EFFECT OF PRESSURE GRADIENT ON FRICTION AND HEAT TRANSFER IN A DUSTY BOUNDARY LAYER

V. M. Agranat

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Approximate analytic expressions for the local friction and heat transfer coefficients in a dusty laminar boundary layer are obtained and tested in the case of an incompressible carrier phase, power-law variation of the external gas flow velocity and small velocity and temperature phase disequilibrium. These expressions supplement the numerical analysis of the dusty boundary layer on a blunt body [1, 2] and the asymptotic calculation of the friction and heat transfer in a quasiequilibrium dusty gas boundary layer on a plate [3]. The combined effect of dustiness and pressure gradient on the friction and heat transfer coefficients is discussed. The results obtained can be used for the practical calculation of the friction and heat transfer in a quasiequilibrium dusty laminar boundary layer and for interpreting the corresponding experimental data.

It is proposed to consider the steady plane flow past a body with constant surface temperature T_w of a uniform monodisperse stream of dusty viscous heat-conducting incompressible gas at large Reynolds numbers. The standard assumptions of the dusty laminar boundary layer model [1–7] are employed and it is assumed that at the outer edge of the boundary layer the gas velocity increases with the longitudinal coordinate in accordance with the power law $u_e = cx^m$ $(m > 0)$ and that at points remote from the body the temperature T_e of the gas and the particles is constant.

The mathematical formulation of the problem of the dynamic and thermal interaction of the wall and the dusty gas essentially depends on the values of the dimensionless parameters $\delta_1 = L/\ell_u$ and $\delta_2 = L/\ell_t$, which determine the relations between the characteristic length of the problem L and the characteristic dynamic and thermal particle relaxation lengths ℓ_u and ℓ_t [1–8]. When the Reynolds numbers for the flow over the particles are small and Stokes phase interaction conditions are realized, for gas suspensions the parameters δ_1 and δ_2 are of the same order [5–8]

$$\frac{\delta_2}{\delta_1}=\frac{2}{3\,\mathrm{Pr}\,\gamma}=O(1),\quad \mathrm{Pr}=\frac{\mu c_p}{\lambda},\quad \gamma=\frac{c_s}{c_p}\quad \delta_1=\frac{18\rho L}{\rho_s^\circ d}\mathrm{Re}_s^{-1},\quad \mathrm{Re}_s=\frac{V_\infty d}{\nu} \tag{1}$$

Here, d, ρ_s°, and c_s are the diameter, density and specific heat of the particles, and the other notation is that usually employed [5].

We will calculate the dimensionless local friction and heat transfer coefficients C_f and Nu_x in the case $Re_s \le 1$, when throughout the flow region it is possible to use relations (1) and the form of the equations of the nonisothermal dusty laminar boundary layer depends only on the value of the parameter δ_1. If the particles are small and the flow velocity is not very great, then $\delta_1 \gg 1$. In this case the velocity and temperature disequilibria of the flow are unimportant and the quasiequilibrium model [3–7] applies. As d and V_∞ increase, the parameter δ_1 decreases and it becomes necessary to use the complete equations of the nonequilibrium dusty boundary layer [1, 2]. We note that at large Re_s relations (1) are not satisfied, δ_1 may considerably exceed δ_2 [8] and in choosing the mathematical model it is necessary to estimate the values of both the parameters δ_1 and δ_2.

When $\delta_1 \gg 1$ the motion of the dusty gas in the boundary layer can be described by means of the following boundary-value problem:

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$$f''' + (1+\varkappa)ff'' = \beta(1+\varkappa)[(f')^2 - 1], \quad \Theta'' + (1+\varkappa\gamma)\Pr f\Theta' = 0$$

$$\eta=0:\ f=f'=\Theta=0;\ \eta=\infty:\ f'=\Theta=1, \quad \varkappa=\frac{\rho_{s\infty}}{\rho}, \quad \beta=\frac{2m}{m+1}, \quad \Theta=\frac{T-T_w}{T_e-T_w}, \quad \eta=y\left[\frac{u_e}{(2-\beta)\nu x}\right]^{1/2} \tag{2}$$

Here, f and η are the dimensionless stream function and self-similar variable [5], Θ is the dimensionless temperature, $\varkappa$ is the relative mass particle concentration in the approach stream, and a prime denotes the derivative with respect to η.

When $\varkappa=0$ the problem (2) coincides with the nonisothermal Falkner—Skan problem for a clean gas [5, 9], and $\beta = 0$ with the Blasius problem for a pseudogas [3] (it should be noted that the energy equation in [3] contains a printing error — the factor f is missing from the second term).

In order to calculate C_f and Nu_x it is sufficient to find the dimensionless quantities $a = f''(0)$ amd $b = \Theta'(0)$, since

$$\tfrac{1}{2}C_f \mathrm{Re}_x^{1/2} = (2-\beta)^{-1/2}a, \quad \mathrm{Nu}_x \mathrm{Re}_x^{-1/2} = (2-\beta)^{-1/2}b \tag{3}$$

By introducing the new variables and parameters

$$\eta_* = \sqrt{1+\varkappa}\,\eta, \quad f_* = \sqrt{1+\varkappa}\,f, \quad \Theta_* = \Theta, \quad \beta_* = \beta, \quad \Pr_* = \Pr\frac{1+\varkappa\gamma}{1+\varkappa} \tag{4}$$

for any $\varkappa$ we can reduce the calculation of a and b from problem (2) to finding the auxiliary quantities a_* and b_* from the nonisothermal Falkner—Skan problem

$$\frac{d^3 f_*}{d\eta_*^3} + f_*\frac{d^2 f_*}{d\eta_*^2} = \beta_*\left[\left(\frac{df_*}{d\eta_*}\right)^2 - 1\right], \quad \frac{d^2\Theta_*}{d\eta_*^2} + \Pr_* f_*\frac{d\Theta_*}{d\eta_*} = 0 \tag{5}$$

$$\eta_*=0:\ f_* = \frac{df_*}{d\eta_*} = \Theta_* = 0; \quad \eta_*=\infty:\ \frac{df_*}{d\eta_*} = \Theta_* = 1$$

$$a = \sqrt{1+\varkappa}\,a_*, \quad b = \sqrt{1+\varkappa}\,b_*, \quad a_* = \left.\frac{d^2 f_*}{d\eta_*^2}\right|_{\eta_*=0}, \quad b_* = \left.\frac{d\Theta_*}{d\eta_*}\right|_{\eta_*=0} \tag{6}$$

Here, β_* is the usual pressure gradient parameter of boundary layer theory, and the part of the Prandtl number is played by the effective Prandl number of the pseudogas $\Pr_*$ [3, 5—7]. When $\gamma \neq 1$ the parameters $\Pr_*$ and Pr are different, the difference increasing with increase in $\varkappa$, and for $\varkappa \geqslant 0$ we have the estimates

$$\gamma < \frac{\Pr_*}{\Pr} \leqslant 1 \quad (\gamma<1), \qquad 1 \leqslant \frac{\Pr_*}{\Pr} < \gamma \quad (\gamma>1) \tag{7}$$

If $|\gamma - 1| \ll 1$, then for any $\varkappa$ it can be assumed that $\Pr_* \approx \Pr$ (for example, for an aluminum particle suspension in air under standard conditions $\gamma \approx 0.9$). When $|\gamma - 1| \sim 1$ for an essentially dusty gas ($\varkappa \sim 1$) the difference between $\Pr_*$ and Pr should be taken into account. For example, in the case of an air suspension ($\Pr \approx 0.7$) when $\varkappa=2$ for water droplets ($\gamma \approx 4.2$) we have $\Pr_* \approx 2.2$, and for copper particles ($\gamma \approx 0.4$) : $\Pr_* \approx 0.4$.

Using a combination of asymptotic methods [10—12], on the basis of (5) we can obtain approximate analytic expressions for a_* and b_* with $0 \le \beta_* \le 2$ and $\Pr_* = O(1)$ that generalize the numerical and asymptotic data on these quantities for fixed β_* derived from the boundary layer theory for a clean gas [5, 9—11]. Finally, using (3), (4), and (6), we arrive at the following expressions for the relative friction and heat transfer coefficients in a quasiequilibrium dusty laminar boundary layer:

$$\frac{C_f}{C_{f0}^\circ} = \alpha_1(\varkappa)k_1(\beta), \quad \alpha_1 = (1+\varkappa)^{1/2}, \quad k_1 = \left(\frac{2}{2-\beta}\right)^{1/2}\frac{a^\circ(\beta)}{a_0^\circ} \tag{8}$$

$$\frac{\mathrm{Nu}_x}{\mathrm{Nu}_{x0}^\circ} = \alpha_1(\varkappa)k_2(\beta, \Pr, \varkappa, \gamma), \quad k_2 = \left(\frac{2}{2-\beta}\right)^{1/2}\frac{b_*}{b_0^\circ} \tag{9}$$

$$a^\circ = a_1^\circ = a_0^\circ\left[{}^1/_2\left(1+\sqrt{1+8.064\beta}\right)\right]^{1/2} \quad (0 \leqslant \beta \leqslant 0.35)$$

$$a_0^\circ = 0.470, \quad a^\circ = a_2^\circ = 1.233\beta^{1/2}, \ (0.7 \leqslant \beta \leqslant 2) \quad a^\circ = a_3^\circ = {}^1/_2(a_1^\circ + a_2^\circ) \quad (0.35 < \beta < 0.7) \tag{10}$$

$$b_* = 0.616(a^\circ \mathrm{Pr}_*)^{1/3}(1+0.153z+0.077z^2), \quad z = \beta(a^\circ)^{-4/3}\mathrm{Pr}_*^{-1/3} \tag{11}$$

Here, the degree symbol superscript relates to the characteristics of the clean gas ($\varkappa=0$), and the zero subscript to the characteristics of the zero-gradient flow ($\beta = 0$).

It should be noted that relations (8) and (9) are exact within the framework of model (2), while the error of the approximate expressions (10) and (11) depends on the values of β and Pr_*. Relations (10) generalize the numerical data of [5, 9, 10] with a maximum error of ±3.5%, and for $\beta = 0$ (longitudinal flow over a plate), 0.5 (flow over the stagnation point of an axisymmetric body) and 1 (flow over the stagnation point of a cylindrical body) the error is tenths of a percent. When $\beta = 1$ the error of relation (11) does not exceed 0.5% over the entire range of Prandtl numbers indicated in [5] (for $0.6 \leq \mathrm{Pr}_* \leq 15$), and when $\beta = 0.5$ it gives results that agree with the numerical results of [10] to within 1.3 and 0.6% for $\mathrm{Pr}_* = 0.7$ and 1, respectively. When $\beta = 1$ and $\mathrm{Pr}_* = \mathrm{Pr}$ relation (11) coincides with the expression given in [11], and as $\beta \to 0$ or $\mathrm{Pr}_* \to \infty$ it takes the form:

$$b_* = 0.616[a^\circ(\beta)\mathrm{Pr}]^{1/3}\left(\frac{1+\varkappa\gamma}{1+\varkappa}\right)^{1/3} \tag{12}$$

Hence for $\varkappa=0$ and $\beta = 1$ in complete conformity with [9] we obtain $b_* = b^\circ = 0.661\mathrm{Pr}^{1/3}$ ($\mathrm{Pr} \to \infty$), and for $\beta = 0$ in conformity with [3]

$$b_* = b_0 = b_0^\circ\left(\frac{1+\varkappa\gamma}{1+\varkappa}\right)^{1/3}, \quad b_0^\circ = a_0^\circ\,\mathrm{Pr}^{1/3} \tag{13}$$

From expressions (8) and (10) it follows that the dustiness and the pressure gradient act independently on the friction, the coefficient α_1 reflecting the increase in friction as a result of the dustiness of the gas having the same form as when $\beta = 0$ [3]. On the other hand, in the context of relations (9) and (11) it is not generally possible to separate the effects of the particles and pressure gradient on the heat transfer rate and, consequently, in estimating the heat transfer coefficient Nu_x it is not generally possible to apply the superposition principle. From (11)–(13) it follows that this principle holds only in particular cases: $\gamma = 1$ ($\mathrm{Pr}_* = \mathrm{Pr}$), $\beta \to 0$, $\mathrm{Pr}_* \to \infty$, when $z = z(\beta, \mathrm{Pr})$ or $z \to 0$. Since for gas suspensions, when (7) is taken into account, we have $\mathrm{Pr}_* = O(1)$, while the case $\beta = 0$ was examined in [3], of the three cases in question when $\beta \neq 0$ only the case $\gamma = 1$ is of practical interest.

When $\gamma = 1$ from (9) and (11) there follows

$$\frac{\mathrm{Nu}_x}{\mathrm{Nu}_{x0}^\circ} = \alpha_1(\varkappa)k_2(\beta, \mathrm{Pr}) \tag{14}$$

so that the coefficient α_2 reflecting the influence of the particles on Nu_x coincides with α_1. This is attributable to the fact that when $\gamma = 1$ ($c_s = c_p$) the specific heat of the pseudogas c_{p*} coincides with the specific heat of the gas c_p and the intensification of friction and heat transfer in the quasiequilibrium dusty laminar boundary layer is caused only by the increase in the density of the pseudogas as compared with the density of the gas ρ by a factor of α_1^2 [5–7]. When $\gamma \neq 1$ and $\beta \neq 0$ the effect of the dustiness on Nu_x is more complex than its effect on C_f.

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HYDRAULIC FRACTURING IN AN INHOMOGENEOUS RESERVOIR

A. V. Domanskii

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A method of estimating the change in well productivity resulting from the hydraulic fracturing of a finite reservoir, piecewise-homogeneous in the horizontal and vertical directions, with an arbitrary number of cracks is proposed. A correction coefficient that enables these estimates to take into account the effect of capillary trapping of mud and fracturing fluid filtrate is derived [1, 2].

Questions relating to the mechanism of formation of artificial cracks in a reservoir were considered, for example, in [3, 4]. In [5] well productivity estimates were constructed for the case of one horizontal or two vertical cracks in a finite homogeneous reservoir, and in [6] the case of an arbitrary number of vertical cracks in an infinite homogeneous reservoir was investigated. The problem of the flow into a well from a horizontal crack was examined in [6, 7].

1. Hydraulic Fracturing in a Homogeneous Reservoir

Let a perfect well of radius r_c lie at the center of a homogeneous reservoir of thickness H having in plan the shape of a circle of radius r_k. By means of hydraulic fracturing N identical, symmetrically distributed vertical cracks of height H are created in the reservoir. These cracks radiate from the walls of the well, extending a distance r_0 from its axis (Fig. 1). The cracks are assumed to be fairly narrow and not to offer hydraulic resistance to the fluid flowing in them. The pressure in a crack is assumed to be everywhere equal to the bottom hole pressure.

By virtue of the chosen geometry of the flow zone the equation for the pressure has the form:

$$r^{-1}(rp_r)_r+r^{-2}p_{\varphi\varphi}=0 \tag{1.1}$$

Here and in what follows the subscripts r, φ, z, x, and y denote differentiation with respect to the corresponding variable.

Neglecting the transverse dimensions of the cracks, we can write the boundary conditions for Eq. (1.1) in the form:

$$p=p_0=\text{const}\ (r=r_k)\ \ p_r=0\ \ (r=r_c,\ 0\leqslant\varphi\leqslant 2\pi),\ \ p|_M=p_c=\text{const},\ p_c<p_0,\ \Delta p\equiv p_0-p_c \tag{1.2}$$

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