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PARTICLE DEPOSITION FROM A TURBULENT FLOW

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The diffusion equations and boundary condition for particle deposition from a turbulent flow are obtained on the basis of the kinetic equation for the probability density of the particle distribution. This approach makes it possible to calculate the deposition fairly simply without introducing additional empirical information relating to the particles (empirical constants are needed only for calculating the characteristics of the turbulent carrier flow).

A considerable number of experimental and theoretical studies (for a review see, for example, [1–3]) have been devoted to the study of particle deposition from turbulent gas streams on the walls of pipes and ducts. For relatively small particles easily entrained into the fluctuating motion of the carrier stream the principal deposition mechanisms are the turbulent diffusion and migration of the particles due to the transverse gradients of the particle concentration and the gas fluctuation intensity, respectively; the influence of such factors as the mean interphase slip, the force of gravity and the Magnus effect is of secondary importance. Most methods of analyzing particle deposition are based on the ordinary diffusion equation; with this approach in order to describe the turbulent migration of the particles effectively it is necessary to introduce the concept of a free-inertial particle path in the boundary zone (see, for example, [4, 5]) or other models without a very sound physical basis. In [2, 6] the migration deposition mechanism is taken directly into account. The formulation of the boundary condition for the diffusion equation is also a complex problem. As the boundary condition on a totally absorbing wall it is usual to take zero particle concentration. The boundary condition for the diffusion equation is discussed in [2, 7, 8]. Particularly interesting is the method of calculating particle deposition based on the solution of the kinetic equation for the probability density of the particle coordinate and velocity distribution [3]. However, for a real turbulent flow the calculations based on the direct solution of the kinetic equation in coordinate and velocity space are rather laborious.

1. The equation of motion of an individual solid spherical particle is written in the form:

$$\frac{dv_{pi}}{dt}=\frac{u_i(\mathbf{R}_p(t),t)-v_{pi}(t)}{\tau}+F_i(\mathbf{R}_p(t),t)+f_i(\mathbf{R}_p(t),t)$$
$$\frac{dR_{pi}}{dt}=v_{pi},\quad \tau=\frac{2\rho_2a^2}{9\rho_1\nu}\frac{1+2\alpha\lambda/a}{1+3\alpha\lambda/a} \tag{1.1}$$

where $\mathbf{R}_p(t)$ and $\mathbf{v}_p(t)$ are the coordinate and velocity of the particle, $\mathbf{u}(\mathbf{x},t)$ is the velocity of the turbulent carrier flow, τ is the dynamic relaxation time of the Stokes particle with allowance for the slip of the gas molecules on the surface, ρ_1 and ρ_2 are are the densities of the fluid and solid phases, ν is the kinematic viscosity of the fluid phase, λ is the mean free path of the molecules in the fluid, a is the particle radius, and α is the slip coefficient (of the order of unity). The first term on the

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right side of (1.1) describes the viscous interaction between the phases in the Stokes approximation; the second term $F_i(\mathbf{x}, t)$ determines the external force acting on the particle (for example, the force of gravity); the last term $f_i(x, t)$ depends on the random force acting on the particle in a random field δ-correlated with respect to time and is associated with the Brownian motion effect.

In writing Eq. (1.1) we assumed that the density of the fluid phase is much less than that of the particle material; accordingly, the forces depending on the pressure gradient in the fluid, the apparent mass and the nonstationarity of the flow (Basset force) can be disregarded. Expression (1.1) is a Langevin equation depending on two uncorrelated random fields describing the velocity of the turbulent carrier flow $\mathbf{u}$ and the Brownian motion $\mathbf{f}$. The mean mass (and a fortiori volume) particle concentration is assumed to be small; accordingly, the reaction of the particles on the carrier flow characteristics and the interaction of the particles themselves as a result of collisions can be neglected.

We introduce the distribution function of the particles with respect to the coordinates $\mathbf{x}$ and velocities $\mathbf{v}$ in phase space

$$p(\mathbf{x}, \mathbf{v}, t) = \langle \delta(\mathbf{x}-\mathbf{R}_p(t))\delta(\mathbf{v}-\mathbf{v}_p(t))\rangle \tag{1.2}$$

where the averaging is carried out over the realizations of the turbulent flow and the random field $\mathbf{f}$.

Using (1.1), we differentiate (1.2) with respect to time:

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x_k}\left\langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)\frac{dR_{pk}}{dt}\right\rangle - \frac{\partial}{\partial v_k}\left\langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)\frac{dv_{pk}}{dt}\right\rangle =$$

$$-\frac{\partial}{\partial x_k}\langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)v_{pk}\rangle - \frac{\partial}{\partial v_k}\left\langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)\left(\frac{u_k-v_{pk}}{\tau}+F_k+f_k\right)\right\rangle \tag{1.3}$$

$$\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\delta(\mathbf{v}-\mathbf{v}_p(t))v_{pk}(t)\rangle = v_k p(\mathbf{x}, \mathbf{v}, t) \tag{1.4}$$

We represent the carrier phase velocity in the form of an averaged and a fluctuation term:

$$u_k(\mathbf{x}, t) = U_k(\mathbf{x}, t) + u_k'(\mathbf{x}, t),\ U_k = \langle u_k\rangle,\ \langle u_k'\rangle = 0 \tag{1.5}$$

Then from expression (1.3), using (1.4) and (1.5), we obtain the following equation for the distribution function:

$$\frac{\partial p}{\partial t} + v_k\frac{\partial p}{\partial x} + \frac{\partial}{\partial v_k}\left(\frac{U_k - v_k}{\tau} + F_k\right)p + \frac{\partial}{\partial v_k}\left(\frac{\langle pu_k'\rangle}{\tau} + \langle pf_k\rangle\right) = 0$$

$$\langle pu_k'\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)u_k'\rangle,\ \langle pf_k\rangle = \langle \delta(\mathbf{x}-\mathbf{R}_p)\delta(\mathbf{v}-\mathbf{v}_p)f_k\rangle \tag{1.6}$$

In order to obtain a closed equation for p, it is necessary to find expressions for the correlations $\langle pu_k'\rangle$ and $\langle pf_k\rangle$. In order to calculate these correlations, we use the Furutsu—Novikov equation [9], assuming that the random fields u_k' and f_k are Gaussian:

$$\langle z(\mathbf{x})R[z]\rangle = \int d\mathbf{x}_1 \langle z(\mathbf{x})z(\mathbf{x}_1)\rangle\left\langle \frac{\delta R[z(\mathbf{x})]}{\delta z(\mathbf{x}_1)}\right\rangle \tag{1.7}$$

where $z(\mathbf{x})$ is a random process in x space; $R|x|$ is a functional depending on the random process; and $\delta R/\delta z$ is a functional (variational) derivative.

In accordance with (1.7) and (1.2), we have

$$\langle pu_i'\rangle = \iint \langle u_i'(\mathbf{x}, t)u_k'(\mathbf{x}_1, t_1)\rangle \frac{\delta p(\mathbf{x}, \mathbf{v}, t)}{\delta u_k'(\mathbf{x}_1, t_1)}d\mathbf{x}_1\,dt_1 \tag{1.8}$$

$$\frac{\delta p(\mathbf{x}, \mathbf{v}, t)}{\delta u_k'(\mathbf{x}_1, t_1)} = -\frac{\partial}{\partial x_j}\left\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\delta(\mathbf{v}-\mathbf{v}_p(t))\frac{\delta R_{pj}(t)}{\delta u_k'(\mathbf{x}_1, t_1)}\right\rangle -$$

$$\frac{\partial}{\partial v_j}\left\langle \delta(\mathbf{x}-\mathbf{R}_p(t))\delta(\mathbf{v}-\mathbf{v}_p(t))\frac{\delta v_{pj}(t)}{\delta u_k'(\mathbf{x}_1, t_1)}\right\rangle \tag{1.9}$$

In order to find the functional derivatives in (1.9), we write the equations of motion of the individual particle (1.1) in integral form:

$$v_{pi}(t)=v_{pi}(0)\exp\left(-\frac{t}{\tau}\right)+\int_0^t\left[\frac{u_i(\mathbf{R}_p(t_1),t_1}{\tau}+F_i(\mathbf{R}_p(t),t_1)+f_i(\mathbf{R}_p(t_1),t_1)\right]\times$$

$$\exp\left(-\frac{t-t_1}{\tau}\right)dt_1,\quad R_{pi}(t)=R_{pi}(0)+\int_0^t v_{pi}(t_1)\,dt_1 \qquad (1.10)$$

We apply to (1.10) the functional differentiation operator, bearing in mind the relation $\delta u_i(\mathbf{x},t)/\delta u_j(\mathbf{x}_1,t_1)=\delta_{ij}\delta(\mathbf{x}-\mathbf{x}_1)\delta(t-t_1)$, the causality principle [9] and the fact that the initial conditions $v_{pi}(0)$ and $R_{pi}(0)$ do not depend on u_i'. In order to obtain a kinetic equation for the particles containing derivatives with respect to the coordinates and velocities of not higher than second order, we neglect the spatial nonuniformity of the carrier flow velocity field in calculating the functional derivatives. As a result we obtain

$$\frac{\delta v_{pi}(t)}{\delta u_j'(\mathbf{x}_1,t_1)}=\frac{1}{\tau}e^{-\beta}\delta_{ij}\delta(\mathbf{x}_1-\mathbf{R}_p(t_1)),\quad \beta=\frac{t-t_1}{\tau}$$

$$\frac{\delta R_{pi}(t)}{\delta u_j'(\mathbf{x}_1,t_1)}=\int_{t_1}^t\frac{\delta v_{pi}(t_2)}{\delta u_j'(\mathbf{x}_1,t_1)}dt_2=(1-e^{-\beta})\delta_{ij}\delta(\mathbf{x}_1-\mathbf{R}_p(t_1)) \qquad (1.11)$$

When (1.9) and (1.11) are taken into account, from (1.8) there follows

$$\langle pu_i'\rangle=\tau g\langle u_i'u_k'\rangle\frac{\partial p}{\partial x_k}-f_1\langle u_i'u_k'\rangle\frac{\partial p}{\partial v_k}$$

$$\frac{1}{\tau}\int_0^\infty\langle u_i'(\mathbf{x},t)u_j'(\mathbf{R}_p(t_1),t_1)\rangle(1-e^{-\beta})dt_1=g\langle u_i'u_j'\rangle,\quad \frac{1}{\tau}\int_0^\infty\langle u_i'(\mathbf{x},t)u_j'(\mathbf{R}_p(t_1),t_1)\rangle e^{-\beta}dt_1=f_1\langle u_i'u_j'\rangle \qquad (1.12)$$

where $\langle u_i'u_j'\rangle$ are the second one-point one-time moments of the carrier flow velocity fluctuations.

Approximating the two-time correlation moments for $\mathbf{R}_p(t)=\mathbf{x}$ by the step function [10]

$$\langle u_i'(\mathbf{x},t)u_j'(\mathbf{R}_p(t_1),t_1)\rangle=\begin{cases}\langle u_i'u_j'\rangle, & |t_1-t|\leqslant T\\ 0, & |t_1-t|>T\end{cases}$$

where T is the particle—turbulent mole interaction time, we obtain the following expressions for the coefficients of entrainment of the particles in the fluctuating motion of the carrier flow:

$$f_1=1-\exp\left(-\frac{T}{\tau}\right),\quad g=\frac{T}{\tau}-1+\exp\left(-\frac{T}{\tau}\right) \qquad (1.13)$$

The random field f_i is assumed to be δ-correlated with respect to time, i.e., when $\mathbf{R}_p(t)=\mathbf{x}$

$$\langle f_i(\mathbf{x},t)f_j(\mathbf{R}_p(t_1),t_1)\rangle=D\delta(t-t_1)\delta_{ij}/\tau,\quad D=\frac{(1+3\alpha\lambda/a)k\theta}{6\pi\rho_1\nu a(1+2\alpha\lambda/a)}$$

where D is the Brownian diffusion coefficient; k is the Boltzmann constant; and θ is temperature.

Then

$$\langle pf_i\rangle=\iint\langle f_i(\mathbf{x},t)f_k(\mathbf{x}_1,t_1)\rangle\frac{\delta p(\mathbf{x},\mathbf{v},t)}{\delta f_k(\mathbf{x}_1,t_1)}d\mathbf{x}_1\,dt_1=-\frac{D}{\tau^2}\delta_{ik}\frac{\partial p}{\partial v_k} \qquad (1.14)$$

Substituting (1.12) and (1.14) in (1.6), we obtain a closed equation for the probability density function of the particle coordinate and velocity distribution in the turbulent flow:

$$\frac{\partial p}{\partial t}+v_k\frac{\partial p}{\partial x_k}+\frac{\partial}{\partial v_k}\left(\frac{U_k-v_k}{\tau}+F_k\right)p=g\langle u_i'u_k'\rangle\frac{\partial^2 p}{\partial x_i\,\partial v_k}+$$

$$\frac{f_1}{\tau}\langle u_i'u_k'\rangle\frac{\partial^2 p}{\partial v_i\,\partial v_k}+\frac{D}{\tau^2}\frac{\partial^2 p}{\partial v_k\,\partial v_k} \tag{1.15}$$

For laminar flow ($\langle u_i'u_k'\rangle = 0$) Eq. (1.15) goes over into the Fokker–Planck equation for the Brownian particle distribution function [9, 11]. When $\tau \gg T$ ($D \approx 0$, $f_1 \approx T/\tau$, $g \approx T^2/2\tau^2 \approx 0$) Eq. (1.15) conforms to the equation for the probability density of the large particle distribution in a turbulent flow [2].

From (1.15) we obtain the equations for the moments. Integrating Eq. (1.15) over the entire volume in velocity space, we obtain the mass balance equation for the solid phase

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial x_k}CV_k=0 \quad C=\int p\,dv, \quad V_k=\frac{1}{C}\int v_k p\,dv \tag{1.16}$$

Here, C is the particle concentration, and V_k is the mean velocity of the solid phase. Multiplying (1.15) by v_i and integrating with respect to v, we obtain the equation for the averaged velocity of the solid phase [12]

$$\begin{gathered}\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}=-\frac{\partial\langle v_i'v_k'\rangle}{\partial x_k}+\frac{U_i-V_i}{\tau}+F_i-\frac{D_{ik}}{\tau}\frac{\partial \ln C}{\partial x_k}\\ \langle v_i'v_k'\rangle=\frac{1}{C}\int v_i'v_k'p\,dv, \quad D_{ik}=\tau(\langle v_i'v_k'\rangle+g\langle u_i'u_k'\rangle), \quad v_i'=v_i-V_i\end{gathered} \tag{1.17}$$

Here, $\langle v_i'v_k'\rangle$ is the solid-phase stress tensor, and D_{ik} is the diffusion tensor. The first term on the right side of (1.17) describes the appearance of stresses in the solid phase as a result of the involvement of the particles in the turbulent motion of the carrier flow and the Brownian motion, while the last term determines the so-called diffusion force. Mulitplying (1.16) by $v_i'v_j'$ and carrying out the integration with respect to v, we obtain the equation for the second moments of the solid-phase velocity fluctuations

$$\begin{gathered}\frac{\partial\langle v_i'v_j'\rangle}{\partial t}+V_k\frac{\partial\langle v_i'v_j'\rangle}{\partial x_k}+(\langle v_i'v_k'\rangle+g\langle u_i'u_k'\rangle)\frac{\partial V_j}{\partial x_k}+\\ (\langle v_j'v_k'\rangle+g\langle u_j'u_k'\rangle)\frac{\partial V_i}{\partial x_k}+\frac{1}{C}\frac{\partial}{\partial x_k}(C\langle v_i'v_j'v_k'\rangle)=\frac{2}{\tau}\left(f_1\langle u_i'u_j'\rangle+\frac{D}{\tau}\delta_{ij}-\langle v_i'v_j'\rangle\right)\\ \langle v_i'v_j'v_k'\rangle=\frac{1}{C}\int v_i'v_j'v_k'p\,dv\end{gathered} \tag{1.18}$$

From Eq. (1.18) in the approximation that disregards the nonstationarity and the nonuniformity of the solid-phase velocity field (more precisely, in the local steady-state and uniform approximation) the following expression for the stresses can be obtained:

$$\langle v_i'v_j'\rangle=f_1\langle u_i'u_j'\rangle+\frac{D}{\tau}\delta_{ij} \tag{1.19}$$

From expression (1.19) it follows that the small particles are completely entrained in the turbulent motion of the carrier flow (i.e., $f_1 \to 1$ as $\tau/T \to 0$), whereas the large particles are not entrained in the fluctuating motion (since $f_1 \to 0$ as $\tau/T \to \infty$).

Without taking the convective terms into account, from Eq. (1.17) we can obtain the following expression for the mean velocity of the solid phase:

$$V_i=U_i+\tau F_i-\tau\frac{\partial\langle v_i'v_k'\rangle}{\partial x_k}-D_{ik}\frac{\partial \ln C}{\partial x_k} \tag{1.20}$$

Using (1.19) and (1.20), we obtain the mass balance equation for the solid phase (1.16) in the form of the diffusion equation

$$\begin{gathered}\frac{\partial C}{\partial t}+\frac{\partial}{\partial x}[(U_k+\tau F_k)C]=\frac{\partial}{\partial x_k}\left[D_{ik}\frac{\partial C}{\partial x_i}+C\frac{\partial}{\partial x_i}(qd_{ik}+D\delta_{ik})\right]\\ D_{ij}=d_{ij}+D\delta_{ij},\ d_{ij}=T\langle u_i'u_j'\rangle,\ q=\tau f_1/T\end{gathered} \tag{1.21}$$

Here, d_{ij} is the turbulent diffusion coefficient, and q is the migration coefficient. As distinct from the ordinary diffusion equation with right side $\partial(D_{ik}\partial C/\partial x_i)/\partial x_k$, in addition to diffusion transport Eq. (1.21) also describes the migration of the particles due mainly to the nonuniformity of the turbulent fluctuation field of the carrier flow. As the particle size increases, the migration coefficient q increases from zero to unity; accordingly, whereas the deposition of the small particles ($\tau/T \ll 1$) is primarily determined by the process of turbulent and Brownian diffusion, as the particle size increases the role of the migration mechanism becomes more important and the deposition of the relatively large particles ($\tau/T \sim 1$) is essentially determined by the turbulent migration process.

2. In order to obtain the boundary condition for Eq. (1.21) on a totally or partially particle-absorbing wall we construct the solution of Eq. (1.15) in a thin kinetic layer near the surface. In Eq. (1.15) as applied to this layer it is possible to retain only the terms relating to the components in the direction y normal to the wall:

$$\frac{(qD_T+D)}{\tau}\frac{\partial^2 p}{\partial v_y^2}+\frac{\partial v_y p}{\partial v_y}=\tau v_y\frac{\partial p}{\partial y}+(U_y+\tau F_y)\frac{\partial p}{\partial v_y}-(1-q)D_T\left(\frac{\partial^2 p}{\partial y\,\partial v_y}\right) \tag{2.1}$$

where $D_T = T\langle u_y'^2\rangle$. We solve Eq. (2.1) by the perturbation method, treating the right side as a perturbing factor, i.e., representing the solution in the series form $p = p_1 + p_2$, where the first (equilibrium) term is given by

$$p_1=C\left[\frac{\tau}{2\pi(qD_T+D)}\right]^{1/2}\exp\left[-\frac{\tau v_y^2}{2(qD_T+D)}\right]$$

The function p_2 is determined from the equation

$$\frac{(qD_T+D)}{\tau}\frac{\partial^2 p_2}{\partial v_y^2}+\frac{\partial v_y p_2}{\partial v_y}=\tau v_y\frac{\partial p_1}{\partial y}+(V_y+\tau F_y)\frac{\partial p_1}{\partial v_y}-(1-q)D_T\frac{\partial^2 p_1}{\partial y\,\partial v_y}=$$

$$\left[\frac{\tau^3}{2\pi(qD_T+D)}\right]^{1/2}\exp\left[-\frac{\tau v_y^2}{2(qD_T+D)}\right]\frac{v_y}{(qD_T+D)}\left\{(D_T+D)\frac{dC}{dy}+\right.$$

$$\left.\frac{C}{2}\left[\frac{\tau(D_T+D)v_y^2}{qD_T+D}-3D_T+2qD_T-D\right]\frac{d\ln(qD_T+D)}{dy}-(U_y+\tau F_y)C\right\} \tag{2.2}$$

The solution of Eq. (2.2) is

$$p_2=-\left[\frac{\tau^3}{2\pi(qD_T+D)}\right]^{1/2}\exp\left[-\frac{\tau v_y^2}{2(qD_T+D)}\right]\frac{v_y}{(qD_T+D)}\left\{(D_T+D)\frac{dC}{dy}+\right.$$

$$\left.\frac{C}{2}\left[(2q-1)D_T+D+\frac{(D_T+D)\tau v_y^2}{3(qD_T+D)}\right]\frac{d\ln(qD_T+D)}{dy}-(V_y+\tau F_y)C\right\}$$

Thus, in the second approximation the solution of Eq. (2.1) takes the form:

$$p=C\left[\frac{\tau}{2\pi(qD_T+D)}\right]^{1/2}\exp\left[-\frac{\tau v_y^2}{2\pi(qD_T+D)}\right]\left\{1-\frac{\tau v_y}{qD_T+D}\left[(D_T+D)\frac{d\ln C}{dy}+\right.\right.$$

$$\left.\left.\frac{1}{2}(2qD_T-D_T+D)+\frac{(D_T+D)\tau v_y^2}{3(qD_T+D)}\right]\frac{d\ln(qD_T+D)}{dy}-(U_y+\tau F_y)\right]\right\}. \tag{2.3}$$

On the basis of solution (2.3) we determine the particle fluxes incident on and reflected from the wall J_f and J_r, respectively:

$$\begin{aligned} J_f&=-\int_{-\infty}^{0} v_y p\,dv_y=\left(\frac{qD_T+D}{2\pi\tau}\right)^{1/2}C-(U_y+\tau F_y)\frac{C}{2}+\frac{(D_T+D)}{2}\frac{dC}{dy}+\frac{C}{2}\frac{d(qD_T+D)}{dy}\\ J_r&=\int_{0}^{\infty} v_y p\,dv_y=\left(\frac{qD_T+D}{2\pi\tau}\right)^{1/2}C+(U_y+\tau F_y)\frac{C}{2}-\frac{(D_T+D)}{2}\frac{dC}{dy}-\frac{C}{2}\frac{d(qD_T+D)}{dy} \end{aligned} \tag{2.4}$$

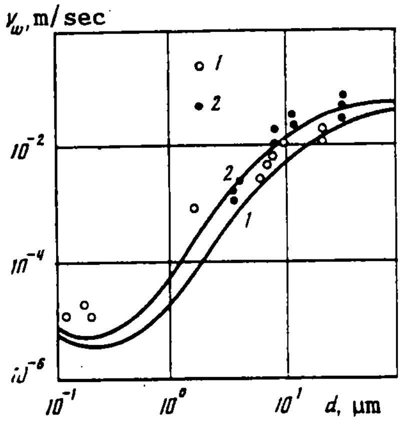

Fig. 1

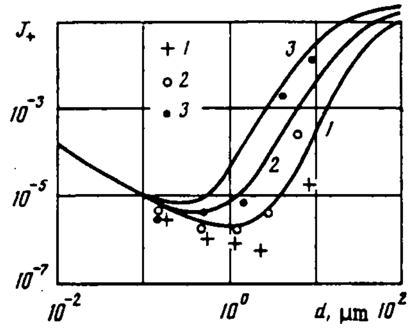

Fig. 2

All the quantities in expressions (2.4) correspond to their values at the wall. From the standpoint of particle reflection or absorption the physical properties of the surface can be characterized by the reflection coefficient χ, equal to the probability of a particle breaking away from the wall after reaching it (return to the flow of a particle colliding with the wall) or by the absorption coefficient $1 - \chi$, equal to the probability of a particle sticking to the wall. Thus, the reflection coefficient is equal to the ratio of the reflected and incident particle fluxes

$$\chi = J_r/J_f \tag{2.5}$$

Substituting expressions (2.4) in (2.5), we obtain the following boundary condition for the particle diffusion equation (1.21):

$$C_w = \frac{1+\chi}{1-\chi}\left(\frac{\pi\tau}{2(qD_T+D)}\right)^{1/2}\left[(D_T+D)\frac{dC}{dy} + C\frac{d(qD_T+D)}{dy} - (U_y+\tau F_y)C\right] = \frac{1+\chi}{1-\chi}\left(\frac{\pi\tau}{2(qD_T+D)}\right)^{1/2} J_w \tag{2.6}$$

where $J_w = J_f - J_r$. The boundary condition relates the particle concentration at the wall C_w and the net particle flux J_w. The terms in square brackets in (2.6) determine the diffusion, migration and convective particle fluxes to the wall, respectively. It follows from (2.6) that, contrary to the opinion widely expressed in the literature, even on a totally absorbing surface ($\chi = 0$) the particle concentration is not equal to zero. Naturally, for a totally reflecting surface ($\chi = 1$) from (2.6) we obtain $J_w = 0$.

3. Let us consider the process of particle deposition from a steady hydrodynamically developed turbulent flow in a plane-parallel or cylindrical channel. In the boundary layer approximation without allowance for body forces the diffusion equation (1.21) takes the form:

$$\frac{\partial}{\partial x}(r^\alpha U_x C) = \frac{\partial}{\partial r}\left\{r^\alpha\left[(D_T+D)\frac{\partial C}{\partial r} + C\frac{\partial}{\partial r}(qD_T+D)\right]\right\} \tag{3.1}$$

where x and $r = 1 - y$ are the coordinates in the longitudinal and transverse directions, and $\alpha = 0$ and 1 for plane-parallel and cylindrical channels, respectively. Integrating (3.1) over the channel cross section, we obtain

$$\frac{dU_m C_m}{dx} = -\frac{2^\alpha}{r_w}J_w, \quad U_m = 2^\alpha\int_0^{r_w} r^\alpha U_x\,dr/r_w^{1+\alpha}, \quad C_m = 2^\alpha\int_0^{r_w} r^\alpha C U_x\,dr/r_w^{1+\alpha}U_m \tag{3.2}$$

where U_m and C_m are the mass-average flow velocity and particle concentration, and r_w is the channel radius. For the region of hydrodynamically stabilized flow in Eq. (3.1) we set $\partial U_x C/\partial x = dU_m C_m/dx$. Then, bearing in mind (3.2), we obtain the following equation for calculating the particle distribution over the channel cross section:

$$(D_T+D)\frac{\partial C}{\partial r} + C\frac{\partial}{\partial r}(qD_T+D) = -\frac{r}{r_w}J_w \tag{3.3}$$

The turbulent particle diffusion coefficient in (3.3) is assumed to be equal to the turbulent viscosity coefficient of the carrier flow $D_T = \nu_T$. The turbulent

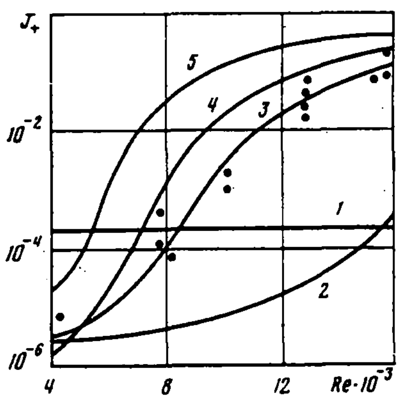

Fig. 3

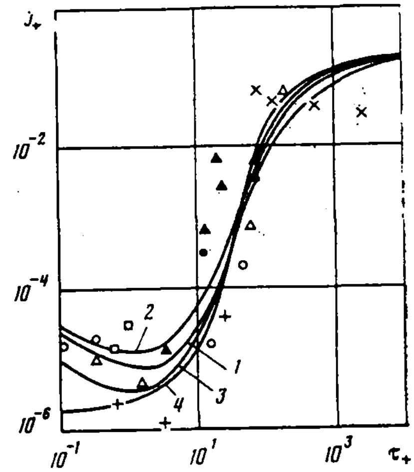

Fig. 4

viscosity coefficient is calculated from the expression

$$\frac{\nu_T}{\nu}=\frac{\varkappa}{3}y_+(2-y_0)\left[\frac{1}{2}+(1-y_0)^2\right]\left[1-\exp\left(-\frac{\varkappa y_+^2}{A^2}\right)\right]$$

which at a point remote from the wall goes over into the Reichardt equation, and as $y \to 0$ into the Van Driest–Deissler relation [13] ($y_0 = y/r_w$, $y_+ = yu_*/\nu$, $\varkappa = 0.4$, $A = 26$). The quantity T is taken equal to the integral time scale of turbulence and in accordance with the experimental data presented in review [14] it is assumed that $T_* = Tu_*^2/\nu = 200$. In solving Eq. (3.3) the boundary condition (2.6), relating the particle concentration at the channel wall with the particle flux through the wall, was written at a distance from the wall equal to the particle radius a.

In Figs. 1 and 2 we have compared the results of calculating the particle deposition velocity $V_w = J_w/C_w$ or $J_+ = V_m/u_*$ (u_* is the dynamic velocity) with the experimental data. Curves 1 and 2 in Fig. 1 correspond to the experimental data of [15] for the Reynolds numbers $Re = 2r_wU_m/\nu = 2.8\cdot10^5$ and $5\cdot10^5$; curves 1–3 in Fig. 2 correspond to the experimental data of [4] for the flow velocities U_m = 7.6, 17.6, and 26.6 m/sec. Attention is drawn to the presence of a minimum in the dependence of the deposition velocity on the particle diameter d. The initial fall in the deposition velocity is attributable to the decrease of the Brownian diffusion coefficient with increase in particle size. In this region (for particles smaller than a micron) the principal deposition mechanism is diffusion. As the particle diameter increases, so does the velocity of turbulent migration due to the nonuniformity of the distribution of the turbulent fluctuation intensity over the channel cross section, which also leads to an increase in the deposition velocity. For particles of the order of 100 μm the dimensionless deposition velocity reaches a maximum $J_+ \approx 0.2$.

As the flow velocity U_m increases, so does the deposition velocity V_w, which is associated with an increase in the intensity of the particle deposition process under the influence of both turbulent diffusion and turbulent migration. In Fig. 3 we have plotted the results of calculating the dependence of the dimensionless deposition velocity J_+ on the Reynolds number for a tube of radius 2.5 mm (curves 1–5 correspond to particle diameters of 0.01, 0.27, 0.81, 2, and 8 μm; the experimental points are from [16] for d = 0.81 μm). It is clear from Figs. 2 and 3 that for submicron particles (d < 0.1 μm) the value of J_+ is practically independent of the Reynolds number, since the velocity of particle deposition V_w as a result of diffusion is proportional to the dynamic velocity u_*.

In Fig. 4 we have compared the results of the calculations with the experimental data represented in the form, often encountered in the literature, of the dependence of J_+ on the dimensionless particle relaxation time $\tau_+ = \tau u_*^2/\nu$ (curves 1 and 2 correspond to a tube radius of 0.015 m and flow velocities of 10 and 30 m/sec; curve 3 corresponds to r_w = 0.3 m, U_m = 30 m/sec and curve 4 to r_w = 0.025 m, U_m = 30 m/sec; the experimental data are taken from [2]).

Clearly, in these variables a relatively good correlation of the calculation results is obtained only for inertial particles ($\tau_+ > 10$), while for small particles the particle size is also an important parameter. This is associated with the fact that the rate of deposition of inertial particles is determined by processes of a turbulent nature — turbulent diffusion and turbulent migration. For the small particles ($\tau_+ < 1$) the deposition velocity is determined not only by the turbulent transfer rate but also by the Brownian diffusion and hence depends significantly on the absolute particle size.

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