INTERPLANETARY SECTOR STRUCTURE AND THE HELIOMAGNETIC EQUATOR

MICHAEL SCHULZ

Space Physics Laboratory, The Aerospace Corporation, El Segundo, Calif., U.S.A.

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Abstract. The magnetohydrodynamics of solar-wind flow lead logically to the formation of one warped annular neutral surface that apparently extends from $\sim 2\,r_\odot$ (solar radii) to the boundary of the heliosphere. The most likely asymptotic configuration for this neutral sheet intersects the heliomagnetic equatorial plane along four corotating arcs. The observer sees a reversal of magnetic polarity on each crossing of the neutral surface, and so interprets each reversal as the crossing of a sector boundary.

1. Introduction

As observed in the ecliptic plane at heliocentric distances $r \sim 1$ AU, the interplanetary magnetic field exhibits a quasi-stationary sector structure that corotates with the Sun (Wilcox and Ness, 1965). The direction of **B** is found to alternate from one sector to the next. A sector in which $B_r < 0$ is designated as a negative sector, and the adjacent sector (in which $B_r > 0$) is known as a positive sector. The conventional interpretation of interplanetary sector structure (Ahluwalia and Dessler, 1962) is that the field consists of flux tubes that emanate from the photosphere and extend deep into interplanetary space by virtue of solar-wind flow (cf. Parker, 1958). Each tube of flux is considered to be surrounded by a neutral surface that is quite stable in the plasma-dynamical sense (Dessler and Michel, 1966).

The purpose of the present work is to promote a substantially different interpretation of the interplanetary sector structure. In the present model, the solar wind extends the heliomagnetic equator into one warped annular neutral surface, which typically intersects the ecliptic along four corotating arcs that an observer interprets as sector boundaries. The present model, which appears to be in good qualitative agreement with observation, follows naturally from a consideration of basic magnetohydrodynamics (MHD), as applied to an underlying spherical-harmonic expansion of the photospheric **B** field. As such, the present model is a more quantitative elaboration of that proposed by Rosenberg (1970) and mentioned in passing by Davis (1972).

The model is based specifically on the principle that the character of the heliomagnetic field at $r \sim r_A$ is 'frozen' into the expanding solar wind. The Alfvén radius r_A is defined as that (latitude-dependent) heliocentric distance at which the Alfvén speed c_A becomes equal to the flow speed u. The magnitude of r_A can be estimated as being $\sim 2r_\odot$ (solar radii) by examining eclipse photographs of the solar corona (e.g., Newkirk, 1967, 1971).

It is interesting to estimate r_A also from a sunward extrapolation of solar-wind observations taken near the Earth's orbit ($r \approx 215\,r_\odot$). Such observations of the quiet

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solar wind (Hundhausen, 1970) indicate that $u \approx 320\ \mathrm{km\ s^{-1}} \approx 8c_A$, with an interplanetary magnetic field $B \approx 5 \times 10^{-5}$ G at $r \approx 215\ r_\odot$. If one assumes radial flow with u independent of r for $r_A < r < 215\ r_\odot$, then the plasma density scales as $1/r^2$, and the field scales according to the relation

$$B^2 \propto (1/r)^4 [1 + (\mathbf{\Omega} \times \mathbf{r}/u)^2], \tag{1}$$

where $\mathbf{\Omega}$ is the angular velocity of the Sun (e.g., Dessler, 1967). Since $(\mathbf{\Omega} \times \mathbf{r}/u)^2 \approx 1.84$ at the Earth's orbit, it seems to follow that $c_A \approx u$ at $r \approx 16\ r_\odot$; moreover, the equatorial field intensity at $r = r_A$ (where $c_A = u$) is denoted B_A and hereby estimated as being $\approx 5 \times 10^{-3}$ G. A dipolar scaling of B for $r < r_A$ thus leads to a polar field intensity ~ 50 G at $r = r_\odot$, as compared with the observation (Newkirk, 1971) that $B \sim 2$ G there. In view of these discrepancies, the use of (1) to extrapolate B from $r = 215\ r_\odot$ to $r = r_A$ must be a serious error.

The essential defect in (1), as an extrapolation to $r = r_A$, is the assumption of purely radial flow at low latitudes. An expansion that is roughly cylindrical at low latitudes and spherical at high latitudes would satisfy the physical constraints equally well. The radial variation of B in a cylindrical expansion is given by an equation similar to (1), but with the exponent 4 replaced by 2. The plasma density then scales as $1/r$ if u is independent of r for $r > r_A$. In this limit it is found (by sunward extrapolation of the observations at $r \sim 1$ AU) that $r_A \approx 1.2\ r_\odot$ with $B_A \approx 5 \times 10^{-3}$ G; a dipolar scaling of B for $r < r_A$ thus leads to a polar field intensity $\sim 2 \times 10^{-2}$ G at $r = r_\odot$.

Evidently the real situation is intermediate between the above-described limits of spherical and cylindrical solar-wind flow at equatorial latitudes. In other words, the Alfvén radius at low latitudes is less than $16\ r_\odot$ but greater than $1.2\ r_\odot$, and the polar field intensity is less than 50 G but greater than 0.02 G. Moreover, a pattern of this intermediate type seems to emerge from a preliminary MHD analysis of solar-wind expansion (Pneuman and Kopp, 1971b). In this configuration (illustrated in Figure 1) the high-latitude streamlines and field lines are nearly radial (neglecting solar rotation), but streamlines and field lines emanating from middle latitudes are deflected equatorward from the radial direction. Thus, the flow beyond $r \sim r_A$ is nearly radial at high latitudes, but roughly parallel to the neutral surface at low latitudes.

The magnetic scalar potential $V(r, \theta, \varphi)$, constructed from the photospheric $\mathbf{B}$ field such that $\mathbf{B} = -\nabla V$ for $r_\odot \lesssim r \lesssim r_A$, typically comprises an elaborate superposition of spherical harmonics extending to very high orders. However, the Alfvén radius r_A may be sufficiently large compared to $r_\odot$ that only the lowest spherical harmonics (i.e., the dipole and quadrupole terms) of the general heliomagnetic field expansion are likely to contribute heavily there. The higher-order harmonics required to characterize extremely localized photospheric and coronal features are of insignificant magnitude at heliocentric distances of more than a few solar radii. Thus, the heliomagnetic field at $r \sim r_A$ is perhaps adequately described (apart from sheet-current effects, as discussed below) by a few low-order moments of the photospheric field. Moreover, the dipole and quadrupole terms of the general field expansion, i.e., the terms most like to survive at $r \sim r_A$, are sufficient to provide a qualitative explanation of the ob-

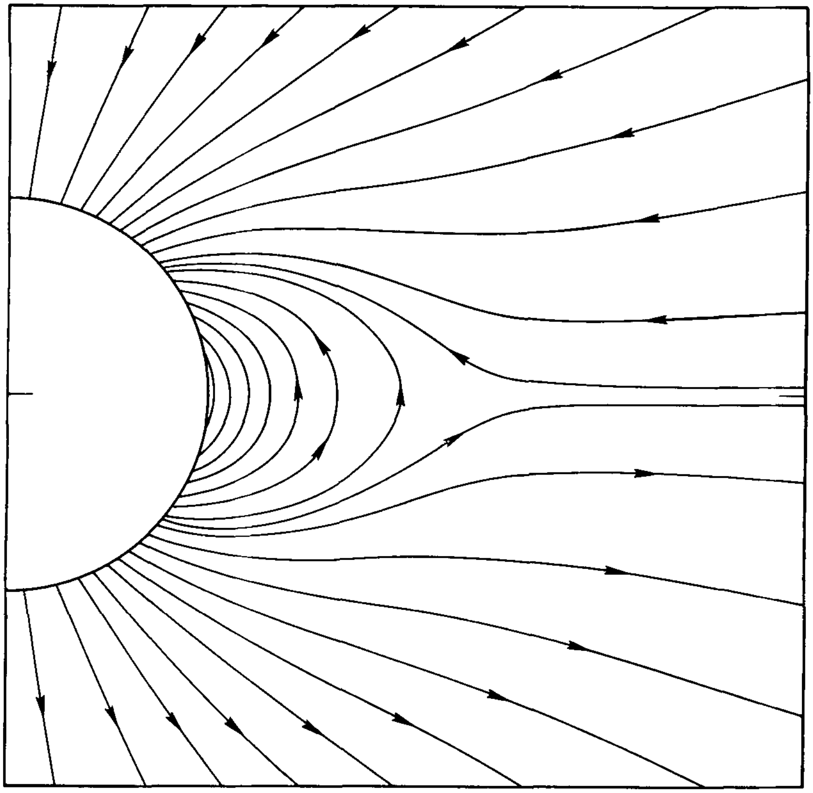

Fig. 1. Deformation of a dipole field from within by an idealized solar wind, including formation of annular neutral sheet. Model is symmetric about the dipole axis, but solar rotation is neglected in order to make B_φ vanish. Field and streamline distribution is based on a numerical MHD calculation by Pneuman and Kopp (1971b). Arrows indicate direction of **B**.

served interplanetary sector structure. The dipole-quadrupole field has such a simple dependence on heliomagnetic latitude that the basic features of its sector structure can be summarized by ordinary analytical expressions.

2. Neutral-Sheet Formation

The formation of a neutral sheet on the equatorial plane of a centered-dipole field in the presence of solar-wind flow is illustrated above in Figure 1. This illustration was generated by Pneuman and Kopp (1971b) in an MHD computation designed to account for the observed structure of a typical coronal steamer (cf. Pneuman, 1966a; Newkirk, 1967). The model would require certain modifications if used to interpret the average solar wind. For example, Figure 1 is based on a model in which the polar magnetic field is equal to 1 G at $r = r_\odot$, while the equatorial field intensity is only 0.5 G at $r = r_\odot$. The model thus conflicts with observational data (Newkirk, 1971) indicating that equatorial field intensities are typically stronger than polar field intensities at $r = r_{\odot}$. Moreover, the solar wind observed at $r \sim 1$ AU ($\approx 215\, r_{\odot}$) is known to be associated with heliographic latitudes $\lesssim 20°$ (Wilcox, 1968) rather than with $\Lambda > 38°$ (as in Figure 1).

Both of these observational objections to a dipolar field at $r \approx r_{\odot}$ can be overcome by enriching the zonal-harmonic content of the model photospheric field, i.e., by representing the field (in the region not distorted by neutral-sheet currents) as the negative gradient of a scalar potential

$$V(r, \theta) = -r_{\odot} \sum_{n=0}^{\infty} B_{2n+1} (r_{\odot}/r)^{2n+2} P_{2n+1}(\cos\theta), \tag{2}$$

where the B_n are expansion coefficients and the $P_n(\cos\theta)$ are Legendre functions of the magnetic colatitude θ, *viz.*,

$$P_n(\cos\theta) \equiv \frac{(-1)^n}{n!\,2^n} \frac{\mathrm{d}^n (\sin^{2n}\theta)}{\mathrm{d}(\cos\theta)^n}. \tag{3}$$

A judicious choice of the coefficients B_n can concentrate the photospheric field within any desired latitude range about the equatorial plane of symmetry. More generally, the expansion coefficients are given by

$$B_n = -\frac{2n+1}{2n+2} \int_{-1}^{+1} \hat{\mathbf{r}} \cdot \mathbf{B}(r_{\odot}, \theta)\, P_n(\cos\theta)\, \mathrm{d}(\cos\theta), \tag{4}$$

for a $\mathbf{B}$ field having azimuthal symmetry. However, the field given by (2) becomes predominantly dipolar at sufficiently large heliocentric distances (e.g., at $r \gtrsim 2r_{\odot}$, depending on the relative magnitudes of $B_1 : B_3 : B_5 : \ldots$).

The formation of a neutral sheet on the equatorial plane of this azimuthally symmetric model field (see Figure 2) follows from the same principles that led to the formation of a neutral sheet in Figure 1. That is to say, coronal conditions yield a supersonic solar wind having flow velocity u. The magnetic field tends to guide the flow of plasma in regions where $c_A \gg u$, and the plasma flow tends to orient the field in regions where $c_A \ll u$ (Parker, 1958). The transition between these two limits along a given field line is rather smooth (see Figures 1 and 2). The neutral sheet forms because field lines on which the flow speed (u) can exceed the Alfvén speed (c_A) will tend to 'break open', thereby allowing the solar wind to escape. Field lines that must remain closed (because equatorial c_A exceeds u) will tend to quench (thermalize) any incipient plasma flow from very low magnetic latitudes (Schulz and Koons, 1972), with the result that steady-state conditions preclude the net flow of plasma along closed field lines and provide for hydrostatic equilibrium there (Pneuman and Kopp, 1971a). The demarcation between closed and open field lines is the surface of rotation generated by that singular field line which forms an equatorial cusp at $r = r_A$, i.e., at the inner boundary of the neutral sheet.

The neutral sheet thus created as a natural MHD consequence of solar-wind flow

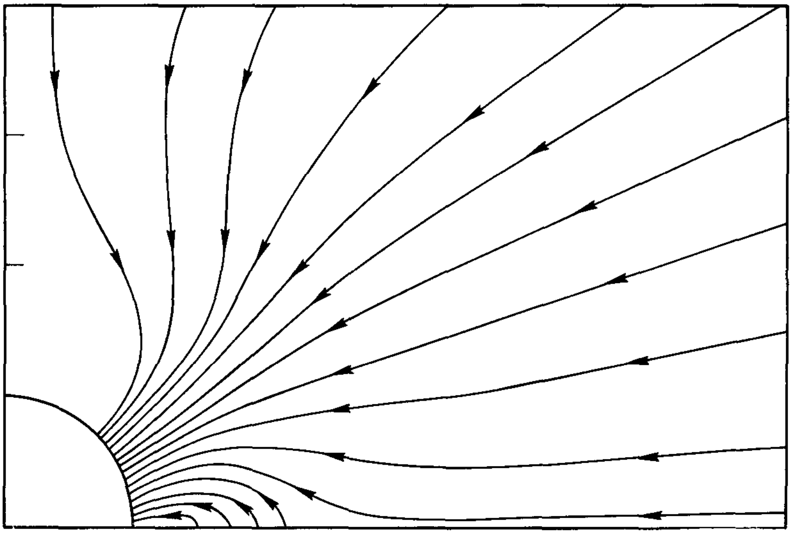

Fig. 2. Deformation of an idealized solar field from within by solar-wind flow, including formation of annular neutral sheet. Model is symmetric about the dipole axis, but solar rotation is neglected in order to make B_φ vanish. Photospheric field is confined to latitudes $\lesssim 45°$ by odd-n zonal harmonics of higher order than the dipole. This schematic field and streamline distribution is based on qualitative considerations only, not on numerical computation. Arrows indicate direction of **B**.

has the shape of an annulus. The outer radius is limited only by the finite radius of the heliosphere, as described by Dessler (1967). The inner radius r_A is determined by the condition $u = c_A$ (Pneuman and Kopp, 1971a). The annulus carries an azimuthal current distribution that cancels out the normal (θ) component of the heliomagnetic-multipole field at $r > r_A$. Moreover, this current introduces a roughly uniform axial magnetic-field perturbation (having intensity $\sim B_A$) in the spherical volume $r < r_A$, so as to distend closed field lines toward the cusp (see Figures 1 and 2). Finally, the neutral-sheet current distribution augments the existing radial component of **B** at $r > r_A$, such that $B_r < 0$ in the northern hemisphere. Thus, the direction of **B** at $r > r_A$ appears to change by 180° as the observer traverses the neutral sheet. The solar wind observed near the neutral sheet at $r \gg r_A$ (e.g., from an Earth-orbiting spacecraft) must have originated from a photospheric latitude slightly above that at which the adjacent singular (cusped) field line enters the Sun. According to Wilcox (1968), the observed solar wind is associated with heliographic latitudes $\lesssim 20°$.

If the Sun's magnetic axis were approximately in coincidence with the Sun's rotation axis (inclined 7.25° to the ecliptic plane), then an observer in Earth orbit would alternate spending six months above a planar neutral sheet ($B_r < 0$) and six months below ($B_r > 0$). There would be no 'sector structure' contingent on solar rotation as such. Any inclination θ_0 of the solar magnetic axis to the rotation axis would break this degeneracy, as viewed by an observer at heliographic latitude λ, provided only that $|\tan\lambda| < |\tan\theta_0|$. Such an inclination of the magnetic axis would cause the neutral plane to wobble with solar rotation, thus creating the two-sector pattern characteristic of the approach to solar maximum (Rosenberg and Coleman, 1969; Svalgaard, 1972).

More precisely, if ϕ is the heliographic longitude in general and ϕ_0 is the longitude of the northern magnetic pole, then the heliomagnetic colatitude θ is given by

$$\cos\theta = \cos\theta_0 \sin\lambda + \sin\theta_0 \cos\lambda \cos(\phi - \phi_0). \tag{5}$$

Thus, the probability (averaged over ϕ) for an observer at heliographic latitude λ to be south of the planar neutral sheet ($\cos\theta < 0$) is given by

$$P(B_r > 0) = (1/\pi)\,\mathrm{Re}\cos^{-1}(\tan\lambda/\tan\theta_0), \tag{6}$$

which is to say that the two sectors have angular breadths equal to $2\cos^{-1}(\tan\lambda/\tan\theta_0)$ and $2\cos^{-1}(-\tan\lambda/\tan\theta_0)$, respectively, if $|\tan\lambda| < |\tan\theta_0|$. The stratagem of taking the real part (Re) of $\cos^{-1}(\tan\lambda/\tan\theta_0)$ in (6) enables this equation to accommodate also the unsectored case, in which the argument of $\cos^{-1}$ is real but greater than unity in absolute value.

The foregoing remarks summarize the idealized case in which the heliomagnetic field is symmetrical about both the dipole axis and the equatorial plane. In this case, as summarized by (2), the neutral sheet created by solar-wind flow must coincide with the magnetic equatorial plane. This idealization evidently has too much symmetry to account for the four-sector pattern characteristic of the approach to solar minimum (Rosenberg and Coleman, 1969; Svalgaard, 1973). Thus, it is instructive to consider the consequences of adding azimuthally asymmetric terms to (2).

If the solar wind is able to 'break open' a field line in the sense described above (cf. Figures 1 and 2), the 'break' seems likely to occur at that point on the field line where B is weakest. This rule holds trivially in the symmetrical case considered above, since the equatorial plane (on which the neutral sheet forms in this case) is clearly identical with the minimum-B surface of the field generated from (2). More generally, however, the configuration (open vs closed) of a field line seems to depend on the maximum value of $(u/c_A)^2$ along the field line, as outlined above. Since particle conservation implies that the plasma density is proportional to B/u for flow along a field line, it follows that $(u/c_A)^2 \propto u/B$. Given a reasonable variation of u with location in space (e.g., Pneuman and Kopp, 1971a), it is eminently plausible that the inner boundary of neutral surface (the cusp) would lie on the minimum-B surface of the field in a genuine MHD calculation. The analytical results obtained below are based on this hypothesis.

It is instructive in this context to consider the magnetic field generated by the scalar potential

$$V(r, \theta, \varphi) = -\,r_\odot B_1 (r_\odot/r)^2 \cos\theta - r_\odot B_2 (r_\odot/r)^3 \sin^2\theta \cos 2\varphi, \tag{7}$$

such that $\mathbf{B} = -\nabla V$. The angles θ and φ are the polar and azimuthal variables with respect to the magnetic (dipole) axis. The expansion coefficients B_1 (dipole) and B_2 (quadrupole) are considered to vary with time over the solar cycle, but not from one rotation to the next.

The minimum-B surface is found by solving the equation $\mathbf{B}\cdot\nabla B = 0$ for $\cos\theta$ as a function of r and φ. The result can be expressed as an odd-power series in

$(r_\odot B_2/rB_1)\cos 2\varphi$, with the lowest-order approximation given (Roederer *et al.*, 1973) by

$$\cos\theta \approx -(13B_2/9B_1)(r_\odot/r)\cos 2\varphi \tag{8}$$

for $r/r_\odot \gg |(B_2/B_1)|$. The presence of the quadrupole thus warps the minimum-B surface relative to the equatorial plane ($\theta=\pi/2$) of the dipole field. Moreover, the minimum-B surface has two 'sectors' in which $\cos\theta$ is positive and two 'sectors' in which $\cos\theta$ is negative, as φ varies from 0 to 2π.

A rough estimate for the resulting interplanetary sector structure can be obtained by inserting $r=r_A$ in (8) and projecting the neutral sheet radially from there. These extrapolations stretch the truth somewhat (as the solar wind stretches the field lines), but are in the spirit of the rather qualitative discussion undertaken here. If it is further assumed that the magnetic axis coincides with the rotation axis (i.e., that $\theta_0=0$), then (8) yields a simple expression for the probability that an observer at heliographic latitude λ is located south of the neutral sheet, viz.,

$$P(B_r>0) \approx (1/\pi)\,\mathrm{Re}\cos^{-1}(|9r_A B_1/13r_\odot B_2|\sin\lambda), \tag{9}$$

which is to say that each positive sector (in which $\sin\lambda<\cos\theta$) has an angular breadth given by the inverse-cosine function appearing in (9) if the argument of $\cos^{-1}$ is less than unity in absolute value. Otherwise (i.e., at latitudes too high to satisfy this condition) the interplanetary magnetic field would appear unsectored.

The interplanetary neutral surface derived from (8) in the manner outlined above is found to have an Archimedean-spiral structure, as observational data seem to require (see Wilcox, 1968). More specifically, within the framework of approximations previously cited (including the approximation that $\sin\theta\approx 1$), the neutral surface satisfies the equation

$$\cos\theta \approx -(13r_\odot B_2/9r_A B_1)\cos[2\varphi + 2(\Omega/u)(r-r_C)] \tag{10}$$

for $r>r_C$, where r_C is the corotation radius. It is reasonable to estimate r_C as the (latitude-dependent) heliocentric distance at which the corotation velocity $\mathbf{\Omega}\times\mathbf{r}$ becomes equal in magnitude to the Alfvén speed c_A.

The magnitude of r_C can be estimated by interpolation between coronal and interplanetary observations. As noted above, the use of (1) leads to an unreasonably large value of r_A ($\approx 16\ r_\odot$), whereas a cylindrical expansion at low latitude implies $r_A\approx 1.2\ r_\odot$ (a value that is somewhat too small). By similar reasoning, the spherical and cylindrical approximations imply $r_C\approx 52\ r_\odot$ and $r_C\approx 31\ r_\odot$, respectively. The latter estimate is compatible with observations (Brandt, 1966; Strong *et al.*, 1967) that the solar-wind velocity has an azimuthal component $u_\varphi\sim 10$ km s^{-1} at $r\sim 1$ AU. This would correspond to a corotation radius $r_C\approx 33\ r_\odot$.

One disturbing feature of both (8) and (10) is the truncation at first order in $(r_\odot B_2/r_A B_1)\cos 2\varphi$. These first-order equations are clearly invalid for $|r_\odot B_2/r_A B_1|\gtrsim 1$. A heuristic expedient to remedy this defect (in lieu of a laborious iteration) consists of replacing $\cos\theta$ on the left-hand side of (8) by $\cot\theta$. This substitution satisfactorily accommodates the limit in which $(r_{\odot}B_2/rB_1)$ approaches infinity, as well as the limit in which $(r_{\odot}B_2/rB_1)$ approaches zero. The conjectured interpolation leads to an interplanetary neutral surface satisfying the weakly transcendental equation

$$\cot\theta \approx -(13r_{\odot}B_2/9r_AB_1)\cos[2\varphi + 2(\Omega/u)(r - r_C)\sin\theta]. \qquad (11)$$

In the limit of a pure quadrupole field $(B_1 = 0)$, the four sector boundaries consist of corotating Archimedean surfaces given by

$$\varphi = \varphi_A - (\Omega/u)(r - r_C)\sin\theta, \qquad (12)$$

where $\varphi_A = \pm\pi/4$ and $\pm 3\pi/4$. In this limit, moreover, the origin of the interplanetary sector pattern closely resembles that postulated by Ahluwalia and Dessler (1962), in that distinct neutral sheets clearly separate interplanetary sectors corresponding to large regions of definite polarity on the solar surface. However, the limit $B_1 = 0$ corresponds to a very singular case of the present model. The more general consequence of (11) is the replacement of $\sin\lambda$ by $\tan\lambda$ in (9).

The observations of Rosenberg and Coleman (1969) indicate a strong correlation between $P(B_r > 0)$ and $\sin\lambda$, with $P \approx \frac{2}{3}$ for $\lambda \approx -7°$ and $P \approx \frac{1}{3}$ for $\lambda \approx +7°$. These results yield estimates for certain parameters of the heliomagnetic field. Specifically, if $B_2 = 0$, then it follows from (6) that $\theta_0 \approx 14°$. Alternatively, if $\theta_0 = 0$, then it follows from (9) that $|B_2/B_1| \approx 0.17\ (r_A/r_{\odot})$, which implies that the interplanetary neutral sheet is confined between heliomagnetic latitudes $\pm 14°$. Figure 3 illustrates the morphology of such a neutral sheet, given algebraically by (11) with $u \approx 320$ km s^{-1} and $r_C = 32\ r_{\odot}$. The spirals are contours of constant $\sin\lambda$ $(= \cos\theta)$ on the neutral surface.

3. Discussion

The foregoing considerations emphasize that the most probable origin of interplanetary sector structure is a single annular neutral sheet lying rather near the dipole equator but warped by a residual heliomagnetic quadrupole surviving at the inner boundary. Such a model is not in any sense contrived to match the observations, but rather stands as the most reasonable expectation for the structure of a stellar magnetic field in the absence of a strongly biased weighting of individual terms in the multipole expansion at $r = r_{\odot}$. The anticipated dominance of the dipole and quadrupole at $r \sim r_A$ follows from the more rapid radial attenuation of the higher-order terms.

The present model (which follows from the reasonable application of MHD concepts) quite naturally accounts for the observed correlation (Rosenberg and Coleman, 1969) between magnetic polarity and heliographic latitude in semi-quantitative terms. Moreover, if the parameters of the model are allowed to evolve appropriately on a 22-yr time scale, the corotating neutral sheet can account qualitatively for the several 'sector' patterns observed in and near the ecliptic plane during the course of a solar cycle. Furthermore, it is easy to understand a common aberration in the interplanetary pattern, namely the 'birth of a new sector' (Wilcox, 1968), as the development of a new wrinkle in the neutral sheet rather than as a totally catastrophic event.

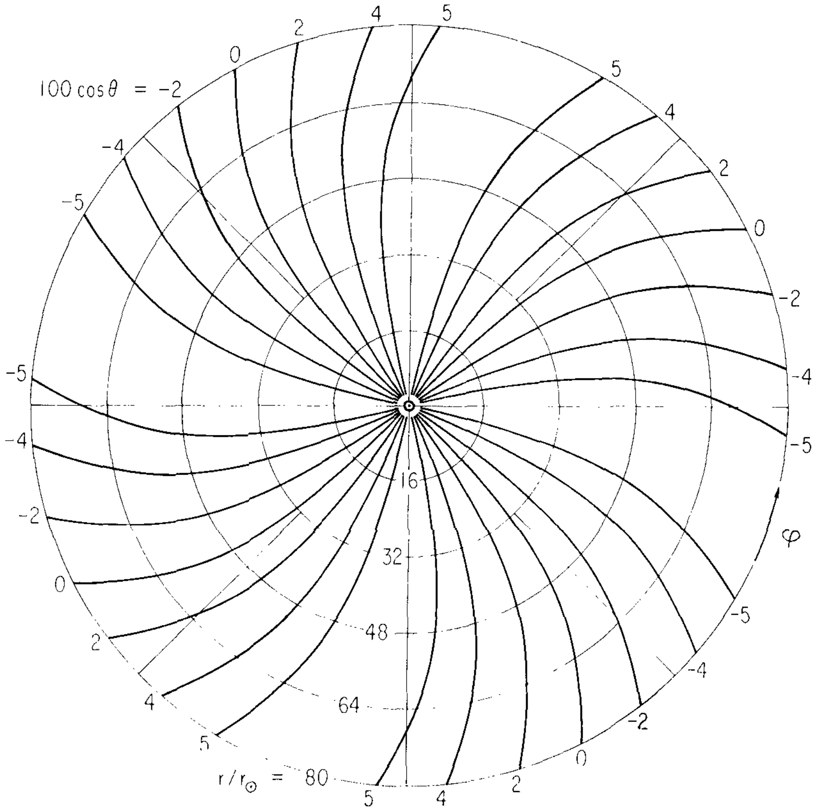

Fig. 3. Schematic spiral contours of constant latitude on heliomagnetic neutral surface, as given by (11) with $r_A = 2\,r_\odot$, $r_C = 32\,r_\odot$, $B_1 = 3B_2$, and $u = 80\,\Omega r_A$. Transition is visually smoothed at $r \sim r_C$.

The present model is somewhat oversimplified in that spherical harmonics higher than the quadrupole are neglected altogether in (7)–(11). To the extent that higher-order multipoles remain appreciable at $r \sim r_A$, they apparently serve only to distort the basic dipole-quadrupole 'sector' structure. Localized magnetic anomalies associated with active regions and individual flares on the Sun can contribute only minor perturbations to $V(r, \theta, \varphi)$ at $r \sim r_A$, where the 'sector' structure of the minimum-B surface is 'frozen' into the neutral sheet by virtue of solar-wind flow. However, the fact that one of the interplanetary sectors is typically only half as wide as the other three when viewed from the ecliptic (Wilcox, 1968) does suggest a modest contribution from one or more of the higher multipoles at $r \sim r_A$ (cf. Roederer *et al.*, 1973). Moreover, this contribution introduces a bias toward observing $B_r > 0$ in the ecliptic plane, since the anomalously small sector is negative (Wilcox, 1968).

It is interesting to compare the above ideas concerning interplanetary manifestations of the Sun's magnetic field with recent quantitative results describing the shape of the Earth's magnetic equatorial surface (Roederer *et al.*, 1973). The relevant geomagnetic results are reproduced in Figure 4, which is a contour plot showing the

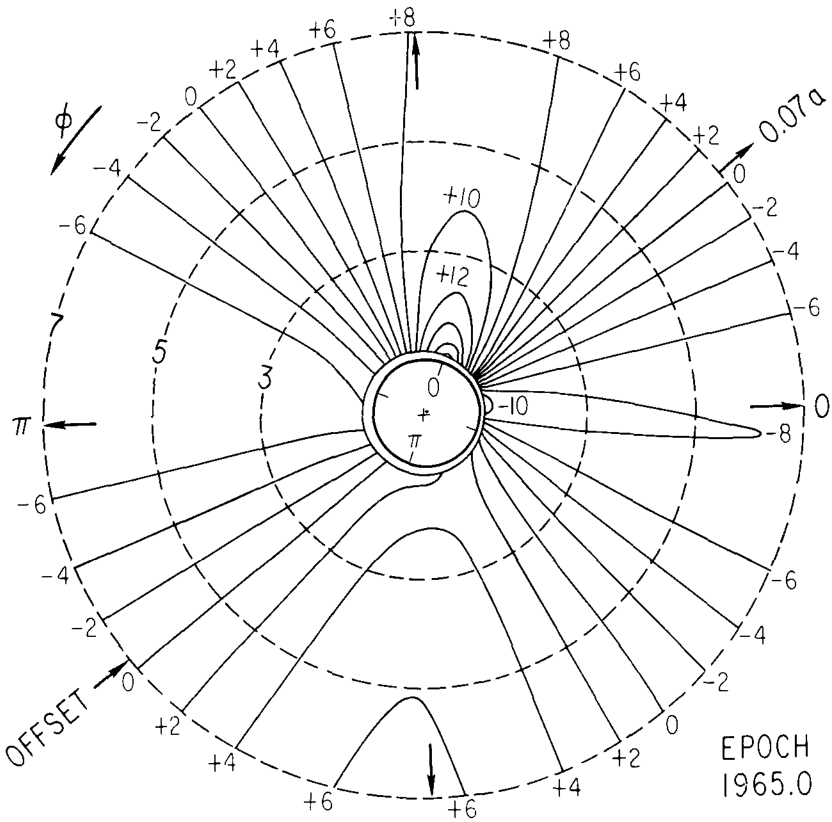

Fig. 4. Contours of constant linear elevation (solid curves), given in hundredths of an Earth radius, on the geomagnetic minimum-B surface. The reference plane for measuring elevation is the dipole equator. Contours of constant $B = L_0^{-3}B_1$ (dashed curves, except for $L_0 = 1.1$) on the same minimum-B surface form roughly circular projections on the equatorial plane. The indicated lateral offset of the geocenter relative to the magnetic center is of no consequence to the sector structure (Roederer *et al.*, 1973).

linear elevation z (solid curves) of the warped minimum-B surface relative to the plane that bisects the dipole axis. These elevations are given in units of $a/100$, where a is the radius of the Earth. It is both interesting and significant that Figure 4 reveals two sectors in which z is positive and two in which z is negative. This is the pattern predicted by (8), with a replacing $r_\odot$ and geomagnetic longitude (ϕ) replacing heliomagnetic longitude (φ). Higher-order ($n > 2$) multipoles substantially perturb the dipole-quadrupole pattern even at altitudes of several Earth radii, but these perturbations do not alter the fact that there are precisely four sectors of roughly similar width in ϕ. The dashed contours represent paths of constant $B = L_0^{-3}B_1$ on the geomagnetic equatorial surface, with $L_0 = 3$, 5, and 7, respectively.

To carry the analogy further, the Earth's magnetic field is known to develop a nightside neutral sheet by virtue of an interaction with the solar wind. The quiet solar wind described by Hundhausen (1970) contains the same energy density at $r \sim 1$ AU as a magnetic field $\sim 4\times 10^{-4}$ G. The Earth's idealized dipole field would attain this magnitude at an equatorial geocentric distance $\sim 9a$, which closely approximates the observed distance (e.g., Ness, 1969) between the geocenter and the inner edge of the Earth's neutral sheet. Moreover, the Earth's neutral sheet is evidently a downstream extension of the geomagnetic equator, as distinguished from the magnetospheric equator (Russell and Brody, 1967). This observation would seem compatible with the formation of a field cusp on the Earth's minimum-B surface. In view of these considerations, the terrestrial analogy apparently offers much toward an understanding of the magnetic sector structure observed in interplanetary space.

Equations (6) and (9), which summarize rather concisely the correlation between $P(B_r>0)$ and heliographic latitude in two limiting cases of interest, do not cover the more general case in which the neutral sheet is warped and $\theta_0\neq 0$. It would be possible, but not very illuminating in the present context, to construct an expression for $P(B_r>0)$ in the more general case. In any event, some further elaboration of the model seems prerequisite to a more detailed and quantitative comparison with the observational data.

The simplified description of the present model, insofar as it involves a juxtaposition of potential and flow-controlled **B** fields, bears a close resemblance to the source-surface model recently analyzed by Schatten (1971). The main conceptual distinction is that in Schatten's model the spherical source surface at $r=1.6\,r_\odot$ carries equivalent currents (cf. Feldstein, 1969) that contribute to the interior solution for **B**, while in the present model (which invokes no such source surface) the annular neutral sheet at $r\gtrsim r_A$ carries true currents that contribute to **B** everywhere.

Of course, the whole idea of selecting a source surface is to approximate the results that a full MHD solution (based on a numerically realistic model of the photospheric **B** field) would produce. The concept of an equivalent-current system is entirely legitimate for this purpose. However, the source-surface model described by Schatten (1971) apparently envisions coronal corotation to terminate at $r=1.6\,r_\odot$, i.e., at the postulated source surface. This would imply an azimuthal component of only $0.02\ \mathrm{km\,s^{-1}}$ for the solar-wind velocity at $r\sim 1$ AU. The observation that $u_\varphi\sim 10\ \mathrm{km\,s^{-1}}$ at $r\sim 1$ AU (Brandt, 1966; Strong *et al.*, 1967) suggests instead that $r_C\sim 30\,r_\odot$, in fairly good agreement with above-described estimates based on the present pseudo-MHD model. Thus, it might be reasonable (in the absence of a full MHD solution) to project the coronal field radially from a source surface at $r\sim 2\,r_\odot$ to a corotation boundary at $r\sim 30\,r_\odot$, and thence in the direction of an Archimedean spiral into interplanetary space.

Ultimately, however, the full MHD solution should verify the formation of a single annular neutral sheet that is a natural extension of the heliomagnetic minimum-B surface. The neutral sheet should bear the imprint of those few terms in the underlying multiple expansion that survive with appreciable magnitude at $r\sim r_A\sim 2\,r_\odot$, and should extend outward from a ring-shaped field cusp located at $r\sim r_A$. The interplanetary flow pattern of the solar wind should correspond to a roughly spherical expansion in the polar regions, but should resemble a cylindrical expansion in the region immediately adjacent to the neutral sheet. This is the configuration that seems most naturally consistent with the observational data. The foregoing considerations do not literally preclude the additional presence of isolated flux tubes emanating from localized coronal features, as envisioned by Ahluwalia and Dessler (1962), but the present results do suggest that such isolated flux tubes need not contribute in an essential way to the magnetic sector structure commonly observed in interplanetary space.

In summary, the present model for interpreting the interplanetary sector structure serves to unify diverse observations variously attributed to the heliomagnetic quadrupole (Wilcox, 1968) and dipole (Rosenberg and Coleman, 1969). The formation of a warped annular neutral surface (inner radius $\sim 2\, r_{\odot}$) is a natural MHD consequence of solar-wind flow out of the corona. The continuous MHD stress thus exerted by the solar wind enables the neutral sheet to be maintained even in the presence of field-line merging there (cf. Dessler and Michel, 1966; Pneuman and Kopp, 1971b). In its present form, the warped-sheet model for interplanetary sector structure offers very few explicitly quantitative predictions by which its merit might be evaluated. This situation could be improved by a numerical solution of the MHD problem for a realistic model of the photospheric **B** field. However, the major contribution of the present model is to provide a self-consistent physical framework for thinking about the interplanetary sector structure and its implications.

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