain-path change. The calculation is done incrementally. For every step, a strain increment ε_1 is imposed to the groove, and the rest of

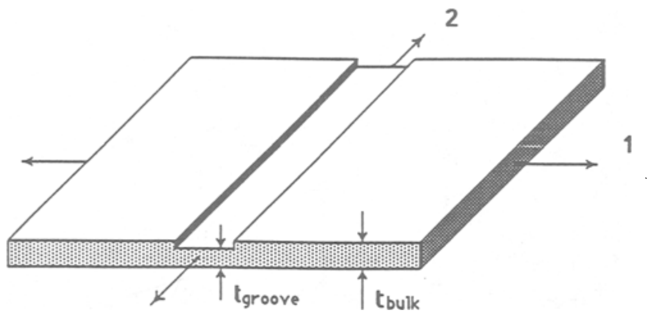

Fig. 1—Schematic illustration of a sheet with pre-existing groove.

the strain increments are calculated to satisfy the geometrical conditions and the force balance across the groove. In the calculations, equilibrium was expressed as

$$\sigma_{1\,\text{bulk}} - f\sigma_{1\,\text{groove}} \leq \text{tolerance} \qquad [4]$$

where f is the current thickness ratio $t_{\text{groove}}/t_{\text{bulk}}$.

The tolerance value determines the precision of the calculation; smaller tolerances result in more precise calculations but with longer computation time. The final results do not depend heavily on the tolerance which, in these calculations, was set at 10^{-7} times the strength coefficient k in Eq. [2].

B. *Abrupt Path Changes*

Only final strain paths for which $\varepsilon_2 \geq 0$ were considered. For these, the final failure is normal to the direction of the major stress, so the orientation of the groove is normal to the 1-axis and does not change during either the initial or the final straining. Consequently, only the relative thickness change of that groove was calculated. This made it unnecessary to calculate during prestrain the evolution of grooves for many different orientations, to calculate the resulting FLDs for each one of them, and finally to select the lowest FLD, as done by Barata da Rocha *et al.*[18] This simplification proved to be valid for most of the cases reported in the companion article.[15] That is, localized necking occurred normal to the direction of the major stress for strain states on the RHS of the FLD. The only exception was for samples with high levels of prestrain in uniaxial tension parallel to the rolling direction (RD) subsequently stretched near balanced biaxial tension. In these, fracture occurred at an angle close to 75 deg from the RD.

Neglecting the tolerance value, Eq. [4] can be transformed into

$$\sigma_{1\,\text{bulk}}/\sigma_{1\,\text{groove}} = f_0 \exp\left(\varepsilon_{3\,\text{groove}} - \varepsilon_{3\,\text{bulk}}\right) \qquad [5]$$

The simultaneous solution of Eq. [1], [2], [3], and [5] along strain paths corresponding to the pre- and final strains is done incrementally using the numerical method outlined in Reference 2. When the desired prestrain level is achieved, the current thickness ratio f and strain states for the groove and bulk are used as starting parameters for the final strain; the material will have different properties according to the severity of the prior deformation. The FLD is then obtained by imposing 10 strain paths ranging from plane strain to equibiaxial strain. Localized necking or failure is assumed when $\Delta\varepsilon_{1\,\text{groove}}/\Delta\varepsilon_{1\,\text{bulk}} \geq 10$ or when the force equilibrium cannot be achieved even for $\Delta\varepsilon_{1\,\text{bulk}} \rightarrow 0$. This latter situation is not uncommon when complex strain paths are imposed; *i.e.*, after a large prestrain in equibiaxial strain, which tends to stabilize the deformation by delaying the necking process, the material will most likely neck when the first strain increment is in plane strain.

C. *Calculated Results*

Figures 2 through 4 show calculated and experimental FLDs corresponding to three types of experiments with abrupt path changes described in the companion article.[15] For this alloy, n was 0.25 and $\bar{R}$ was 0.78. For all the calculations, the value of f_0 in Eq. [5] was 0.992, chosen to match the experimental FLD of the as-received material at plane strain ($\varepsilon_2 = 0$). The FLDs calculated using $a = 2$ in Eq. [1], which reduces to Hill's 1948 anisotropic yield criterion, are plotted for comparison. Comparison of experimental and calculated curves shows that in general the calculated curves lie above the experimental ones. Several factors which may contribute to this are as follows: the negative strain-rate sensitivity was neglected in the calculations; the calculations assumed linear strain paths outside the groove, whereas in the experiments, the paths were curved (Section D); and perhaps the limits were controlled by the occurrence of fracture in the groove before the assumed failure condition $\Delta\varepsilon_{1\,\text{groove}}/\Delta\varepsilon_{1\,\text{bulk}} \geq 10$ was reached (*i.e.*, calculations assuming a maximum thickness strain of 0.6 inside the groove predict a decrease of the forming limits close to equibiaxial tension). Nevertheless, the calculations reproduce the general trends.

The model cannot predict the differences found between experimental FLDs after prestraining in uniaxial tension along different directions; *i.e.*, the level of FLDs obtained after a certain amount of uniaxial prestrain is different depending on whether the axes of major strains during prestrain and final testing were normal or parallel to the RD. The same can be said for the predicted FLDs after balanced biaxial prestrain, in which the calculated positions of the minima follow a constant effective strain line, but the experiments deviate from this trend for high prestrain levels. Part of the differences can be attributed to the fact that the material is deformed during pre and final straining with different methods (in-plane stretching followed by punch stretching). Also, the model does not take into account the occurrence of ridging parallel to the RD[5] or the effects of strain reversal and the associated transient hardening effects.[19,20]

D. *Gradual Path Changes*

It is known that strain paths during stretching over a dome are gradually curved toward plane strain.[21] Figure 5 shows strain paths measured on specimens from two lots of low-carbon steel stretched over a hemispherical punch of 101.6-mm diameter. To investigate the effect of such curvature on the forming limits, FLDs were calculated, with the bulk material (outside the groove) assumed to undergo gradual strain-path changes instead

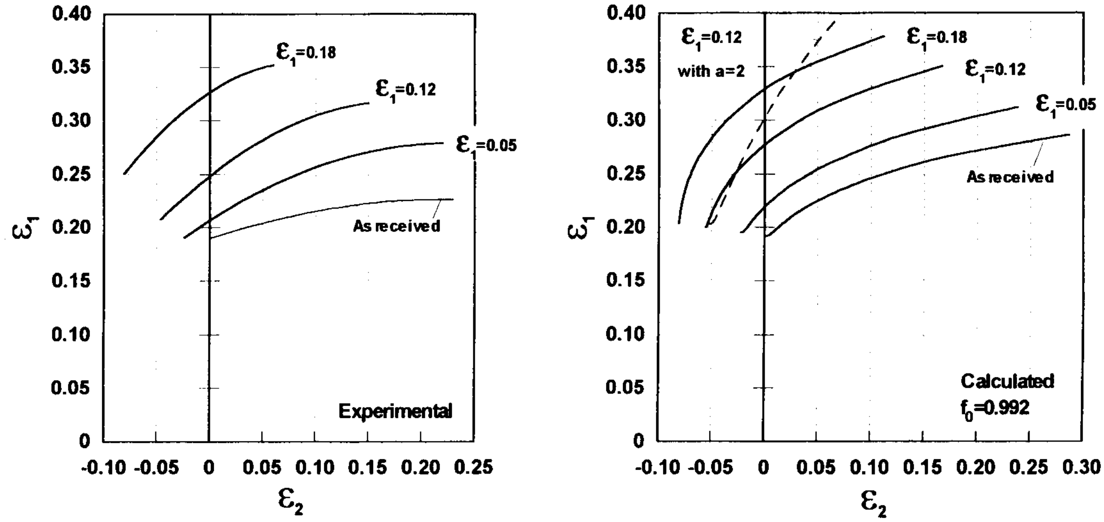

Fig. 2—Calculated and experimental FLDs for Al 2008 T4 prestrained to different levels in uniaxial tension.

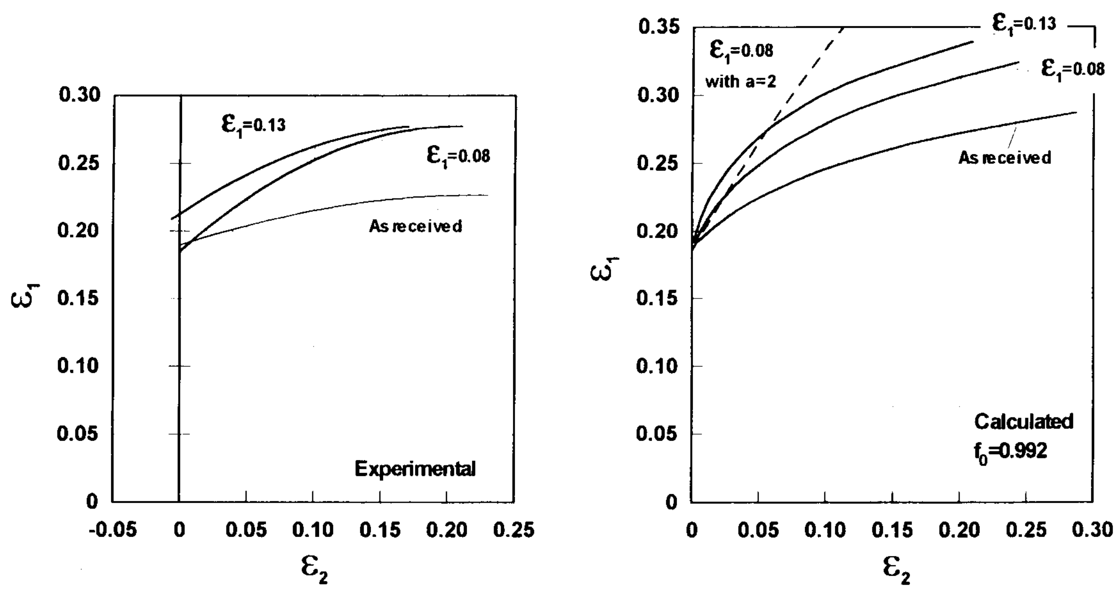

Fig. 3—Calculated and experimental FLDs for Al 2008 T4 prestrained to different levels near plane-strain tension.

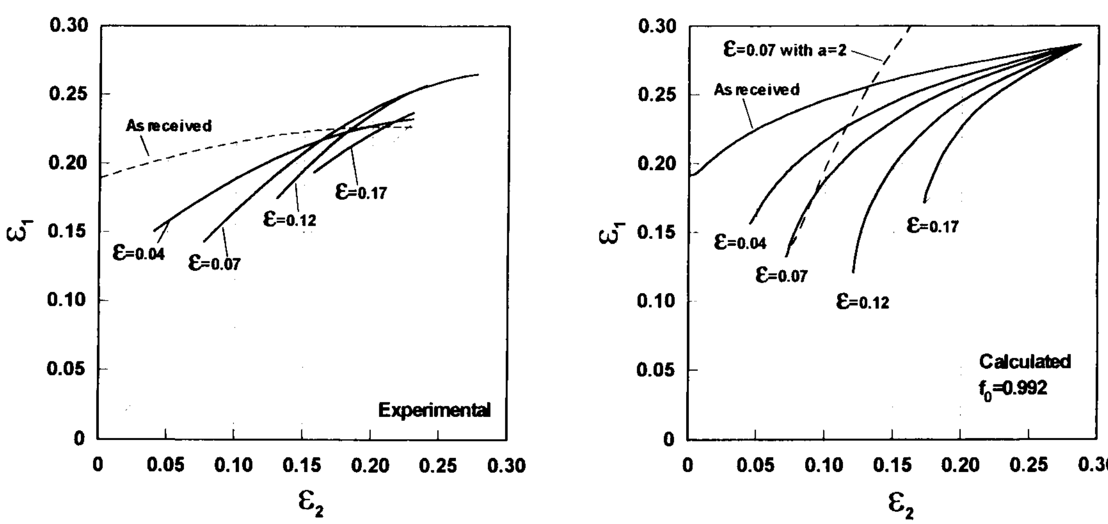

Fig. 4—Calculated and experimental FLDs for Al 2008 T4 prestrained to different levels in balanced biaxial tension.

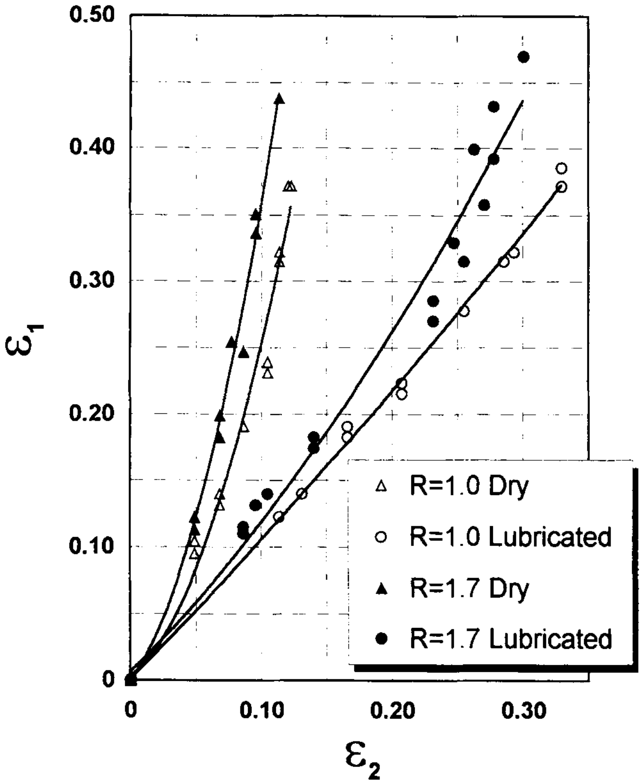

Fig. 5—Strain paths obtained in dome stretching of two low-carbon steels with two lubrication conditions.

of linear straining. Curved paths were introduced into the calculations by assuming that straining initiates in equibiaxial tension, as the sheet conforms to the punch in the initial stage of the deformation, and then departs from that condition with different curvatures. At each step of the deformation for a given strain path, the stress ratio is adjusted such that

$$\alpha^{i+1} = \alpha^i - c(\alpha^i)^8(\alpha^i - \alpha_{ps})\Delta\varepsilon_1 \qquad [6]$$

where α^i and α^{i+1} are the stress ratios for the current and next strain increment, respectively, α_{ps} is the stress ratio required to produce plane strain, and $\Delta\varepsilon_1$ is the strain increment in the groove. Different strain paths are generated by changing the value of c, with higher values resulting in paths closer to plane strain (evidently $c = 0$ generates a linear strain path). The general shape of this equation is such that it produces a high curvature at the beginning of the deformation (when $\alpha \approx 1$) and nearly linear strain paths for $\alpha^i \rightarrow \alpha_{ps}$. The exponent 8 was chosen to fit the overall shape of the strain paths of Figure 5.

Figure 6 shows calculated FLDs for linear strain paths and for curved paths generated with Eq. [6]. The calculations predict a lower FLD for punch stretching (curved paths) than for in-plane stretching (linear paths). This is not in agreement with experimental observations,[22,23] which suggests that other factors may come into play. It is interesting that the calculations predict a minimum in the FLD to the right of plane strain, in accordance with many experimental FLDs for steel.[24]

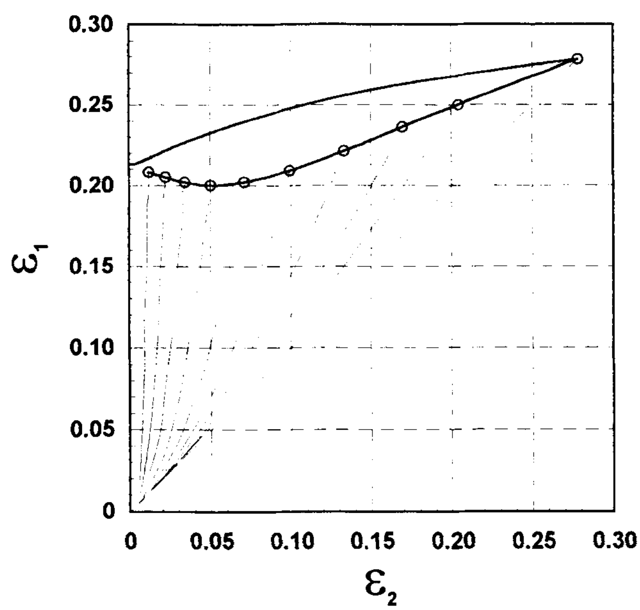

Fig. 6—Calculated FLDs following linear and nonlinear strain paths. Note the shifting of the minimum to the right for the FLD for nonlinear paths.

CONCLUSIONS

1. The FLDs have been calculated for changing strain paths using the M-K analysis with a high-exponent yield criterion.
2. The calculations reproduce the general trends of the experimental results for abrupt path changes, although not all curves match exactly. The agreement between experiments and theory is poorest for biaxial prestraining.
3. The calculations with curved strain paths, introduced to simulate punch stretching, predict the minimum of the FLD shifting to the right and lower strain limits for intermediate strain states between plane strain and equibiaxial tension.

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REFERENCES

1. Z. Marciniak and K. Kuczynski: *Int. J. Mech. Sci.*, 1967, vol. 9, pp. 609-20.
2. A. Barata da Rocha, F. Barlat, and J.M. Jalinier: *Mater. Sci. Eng.*, 1984–1985, vol. 68, pp. 151-64.
3. A. Barata da Rocha: in *Forming Limit Diagrams: Concepts, Methods and Applications*, R.H. Wagoner, K.S. Chan, and S.P. Keeler, eds., TMS-AIME, Warrendale, PA, 1989, pp. 184-202.
4. A. Parmar and P.B. Mellor: *Int. J. Mech. Sci.*, 1982, vol. 24, pp. 385-91.
5. D. Lee and F. Zaverl, Jr.: Int. *J. Mech. Sci.*, 1982, vol. 24, pp. 157-73.
6. Sowerby and Duncan: *Int. J. Mech. Sci.*, 1971, vol. 13, pp. 217-29.
7. D.D. Olander and A.K. Miller: in *Controlling Sheet Metal*

Forming Processes, IDDRG, ed., ASM INTERNATIONAL, Warrendale, PA, 1988, pp. 133-44.
8. K.S. Chan: *Metall. Trans. A*, 1985, vol. 16A, pp. 629-39.
9. J. Lian, F. Barlat, and B. Baudelet: *Int. J. Plast.*, 1989, vol. 5, pp. 131-47.
10. A. Graf and W.F. Hosford: in *Forming Limit Diagrams: Concepts, Methods and Applications*, R.H. Wagoner, K.S. Chan, and S.P. Keeler, eds., TMS-AIME, Warrendale, PA, 1989, pp. 153-63.
11. A. Graf and W.F. Hosford: *Metall. Trans. A*, 1990, vol. 21A, pp. 87-94.
12. A.K. Ghosh: *Met. Eng. Q.*, 1975, vol. 15, pp. 53-64.
13. R. Hill: *Proc. R. Soc.*, Published by the Society of Manufacturing Engineers. Meeting: May 1979, Dearborn, MI, 1948, vol. 193A, pp. 281-97.
14. W.F. Hosford: *Proc. 7th NAMRC*, Dearborn, MI, May 1979, pp. 191-96.
15. A. Graf and W.F. Hosford: *Metall. Trans. A*, 1993, vol. 24A, pp. 2503-12.
16. K. Narasimhan and R.H. Wagoner: *Metall. Trans. A*, 1991, vol. 22A, pp. 2655-65.
17. R. Logan and W.F. Hosford: *Int. J. Mech. Sci.*, 1980, vol. 22, p. 419.
18. A. Barata da Rocha and J.M. Jalinier: *Trans. Iron Steel Inst. Jpn.*, 1984, vol. 24, pp. 132-40.
19. J.H. Schmitt, J. Raphael, E. Rauch, and P. Martin: *Computational Methods for Predicting Material Processing Defects*, Elsevier, New York, NY, 1987, pp. 309-18.
20. M. Zandrahimi, S. Platias, D. Price, D. Barrett, P.S. Bathe, W.T. Roberts, and D.V. Wilson: *Metall. Trans. A*, 1989, vol. 20A, pp. 2471-82.
21. S.S. Hecker: *7th Biennial Congress of the IDDRG*, Amsterdam, Oct. 1972, Paper 5C.
22. A.K. Ghosh and S.S. Hecker: *Metall. Trans.*, 1974, vol. 5, pp. 2161-64.
23. K.S. Raghavan: *A Simple Technique to Determine In-Plane Forming Limit Curves*, presented at the 1992 NADDRG Spring Meeting, April 28–30, 1992, Dearborn, MI.
24. K.S. Raghavan, R.C. Van Kuren, and H. Darlington: SAE Paper No. 920437, published in: Autobody Stamping Applications and Analysis SOC, Warrendale, PA, 1992.