A Geometric Model for Fatigue Crack Closure Induced by Fracture Surface Roughness

S. SURESH and R. O. RITCHIE

Mechanisms for fatigue crack closure under plane strain conditions have recently been identified at very low (near-threshold) stress intensities in terms of effects of excess corrosion deposits or fracture surface roughness in promoting premature closure of the crack. In the present paper, a geometric model is presented for crack closure induced by fracture surface roughness. This model specifically addresses the contribution from both Mode I and Mode II crack tip displacements in addition to considering the nature of the fracture surface morphology. The implications of this model are briefly discussed in light of the roles of grain size, yield strength, microstructure, and crack size in influencing near-threshold fatigue behavior in engineering alloys.

I. INTRODUCTION

CRACK closure[1] is generally considered to arise from the fact that during fatigue crack advance, material is plastically strained at the crack tip and due to the restraint of surrounding elastic material on this residual stretch, some closure of the crack surfaces occurs above the minimum load of the fatigue cycle. Since the crack cannot propagate while it is physically closed, the stress intensity range experienced at the crack tip is reduced from the nominally applied value ($\Delta K = K_{max} - K_{min}$) to some effective value ($\Delta K_{eff} = K_{max} - K_{cl}$), where K_{cl} is the stress intensity to close the crack. Such *plasticity-induced closure,* however, is most prevalent under essentially plane stress conditions,[2,3] and yet recent studies on fatigue crack propagation at ultralow growth rates ($da/dN < 10^{-6}$ mm per cycle), approaching the threshold for fatigue crack growth (ΔK_0), have clearly shown that very significant closure effects can also occur in plane strain.[4–18]

Two mechanisms have been proposed to account for such closure in plane strain, based on the role of crack surface corrosion deposits[5–11] and fracture surface roughness or morphology[4,8–15,17,18] (Figure 1). The first of these mechanisms, so-called *oxide-induced crack closure,*[5–8] arises from the fact that when oxide deposits, formed on freshly exposed surfaces at the crack tip in moist environments, reach a thickness comparable to crack tip opening displacements, the crack can become effectively wedged-closed at stress intensities above K_{min}. This concept, which has proved to be particularly effective in explaining certain aspects of the role of environment in influencing near-threshold fatigue crack growth, has been described in detail elsewhere[5–11] and will not be considered further in this paper. The second mechanism, termed *roughness-induced crack closure,*[9,17] arises in situations where the size-scale of the fracture surface roughness is comparable to crack tip opening displacements and where significant Mode II displacements exist, *e.g.*, at near-threshold levels. As shown in Figure 1, closure can again be promoted since the crack can become wedged-closed at discrete contact points along crack faces.[4,9,12,15,17,18]

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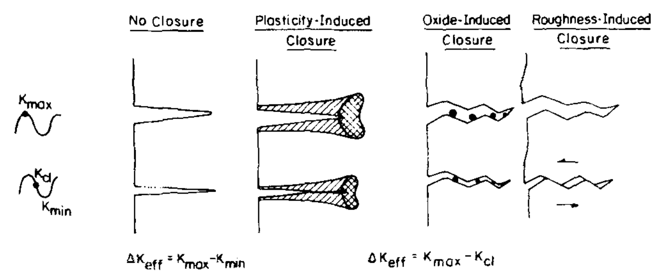

Fig. 1—Schematic illustration of mechanisms of fatigue crack closure. ΔK_{eff} is the effective stress intensity range, given by the difference between the maximum stress intensity K_{max} and the stress intensity to close the crack K_{cl}. ($K_{cl} \geq K_{min}$, the minimum stress intensity.)

Recent studies have indicated that roughness-induced closure is most prevalent at near-threshold levels where maximum plastic zone sizes (r_y) are typically less than the grain size (d_g).[15,16,17] In such instances, the low restraint on cyclic slip will primarily promote crack extension along a single slip system, akin to Forsyth's Stage I mechanism,[19] resulting in serrated or zig-zag fracture paths[15,17] (Figure 2). Such faceted fracture morphologies, which have been termed crystallographic or microstructurally-sensitive growth, are evident in Figures 2 and 3 for metallographic sections through near-threshold cracks in steels,[15] aluminum alloys,[20,21] and titanium.[4] The large Mode II crack displacements accompanying such crack advance, which have been confirmed experimentally using stereoimaging studies,[22] thus provide the mechanism for contact between crack surfaces, as shown in Figure 3. However, at larger stress intensity ranges, where maximum plastic zone sizes exceed the grain size, the increasing restraint on cyclic plasticity activates more than one slip system at the crack tip leading to the more planar striation mode of crack advance by alternating or simultaneous shear, akin to Forsyth's Stage II mechanism (Figure 2(b)).[17] The predominately Mode I character of such noncrystallographic crack growth, coupled with its less faceted morphology (Figures 2(d) and (f)), thus results in a marked reduction in (roughness-induced) closure at higher growth rates.[15,16,17]

The only analysis to *quantify* the effect of fracture surface roughness on fatigue crack closure was presented by Purushothaman and Tien[23] in 1979. These authors suggested simply that the closure stress intensity K_{cl} could be estimated

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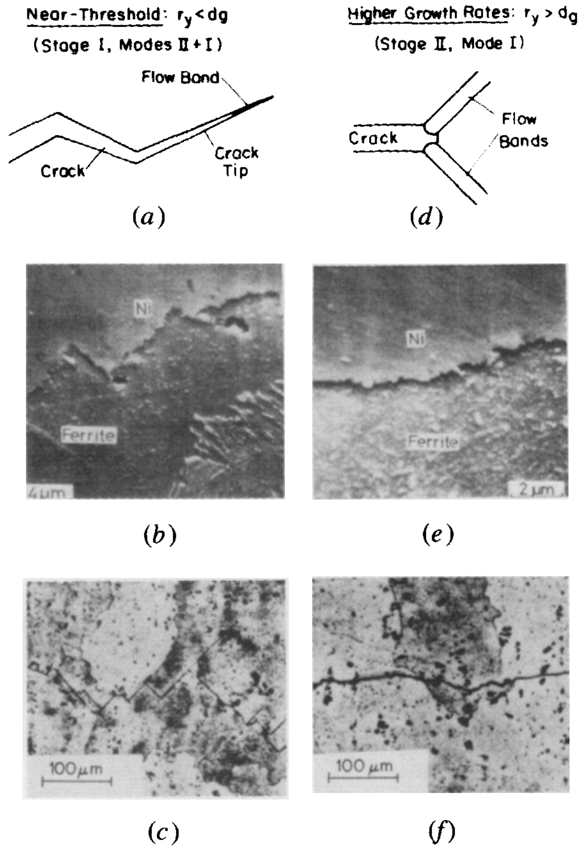

Fig. 2—Crack opening profiles and resulting crack path morphologies corresponding to (*a*), (*b*), (*c*) near-threshold (Stage I) and (*d*), (*e*), (*f*) higher growth rate (Stage II) fatigue crack propagation. (*b*) and (*e*) are nickel-plated fracture sections of fatigue crack growth in 1018 steel (after Ref. 15), and (*c*) and (*f*) are metallographic sections of crack growth in 7075-T6 aluminum alloy (after Ref. 20). r_y is the maximum plastic zone size, and d_g the average grain size.

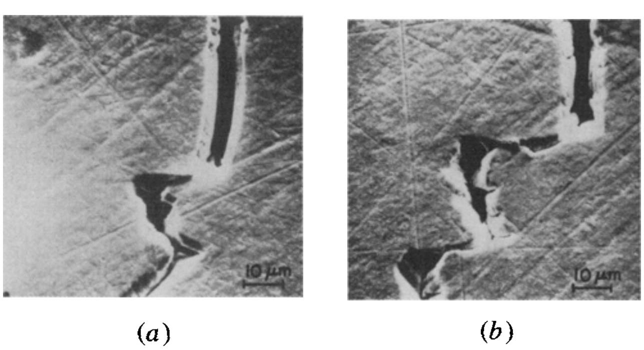

Fig. 3—Replicas of a fatigue crack in α-titanium ($\Delta K \approx 9$ MPa$\sqrt{\text{m}}$, $R = 0.1$), taken at minimum load of fatigue cycle at (*a*) 9.2 mm and (*b*) 14.5 mm from the crack tip. Note contact between crack surfaces at discrete points due to roughness-induced closure (after Ref. 4).

by equating the change in fracture surface asperity height (taken as a function of true fracture ductility) to the crack opening displacement. While perhaps providing a lower bound solution, this model severely underestimates closure stress intensities since it does not incorporate the role of crack tip Mode II displacements which are clearly very relevant to the extent of fracture surface interference at near-threshold levels. In the current work we propose an alternative description for roughness-induced crack closure in the form of a geometric model which specifically addresses the contribution from both Mode I and Mode II displacements. The results of this model are briefly discussed in light of the effects of grain size, yield strength, microstructure, and crack size on the behavior of fatigue cracks at ultralow, near-threshold growth rates.

II. GEOMETRIC MODEL

Fatigue crack closure induced by fracture surface roughness[4,8,9,12,15,17] is modeled as a two-dimensional problem by specifically incorporating both Mode I and Mode II crack tip displacements which are characteristic of near-threshold crack advance (Figure 2(a)). Fracture surface roughness is idealized in terms of asperities, assumed to be of triangular cross section and roughly equal in size (Figure 4(a)), consistent with the metallographic sections through near-threshold cracks shown in Figures 2 and 3. The base of the fracture surface asperity is of length w, taken as a fraction α of the mean grain diameter d_g, *i.e.*,

$$w = \alpha d_g \qquad [1]$$

Asperities have an average height h, with a semicone angle θ at the tip.

In Figure 4 we consider the crack tip opening displacement δ_{max} at the peak of the loading cycle at $K = K_{max}$, and allow the crack to unload to the point of closure at $K = K_{cl}$. Assuming the crack unloads in Mode I with an accompanying Mode II displacement (u_{II}) taken as a fraction x of the Mode I displacement (u_I), the two crack faces will first come into contact at a tensile crack opening displacement δ_{cl} given by

$$\delta_{cl} = \delta_{max} - u_I \qquad [2]$$

with

$$x = u_{II}/u_I \qquad [3]$$

From geometrical considerations (Figure 4),

$$\cot\theta = h/(w/2) = \delta_{cl}/u_{II} \qquad [4]$$

By combining Eqs. [1] to [4], we find

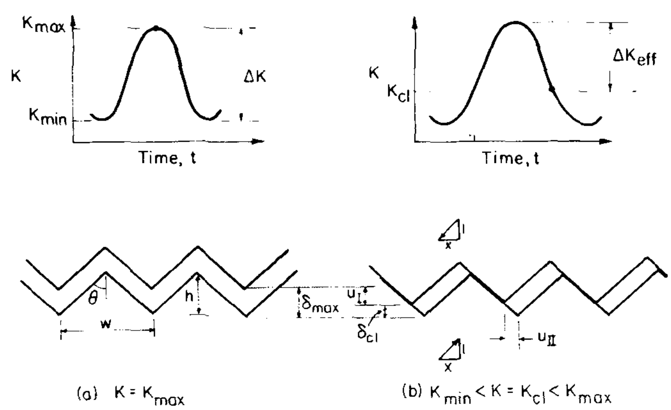

Fig. 4—Schematic representation of a near-threshold Stage II fatigue crack (*a*) at the maximum crack opening displacement δ_{max}, where $K = K_{max}$; and (*b*) after unloading by Mode I and Mode II displacements to a point where the crack faces first come into contact at the closure crack opening displacement δ_{cl}, where $K = K_{cl}$.

$$\delta_{cl} = \frac{2hx\,\delta_{max}}{\alpha d_g + 2hx} \quad [5]$$

The magnitude of the closure effect, expressed as the ratio of the closure to maximum stress intensities, can thus be represented as

$$\frac{K_{cl}}{K_{max}} = \sqrt{\frac{\delta_{cl}}{\delta_{max}}} = \sqrt{\frac{2hx}{\alpha d_g + 2hx}} \quad [6a]$$

or in nondimensional form as

$$\frac{K_{cl}}{K_{max}} = \sqrt{\frac{2\gamma x}{1 + 2\gamma x}} \quad [6b]$$

where $\gamma = h/\alpha d_g$ is a nondimensional fracture surface roughness factor.

III. RESULTS

From the above simple analysis, it is apparent that the extent of crack closure is a strong function of the fracture surface roughness (γ) and the proportion of Mode II crack tip displacements (x). This can be appreciated from Figure 5 where the predicted closure ratio (K_{cl}/K_{max}) is plotted as a function of the roughness factor γ for various values of x from Eq. [6b]. Experimental values of K_{cl}/K_{max},[15,24] measured using compliance techniques, and estimates of γ, obtained from surface coating[24] and profilometric studies[15] in steels, when substituted into Eq. [6b], indicate that the extent of the Mode II component ($u_{II} = xu_I$) may be as much as 30 pct of the Mode I displacement at near-threshold stress intensities, as shown by the data points in Figure 5. Direct experimental examination of crack tip motion using stereo-imaging procedures in fact reveal significant Mode II displacements at low stress intensities consistent with such predictions.[22]

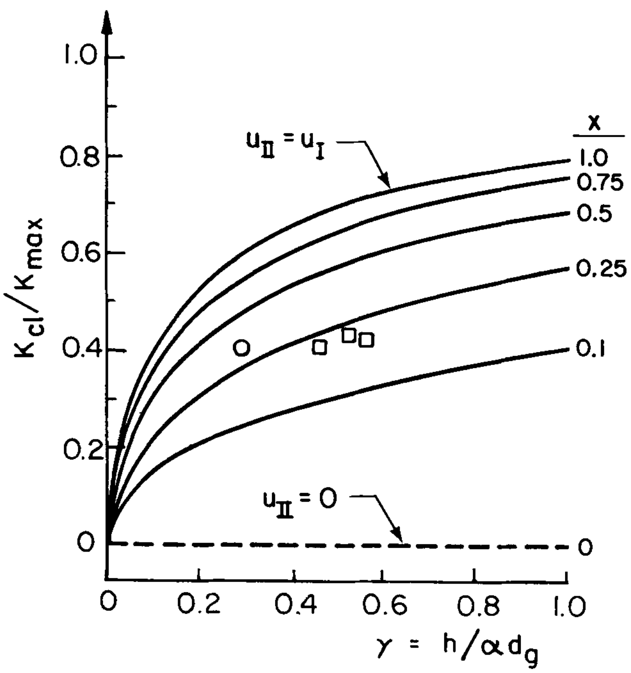

Fig. 5—Predicted variation from Eq. [6b] of K_{cl}/K_{max} with the fracture surface roughness factor γ as a function of x, the ratio of unloading Mode II and Mode I displacements (u_{II} and u_I, respectively). Experimental data points, derived from K_{cl}/K_{max} measurements and estimates of γ from fracture surface profile measurements, are taken from Ref. 15 on 1018 steel (○) and Ref. 24 on fully pearlitic rail steel (□).

It should be noted that such roughness-induced crack closure will be predominant only at low load ratios where premature contact between mating fracture faces can occur *via* the crack tip shear displacements. In the limit, no crack closure due to surface roughness is possible when the minimum crack tip opening displacement exceeds the scale of roughness, *i.e.*, when $\delta_{min} \geq h$. It follows then that roughness-induced crack closure can occur (for a given δ_{max}) only below a critical value of the load ratio, given by

$$R_{cr} = \sqrt{h/\delta_{max}} \quad [7]$$

Since the effect of load ratio R on fatigue crack growth threshold values (ΔK_0) is generally interpreted in terms of closure arguments,[25,5] the fact that ΔK_0 values are often found to be independent of R at high load ratios and to decrease with R at low load ratios[25] is consistent with this concept.

IV. DISCUSSION

The geometric model for roughness-induced crack closure, presented above, principally serves to quantitatively illustrate the marked importance of Mode II displacements in promoting crack closure at near-threshold stress intensities. Even with low values of fracture surface roughness (for example, with $\gamma = 0.2$), substantial levels of crack closure, *i.e.*, up to 35 pct reduction in effective stress intensity values, can arise if the shear displacements equal 30 pct of the Mode I crack opening. Such values are in agreement with the limited experimental data[15,24] in the literature, plotted in Figure 5, and with other closure[12,16] and Mode II shear displacement[22] measurements* on near-threshold cracks where crack advance occurs *via* the single shear (Forsyth's Stage I) mechanism (Figure 2(a)). However, when the maximum plastic zone sizes exceed the grain size (*i.e.*, with increasing ΔK levels), marked reductions in closure levels are observed[12,15,16] with the ratio K_{cl}/K_{max} approaching a value of about 0.2. Such behavior is to be expected since, under conditions of higher ΔK levels, crack advance occurs primarily *via* simultaneous or alternating slip (Forsyth's Stage II) mechanism (Figure 2(a)) with a corresponding reduction in the Mode II contribution (*i.e.*, reduced x) and in the roughness of the fracture surface (*i.e.*, reduced γ).

*Such additional data are not plotted in Figure 5 due to the lack of information on fracture surface roughness measurements.

The concept of roughness-induced closure may also have implications in interpreting the large effects of grain size and yield strength on near-threshold fatigue crack growth behavior.[13,17,26] Many investigators have reported decreased near-threshold growth rates and higher ΔK_0 values in coarser-grained or lower strength materials.[8,9,13-15,17,27] Since both coarser microstructures and lower strength would be expected to increase γ, such effects may be traced to increased crack closure levels. The fact that at high load ratios, where in general closure effects are minimal, the beneficial influence of grain size and strength is much reduced[14,27] is certainly consistent with this explanation. In a similar vein, it is reasonable to expect improved near-threshold crack growth resistance in dual-phase materials by incorporating a soft phase in the microstructure to promote crack path meandering. Here the accompanying increase in fracture surface roughness would be expected to enhance closure levels leading to a reduction in ΔK_{eff}. Preliminary experiments in low strength steels with duplex microstructures show promise in this regard.[28]

Finally, one may speculate on the concept of roughness-induced crack closure with regard to the problem of crack size.[29,18] There is now a growing body of evidence[18,29-36] indicating that crack growth rates and in particular threshold ΔK_0 values measured on small cracks may be substantially different from conventional long crack data at the same nominal driving force (*i.e.*, at the same ΔK). Here by small cracks we refer to flaws which are small compared to the scale of microstructure (*i.e.*, of the order of the grain size), or small and comparable in size with the scale of plasticity, or simply physically small (*i.e.*, <0.5 mm). Results in steels, aluminum, titanium, and nickel-base alloys[18,29-36] have shown the growth rates of short cracks to be significantly higher, and threshold ΔK_0 levels to be significantly lower, than those corresponding to long cracks (*i.e.*, >10 to 20 mm long) at equivalent stress intensity ranges. While this discrepancy between long and short crack behavior may be a function of a number of factors, such as differing crack tip plasticity[31] or local crack tip environments,[34] a reduced contribution from crack closure cannot be ruled out.[18,29] This can be simply appreciated with reference to Figure 6 by examining the behavior of a long and a microstructurally-small crack. The long crack, which encompasses several grains, will at near-threshold levels have developed a faceted morphology and accordingly be subject to roughness-induced closure in the manner depicted in Figure 1. The short crack, however, will be unable to develop such closure while its length remains less than a grain diameter despite the fact that it is advancing *via* the same single shear growth mechanism. Recent measurements[18] of the crack opening displacement (at zero load) of short surface cracks in titanium alloys are consistent with this notion, although the transition crack size, below which the extent of crack closure decreases with decreasing crack length, was found to be somewhat larger than one grain diameter. Thus, when comparing the propagation rates of long and short cracks, the short crack is likely to experience a larger driving force at the crack tip due to a smaller contribution from (roughness-induced) crack closure. While this is clearly an over-simplification of the actual phenomenon, since short crack growth may also be impeded by the presence of grain boundaries,[35,36] it does provide a partial explanation as to why threshold ΔK_0 values measured for short flaws are often significantly smaller (at low load ratios) than conventionally measured thresholds on long cracks.

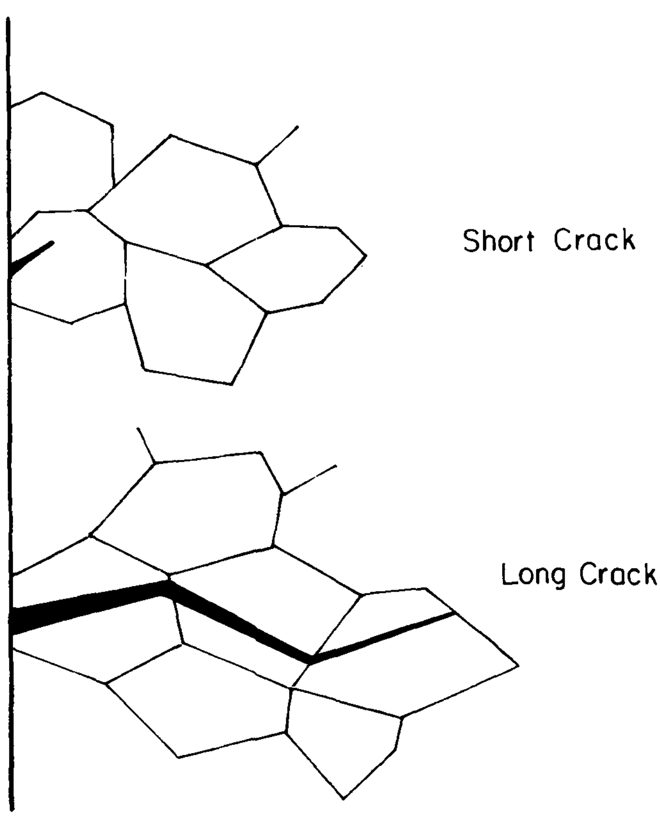

Fig. 6—Idealization of a microstructurally-short and a microstructurally-long crack propagating at near-threshold levels by a single-shear Stage I mechanism.

V. CONCLUSIONS

A simple geometric model is presented for crack closure induced by fracture surface roughness, which is applicable to the growth of fatigue cracks at near-threshold stress intensities. The model predicts that the extent of crack closure, defined in terms of the ratio K_{cl}/K_{max}, will be a strong function of the degree of surface roughness and the magnitude of the Mode II crack tip displacements. Limited experimental data reported in the literature are found to be consistent with the model predictions. The marked influences of grain size, yield strength, and crack size on near-threshold fatigue crack growth behavior are attributed, at least in part, to a differing contribution from such roughness-induced crack closure.

NOMENCLATURE

a	Crack length
d_g	Grain size
da/dN	Fatigue crack growth rate per cycle
h	Height of fracture surface asperity
K	Mode I stress intensity factor
K_{cl}	Stress intensity to close crack
K_{max}	Maximum stress intensity
K_{min}	Minimum stress intensity
ΔK	Alternating stress intensity ($K_{max} - K_{min}$)
ΔK_{eff}	Effective stress intensity range ($K_{max} - K_{cl}$)
ΔK_0	Threshold stress intensity range for fatigue crack growth
R	Load ratio ($= K_{min}/K_{max}$)
R_{cr}	Critical load ratio above which roughness-induced crack closure cannot occur (at given δ_{max})
r_y	Maximum plastic zone size
t	Time
u_I	Mode I crack tip displacement
u_{II}	Mode II crack tip displacement
w	Width of fracture surface asperity

x Ratio of Mode II to Mode I displacement (u_{II}/u_I)

α Ratio of asperity width to grain size

γ Nondimensional surface roughness factor $(h/\alpha d_g)$

δ_{cl} Tensile crack tip opening displacement at $K = K_{cl}$

δ_{max} Maximum crack tip opening displacement at $K = K_{max}$

δ_{min} Minimum crack tip opening displacement at $K = K_{min}$

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