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Principal Developments in Band Saw Vibration and Stability Research

A. G. Ulsoy *, **C. D. Mote, Jr.** **, and **R. Szymani** ***
Department of Mechanical Engineering, University of California, Berkeley, Cal. 94720, U.S.A.

1 Introduction

This paper reviews fundamental investigations of band saw vibration and stability. The literature is spread out in journals and reports in a variety of languages, and considerable duplication of effort exists. It is this diffusion of references which makes a review necessary. The authors have reviewed all pertinent papers known to them, and apologize to both authors and readers for significant work that is missing. It should be noted that a critical review such as this is necessarily tinged by the authors' interpretation of what is fundamental knowledge in a particular field.

The review has been limited to topics of direct, fundamental importance to band saw vibration and stability. Topics in which vibration is not the main consideration, such as tooth design, power requirements, fatigue cracking, etc., are not explicitly discussed here.

Band saw vibration belongs to a class of vibration problems, called axially moving material vibrations, first investigated by Rudolf Skutch [1897]. This class contains research on magnetic and paper tape vibrations [Miranker 1960], moving belt dynamics [Chubachi 1957, 1958; Doyle, Hornung 1969; Rhodes 1970; Lai, Chen 1971], chain drive vibrations [Mahalingam 1957], vibration of pipes transporting fluids [Housner 1952; Niordson 1953; Naguleswaran, Williams 1968b; Thurman, Mote 1969b; Liu, Mote 1974; and many others], moving strings and fibers [Sack 1954; Archibald, Emslie 1958; Swope, Ames 1963; and others], cables and catenaries [Simpson 1972], and of course band saw vibrations. The problems listed above are analogous as a first approximation, because the linearized equation of motion and boundary conditions are identical [Mote 1972]. Another class of problems relating to band saw vibrations is the response of elastic solids under moving loads. The reader is referred to the book by Fryba [1972].

* Graduate Student, Department of Mechanical Engineering. Research Assistant, Forest Products Laboratory, Richmond
** Professor and Vice Chairman, Department of Mechanical Engineering. Research Engineer, Forest Products Laboratory, Richmond
*** Asst. Professor, Department of Forest Products, Oregon State University, Corvallis, Oregon. Formerly Assistant Specialist, Forest Products Laboratory, Richmond

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2 The Importance of Vibration in the Sawing Process

In recent years, due to rising raw material and labor costs, increasing attention is being devoted to improving wood sawing practices. While a high cutting rate is still among the most desired characteristics, other factors are becoming increasingly important. These factors include improved cutting accurracy and surface quality, and reduced kerf losses, noise, downtime and maintenance.

Band saws have advantages over circular saws which include i) higher cutting speeds, ii) lower kerf waste, and iii) typically lower noise levels. The use of band saws in the wood industry has increased steadily since 1920 [Pahlitzsch 1962]. Today band saws of different types and sizes are employed in many areas from primary log conversion to furniture manufacturing.

Band saw kerf losses can be reduced further through thinner blades, as long as blade vibration is controlled and blade stability maintained. Blade instabilities lead to poor cutting performance, and catastrophic blade failures. Although some vibrating cutter techniques have been proposed, one can conclude that saw vibration in general is undesirable. Several investigations of the effects of blade vibration during sawing on cutting accuracy and surface quality have been published. Birkeland [1968] concluded that most sawing inaccuracies were traceable to carriage motion and the depth of cut did not influence cutting accuracy, but he did not investigate blade vibration. Thunell [1971] noted that the effect of feed speed on dimensional accuracy is fairly small until a critical feed speed is reached above which accuracy deteriorates rapidly. This observation may be due to blade buckling at a critical edge load as discussed in Section 6 but the mechanism of instability has not been researched. Breznjak and Moen [1972] observed during sawing experiments that: i) lumber thickness variation, energy consumption, and kerf losses increased with increasing vibration amplitudes, ii) lateral vibration increased with increasing clearance between blade and guides and with increasing feed speed, iii) surface quality improves with increasing high frequency vibration due to a polishing effect.

3 A Description of Band Sawing

A modern bandmill and a schematic of a band saw indicating the important geometric and process parameters are shown in

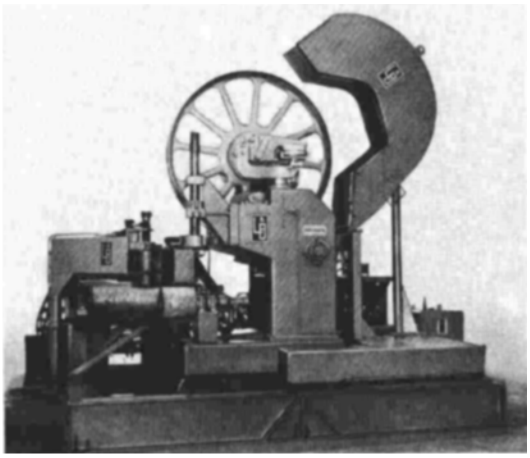

Fig. 1. A production bandmill. Kockums Letson and Burpee Ltd.
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Table 1. Stresses in headrig band saws

Stress component	Stress magnitude, N/mm^2		
	Normal strain	High strain	
Bending σ_B	210—240	110—130	Porter [1971]; Krilov [1975]; Pahlitzsch, Puttkammer [1972]
Centrifugal σ_C	7— 20	7— 20	Porter [1971]; Pahlitzsch, Puttkammer [1972]
Strain or axial tension σ_S	40— 80	80—140	Porter [1971]; Krilov [1975]; Pahlitzsch, Puttkammer [1972]
Tensioning or pre-stressing σ_T	35— 70	35— 50	Porter [1971]; Krilov [1975]

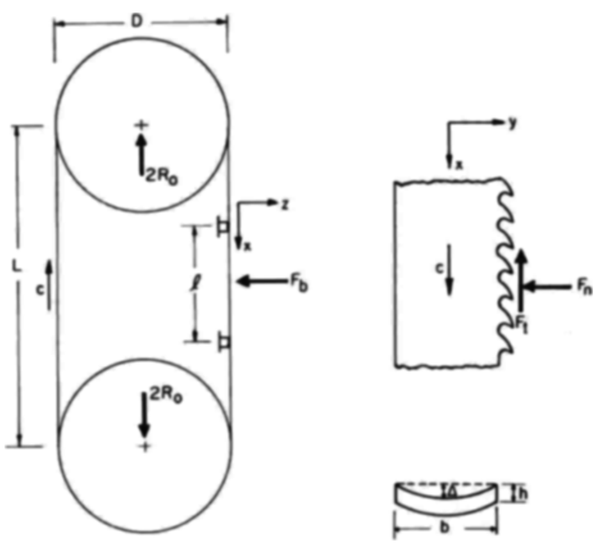

Fig. 2. Schematic representation of a bandsaw.
Schema der an einer Bandsäge wirkenden, hauptsächlichen Kräfte und Abmessungen

Figures 1 and 2 respectively. From the point of view of stability, the important parameters are the band axial tension (R), the free length between guides (l) and the types of guides, the cutting force components (normal, F_n, tangential, F_t, and transverse, F_b), the band thickness (h) and width (b), as well as lateral forces due to blade-workpiece interactions, wheel irregularities and eccentricities, band irregularities, and the state of stress of the blade during operation. Further discussion of the effects of these parameters on vibration and stability is presented in Sections 4, 5 and 6.

When the production objective is minimum kerf and high sawing accuracy, the sawing system dynamics and vibration must be considered. There has been considerable research investigating the relationships between parameters such as cutting forces, feed rate, power consumption, gullet feed index, etc. and cutting accuracy. Sugihara [1953] found experimentally that the normal cutting force (F_n) increases linearly with feed. Pahlitzsch and Dziobek [1959] observed a minimum tangential cutting force (F_t) for band velocities of approximately 30 m/s. Allen [1975, 1976] presents experimental data showing that the thickness standard deviation within boards decreases with increasing band axial tension, commonly referred to as "strain", and increases with increasing gullet feed index. The trend toward high strain bandmills results from the desirability of increased effective blade stiffness [Porter 1971; Allen 1972, 1973]. It is also well known that the effective blade stiffness increases as the free length between guides is reduced.

Allen [1976] points out, "the most important element of any bandmill operation is the band saw blade itself, all other elements are simply there to make it work." The state of stress of the blade is of particular importance to vibration and stability. The stresses in the blade arise from band axial tension [Porter 1971; Krilov 1975], bending over the wheels [Thunell 1972; Pahlitzsch, Puttkammer 1972], pre-stressing or "tensioning" operations [Trubnikov 1965a; Wüster 1966], centrifugal forces [Porter 1971; Pahlitzsch, Puttkammer 1972], cutting forces [Thunell 1972; Krilov 1975], thermal effects [Sugihara 1965; Sanev 1969], and cutting or punching the teeth [Yurev, Veselkov 1972]. Typical magnitudes of stresses in an idling band saw blade are summarized in Table 1.

The bending stresses, shown in Table 1, are cyclic and quite large. They are known to be the main cause of band saw fatigue failures and gullet cracking problems [Jones 1968; Trubnikov 1965b; Porter 1971]. The maximum bending stress is approximately proportional to the band thickness-wheel diameter ratio (h/D), thus these stresses are reduced by the use of larger diameter wheels and thinner blades.

Tensioning is a saw pre-stressing procedure commonly used in the forest products industry to stiffen the saw blade by introducing favorable in-plane residual stresses. Local plastic deformation in a blade is produced by hammering, rolling or heating. Though band saw tensioning has been used as early as the end of the nineteenth century, the control of the tensioning process is still an unresolved problem. It is important to be able to estimate and introduce the correct tension in order to obtain maximum cutting performance without causing fatigue and gullet cracking problems because of overstressing the blades. Accurate measurement of tensioning stresses is difficult, and the values reported in Table 1 should be considered as approximate. Tensioning results in a transverse deflected shape of the blade which is related to the distribution of residual stresses [Eklund 1972]. Aoyama [1970, 1971, 1974] concludes that this deflected shape or "light gap" depends upon the saw blade geometry, number of rolling passes, their location, and the roller geometry and pressure. Even though transverse deflection profiles of two blades are similar, the corresponding band stress distributions may be very dissimilar. Thus the light gap technique cannot give a very accurate estimation of residual stresses [Foschi 1975].

Tensioning stresses also counteract unfavorable heating effects and help keep the band on the wheels. The wheels usually have a small "crown" for a better geometric match with the blade. The tendency of the blade to be pushed off the wheel is not considered as a transverse stability problem here. The reader is referred to Doi et al. [1970] and Sugihara [1977] for a discussion of this problem.

The blade-workpiece interaction during vibration heats the blade by friction; blue spots or streaks are formed at approximately 250 °C. This problem may in some cases be aggravated by the workpiece pinching the blade due to residual stresses in the workpiece (e.g., due to tension wood in hardwoods). Tilting of the wheels so that the gravitational force pulls the workpiece away from the blade is sometimes used to counteract this problem. Tilting of the blade in the vertical plane is also used to allow the blade to enter and leave the workpiece with a smaller depth of cut.

4 Formulation of the Band Saw Equations

The earliest analyses of the band saw vibration problem modeled the blade as an axially moving beam [Mote 1965a,b; Mote, Naguleswaran 1966]. The undamped equation of transverse motion is

$$m w_{,tt} + 2mc\, w_{,xt} + (mc^2 - R(c,t))\, w_{,xx} + EI\, w_{,xxxx} = 0 \qquad (1)$$

where the subscripts denote partial derivatives with respect to time and length, m is the mass per unit length, c is the constant band transport velocity, $R(c,t)$ is the velocity dependent band axial tension or strain, and EI is the flexural stiffness. A physical interpretation of the terms in Equation (1) is given in Mote [1972]. Equation (1) is similar to the equation of motion derived for axially moving strings when $EI w_{,xxxx} = 0$ [Skutch 1897; Sack 1954; Swope, Ames 1963; Miranker 1960; Archibald, Emslie 1958], for axially moving rods [Chubachi 1958; Mote 1965a; Barakat 1968], for belts [Doyle, Hornung 1969; Lai, Chen 1971], and plates [Mote, Naguleswaran 1966; Labra 1974]. Other linear formulations include a Timoshenko beam model by Simpson [1973] and a technical discussion of torsional vibration by Alspaugh [1967] and Soler [1968].

The boundary conditions accompanying Equation (1) are conditions acting at the wheels, or guides when they are present, and they are usually chosen to be the classical pinned-pinned or clamped-clamped conditions. Disturbances and band motions outside the cutting region between the guides as well as the wheel shape, influence the band response by propagation of energy into and out of the cutting region. Thus the band response is determined by the design and excitation of the entire band, not just the segment between the guides. Anderson [1974] studied the multiple support problem; he also noted that the pinned-pinned vs. clamped-clamped support assumptions do not greatly influence the first few natural frequencies of the band.

The analytical techniques used in the solution of moving band transverse motion have been both classical and original. The method of characteristics and superposition of D'Alembert travelling waves were used by Swope and Ames [1963], and Sack [1954] in the response of travelling strings. Exact solutions for natural frequencies and response of moving belts and band saws have been determined by Chubachi [1958], Mote [1965a, b], and Mote, Naguleswaran [1966]. A number of approximate methods for analysis of band saw natural frequencies and band response have been developed which include frequency bounding methods and the Galerkin method [Mote 1965a, b; Mote, Naguleswaran 1966; Anderson 1974], harmonic balance [Sinha, Srinivasan 1975], finite element [Anderson 1974], transform methods [Labra 1974; Adler, Reismann 1974; Sagartz, Forrestal 1975], and Fourier series [Chubachi 1958]. The papers on moving strings (e.g., Sack 1954; Swope, Ames 1963; Mote, Thurman 1971] have investigated their forced and free vibration response including the influence of transport velocity (c) in Equation (1). The band saw beam model papers usually relate their natural frequencies to the axial transport velocity without explicit determination of the transverse response. Experimental studies show good agreement with theory at low to moderate transport velocities [Mote, Naguleswaran 1966; Kirbach, Bonac 1977b]. The band saw does not possess natural frequencies in the classical sense. That is, no single frequency in phase oscillation is possible for $c \neq 0$. Excitation frequencies do exist which cause solutions of the linear, undamped equation of band transverse motion to become unbounded for finite excitation amplitudes, and these are referred to as the natural frequencies of the band saw in the literature.

Consideration has also been given to band saw transverse vibration induced by a periodic axial tension or "strain" variation during operation. This is called parametric excitation [Mote 1965a; 1968a,c; Naguleswaran, Williams 1968a; Rhodes 1970; Lai, Chen 1971]. Band saw buckling by cutting edge loads has also been investigated [Alspaugh 1967; Soler 1968; Mote 1968b; Foschi, Porter 1970]. Parametric excitation and edge load buckling are discussed in Sections 5 and 6.

The extensions of the equation of motion, Equation (1), to include additional nonlinear terms have been predominantly for moving string models ($EIw_{,xxxx} = 0$). Hsiang [1966] replaced the curvature approximation $w_{,xx}$ in Equation (1) by $w_{,xx}/(1 + w_{,x}^2)^{1/2}$ and used the method of characteristics and the perturbation method to analyze the string response. Ames, Lee and Zaiser [1968] examined the nonlinear string under planar excitation; experiments showed the motion to be periodic, in either a planar or three dimensional "balooning" motion. Bapat and Srinivasan [1967] analyzed the planar nonlinear travelling string problem by the method of harmonic balance, and Kim and Tabarrok [1972] used the method of characteristics for the same problem. Shih [1971] analytically treated the general case of three dimensional nonlinear string motion; his results agree with experimental observations [Ames et al. 1968]. Moustafa and Salman [1976] presented an algorithm for a numerical solution to the nonlinear threadline equation. Some attention has been given to nonlinear axially moving beam problems ($EI \neq 0$) by Thurman and Mote [1969a, b], Tabarrok, Leech and Kim [1974], and Sinha and Srinivasan [1975]. A principal contribution of the nonlinear analyses has been to show that the relative importance of the nonlinear terms in the equation of motion increases as the transport velocity (c) increases. Thus the linear analyses are restricted to small transverse displacements and to a low to moderate transport velocity range [Mote 1966]. Curves are available for estimation of the linear period accuracy, and for comparing linear and nonlinear modes of oscillation in strings and beams [Thurman, Mote 1969a, b; Mote, Thurman 1971].

Linear damping contributions to the response of an axially moving string have been studied by Sack [1954] and Mahalingam [1957]. Mahalingam [1957] concluded that a linear damping term $\beta(cw_{,x} + w_{,t})$ gave results closer to experimental observations than $\beta w_{,t}$, but little attention has been directed to the whole question of damping.

The band saw axial tension, as formulated by Chubachi [1958] and Mote [1965a, b] is dependent on the band axial velocity (c) and possibly time;

$$R(c,t) = R_0(t) + \eta mc^2 = R_0(t) + (1 - \kappa) mc^2 \qquad (2)$$

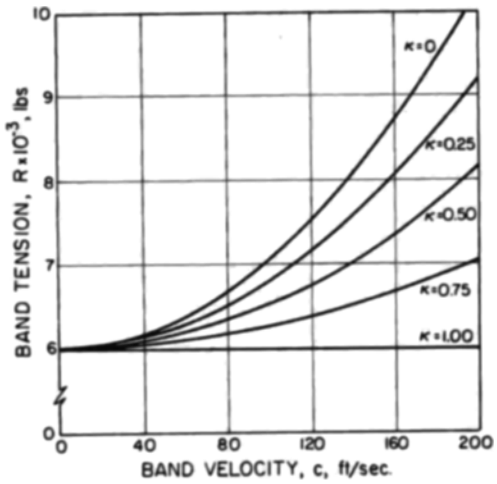

Fig. 3. Band tension-velocity dependence for five pulley mounting systems [Mote 1965b]
Zusammenhang zwischen Bandgeschwindigkeit und Bandspannung für fünf verschiedene Systeme der Rollenanordnung [Mote 1965b]

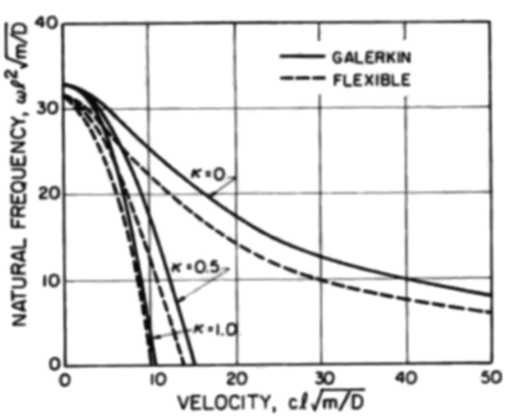

Fig. 4. Theoretical fundamental frequency bounds vs. band velocity for a wide bandsaw ($l = 3b$; $R_0 = 100\, D/l$) [Mote, Naguleswaran 1966]
Theoretische Bandbreiten von verschiedenen Grundfrequenzen in Abhängigkeit von der Bandgeschwindigkeit für ein breites Bandsägeblatt ($l = 3b$; $R_0 = 100\, D/l$) [Mote, Naguleswaran 1966]

$R_0(t)$ is the tension caused by wheel position and wheel shape. It is the initial static tension in the band for $c = 0$, and it is a periodic function of time for rotating eccentric or noncircular wheels. The second term in Equation (2), involving η or $\varkappa$, results from the normal acceleration of the band at the wheels. The physical phenomenon is as follows. A portion of the band tension, a dynamic component, is required to accelerate the band around the wheels. This dynamic tension component does not cause interaction between the band and the contiguous wheel. The static tension component is induced by band-wheel interaction. Therefore the dynamic tension is not supported at the wheel axle. For wheels loaded with a weight and lever system, which is mechanically equivalent to applying tension with an infinitely soft spring, the dynamic tension is superimposed on the initial static tension R_0. The tension increase with speed is a maximum with $\eta = 1$ or $\varkappa = 0$. In general, for tension applied through springs, the static tension component is reduced as indicated (Fig. 3) with κ in the range $0 \le \kappa \le 1$, where 0 corresponds to an infinitely soft spring mounting and 1 corresponds to a rigid spring [Mote 1965a; b; Mote, Naguleswaran 1966]. The lever tension system has probably been successful in practice because it results in a maximum operating tension, and accordingly maximum band natural frequencies, for a given R_0. Typical band tension, band velocity relationships are shown in Figure 3.

The blade-guide interactions in band saws are not well understood, and this is an area requiring further investigation. Contacting pressure guides are widely used in saw mills but guides with a small clearance and roller type guides are also in use [Eklund 1974]. Other guide designs are also being investigated (see Sect. 7). The simple support boundary condition assumption at the guides, used in conjunction with Equation (1), is notably approximate since the bending moment at the guides does not vanish.

There exists an extensive literature on the problem of an elastic strip or band responding to moving loads such as guides, cutting forces, and saw-workpiece interaction [e.g., Fryba 1972; Labra 1974; Flaherty 1968; Ting et al. 1974; Adler, Reismann 1974]. While not specific to band saws, this literature is directed toward physical mechanisms occurring in band saw motion.

The total mechanical energy of the portion of the moving band between the guides, which is the sum of the kinetic energy of vibration and the strain energy of deformation of the segment of the band, is never constant even for a model with conservative external loading and zero damping. During periodic motion there is a continual, periodic transfer of energy into and out of the region between the guides as shown by Chubachi [1958] and Miranker [1960]. This means that the band transverse response amplitude for $c \neq 0$ in Equation (1) can be much larger than with $c = 0$ for the same initial conditions. Sack [1954] demonstrated this quite clearly for the case of a moving string. The importance of this phenomenon is that a relatively small excitation, say from an eccentric wheel, can be amplified in the band response between the guides.

The transport velocity (c) has been restricted to be constant in most studies. Miranker [1960] introduced variable velocity in his analysis but did not develop the solutions for these cases. Liu and Mote [1974] experimentally observed transverse excitation induced by a variable mass transport velocity in pipes transporting oil; and Mote [1975] presented an approximate analysis indicating acceleration of the transport mass is a stabilizing and deceleration of this mass is a destabilizing process. The practical importance of the phenomenon to band saws appears minimal. The motion of a string under an accelerating force or mass is discussed by Flaherty [1968] and Sagartz and Forrestal [1975]. This type of approach can be used to investigate accelerating guide or cutting forces on a band.

Mote and Naguleswaran [1966] presented approximate, bounding natural frequency solutions for an axially moving wide band or plate which is probably realistic for the large headrig band saws. The undamped equation of motion is,

$$D \nabla^4 w + 2mc\, w_{,xt} + (mc^2 - R(c,t))\, w_{,xx} + m\, w_{,tt} = 0 \tag{3}$$

where D is the flexural stiffness and ∇^4 is the biharmonic operator. The first natural frequency upper and lower bounds versus band velocity for wide bands are shown in Figure 4.

As discussed previously, the state of stress of the band saw blade is known to have significant effect on vibration and stability [Thunell 1972]. These stresses do mechanical work during deformation of the blade, alter saw transverse displacement under a given load, and in effect change the blade stiffness. Initial stresses purposely introduced into the blade by rolling, hammering or heating are known from experience to significantly affect stability [e.g., Kirbach, Bonac 1977a]. In recent years the tensioning practice has been evolving from an art into a more science based technique; automated tensioning and tension measuring devices have been developed [Foschi 1975; Allen 1976]. Kirbach and Bonac [1977a] observed in experiments that a tensioned stationary band has a higher torsional fundamental frequency than an untensioned band; tensioning had little effect on the transverse bending fundamental frequency. The effects of tensioning on the band saw vibration problem have not been thoroughly investigated. The subject awaits a formal investigation of the optimum tensioning criterion and the band stability criterion. It is clearly difficult to address the question of theoretically desirable tensioning stresses until a criterion of evaluation has been established.

Thermal effects during cutting alter the state of stress in the band [Sugihara 1965; Thunell 1972]. The state of thermal stress depends on the temperature distribution and the coefficient of thermal expansion, and it can be calculated once the blade temperature distribution is known. The effects of feed speed and band axial velocity on heat generation during cutting was investigated by Aoyama [1958] and Sanev [1969]. Sanev [1969] used a semiconductor type bolometer for non-contacting temperature measurements and reports typical gradients of approximately 30 °C, across the width (100 mm) of the blade just downstream from the workpiece, during sawing. Sanev and Plujusnin [1969, 1970] theoretically analyzed the stresses arising in band saw blades due to temperature gradients. The effects of thermal stresses on the band saw vibration problem have not been investigated.

5 Band Excitation Mechanisms

Vibration in band saw blades is caused by normal disturbances of the production environment and will always be present. Band saw excitation results from the complete cutting forces, the interaction between the blade lateral surfaces and the workpiece, the wheel eccentricities and irregularities, the band stiffness variations, the disturbance caused by the weld, and the guides. These excitation mechanisms can be classified as either direct or parametric. During cutting there are direct periodic forces, such as those at the tooth passage frequency, and there are direct random forces, such as those caused by interactions of the blade surfaces with the workpiece. Direct forces drive the band in transverse-torsional motion. When there is sufficient direct excitation energy at forcing frequencies that are at or near the band saw natural frequencies, resonant amplification occurs producing large amplitude band vibrations. The cutting force can be represented by its normal, tangential, and transverse components as shown in Figure 2. The early investigations of cutting forces are summarized by Pahlitzsch [1962]. More recent studies include comprehensive theoretical and experimental investigations by Pahlitzsch and Puttkammer [1972—1976]. Typical magnitudes of the cutting force components as reported by Pahlitzsch and Puttkammer [1975, 1976a] are presented in Table 2.

Alspaugh [1967] first investigated the purely torsional response of an axially moving band with a concentrated edge load. Soler [1968] demonstrated the importance of coupled bending-torsion of band saws to the edge buckling load. Mote

Table 2. Cutting force in head rig band saws [Pahlitzsch, Puttkammer, 1975, 1976a]

Cutting force component	Magnitude, N	
	Range	Typical
Tangential F_t	100—1000	500
Normal F_n	0—600	250
Transverse F_b	0—25	15

[1968a, b] treated the coupled bending-torsion vibration and buckling problems for general concentrated and distributed edge loading. These buckling and vibration problems have not been treated in sufficient depth and are without experimental verification. It still remains to relate specific process and design parameters to these excitation mechanisms. The interaction between the band surfaces and the workpiece is also an important area for investigation, but it appears difficult to pursue.

Periodic variation of the band saw tension can induce transverse instability when the frequency of tension variation is twice any natural frequency of the band saw, particularly the band fundamental frequency. This is termed parametric vibration because the band vibration is induced by an oscillating stiffness rather than any direct driving of the band by external forces. Studies by Mote [1965a, 1968a,c], Naguleswaran and Williams [1968a], Doyle and Hornung [1969], Rhodes [1970], and Lai and Chen [1971] indicate that the amplitude of parametrically induced vibrations may exceed those from direct excitation. The periodic tension variations result from the wheel eccentricities and irregularities, or band irregularities. For parametric excitation, $R_0(t)$ in Equation (2) becomes $R_0(t) = R_0' + \varepsilon \cos 2\omega_\varepsilon t$, where ε is the band tension variation and ω_ε is one half the band tension variation frequency [Mote 1965a]. When $\varepsilon = 0$, $R_0(t) = R_0'$ is a constant. Rhodes [1970] studied the problem of band tension stored on the wheels on the high tension side by friction and then "played back" on the slack side of the wheel. Naguleswaran and Williams [1968a] obtained a coupled Mathieu-equation sequence using the Galerkin method. Mote [1968a] studied parametric instability of coupled bending-torsion oscillation. The theoretical developments require an accurate knowledge of the band axial tension, which is generally not known, to predict instability conditions. Experimental investigations of parametric excitation included in Doyle and Hornung [1969] and Naguleswaran and Williams [1968a] support the theoretical developments; a wheel rotation speed at twice the fundamental frequency can be expected to cause transverse instability.

The parametric and direct excitation forces drive the band and are prominent in the prediction of band transverse motion. It is surprising that more effort has not been concentrated on developing analytical models which more completely describe the total excitation of the band. The explanation may lie in the difficulty in conducting experiments to identify loading mechanisms.

6 Criteria of Band Saw Instability

When a small disturbance produces a large amplitude response from the equilibrium position, the band saw is said to be unstable. Because instability is necessarily associated with poor cutting performance, considerable research effort has been directed to investigating instability criteria [Pahlitzsch, Puttkammer 1974a; Thunell 1970; Foschi, Porter 1970; Mote 1965—1975; Naguleswaran, Williams 1968a; Mote, Naguleswaran 1966].

The static or divergence type of instability occurs at an edge load depending upon the band geometry, material, supports, tension, and transport velocity. The trend toward higher speed and thinner band saw blades reduces the load at which edge buckling occurs. This buckling load (F_{ncr}) reduction is approximately proportional to the velocity squared (c^2), until instability occurs for virtually zero edge load at the critical band velocity (c_{cr}) [Sack 1954; Archibald, Emslie 1958; Mote 1965a, 1968b]. A typical example of the dependence of the edge buckling load on the band axial velocity is shown in

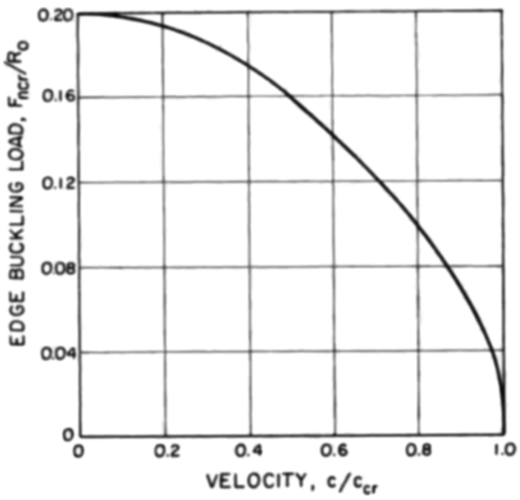

Fig. 5. Edge buckling load-axial velocity relationship ($R_0 = 22.241$ kN; $b \times h = 8.065$ cm^2; $l = 1.524$ m) [Mote 1968b]
Zusammenhang zwischen Schneiden-Knicklast und Bandgeschwindigkeit ($R_0 = 22{,}241$ kN; $b \times h = 8{,}065$ cm^2; $l = 1{,}524$ m) [Mote 1968b]

Figure 5. The band axial or transport velocity cannot be neglected in the prediction of the buckling load as presented by Foschi and Porter [1970] and Porter [1971]; this is shown in the investigations for concentrated and distributed edge loads by Mote [1968b]. Feoktistov [1960] concluded that doubling the axial tension only increases the edge buckling load by about 10%; however, results presented by Mote [1968b], Foschi and Porter [1970] and Porter [1971] do not support this conclusion and indicate increases in the order of 100%.

The modes or types of band saw instability can be bending or torsion but in general a coupled bending-torsion motion is expected and such a theoretical model must be developed to study band saw instability [Soler 1968; Mote 1968a, b]. The available experimental studies by Ames, Lee and Zaiser [1968], and Liu and Mote [1974], reported that coupled bending-torsion and out of plane motion occurs at all transport velocities and that the coupling increases with transport velocity. Practical investigation of the instability modes occurring in the cutting process apparently has not been conducted, but identification of these modes is critical to determination of the instability criteria.

The instability described above is of the static type, meaning that the band in the particular environment will seek a different equilibrium shape determined by the bending-torsion buckling mode. This new equilibrium, being non-flat, is undesirable. There exists another characteristic type of instability termed "dynamic instability." This instability is characterized by a large amplitude band saw oscillation about the equilibrium state, rather than a shift in the equilibrium. Flutter in aircraft wings is an example of dynamic instability. Dynamic stability in band saws has not been researched but the observed motions in band saws indicate that such instabilities occur.

Noise generated by band saws is known to exceed 95dBA [Thunell 1973]. Noise mechanisms in circular saws have received some attention as outlined by Mote and Szymani [1977] and Szymani and Mote [1977], but there have been relatively few investigations of the sources of band saw noise [Kitayama, Sugihara 1968; Pahlitzsch, Eckert 1975]. Pahlitzsch and Eckert [1975] conclude that aerodynamic excitation by the saw teeth does not contribute significantly to band saw noise; this result is in contrast with circular saw noise where aerodynamic excitation can be significant [Cho, Mote 1977].

7 Vibration Control

To achieve effective vibration control the relationship between the band excitation and instability mechanisms and the design and operating parameters must be established. However, some techniques for vibration reduction and control are currently available. The oldest and most widely used is blade stress modification by tensioning.

Tensioning was done in the past by hammering, today it is often done by manually operated rolling machines [Aoyama 1970—1974]. In recent years automatic roller tensioning devices and tensioning by thermal methods [Doi 1964; Hsu, Trasi 1976] have been in use as well. Kirbach and Bonac [1977a] have experimentally confirmed the benefits of roller tensioning for vibration control.

Vibration feedback control by on-line heating [Mote, Holøyen 1977] and by using active electromagnetic guides [Ellis, Mote 1977] have been demonstrated for circular saws, and these concepts appear to be directly applicable to band saw vibration control as well. Automatic on-line adjustment of axial tension [Novosel'tsev, Seleznev 1974], feed rate [Shtol'tser 1976], and guide position [Pahlitzsch, Puttkammer 1976a] have also been suggested and appear promising. The use of novel guides, such as aerostatic guides [Berlin 1969; Pahlitzsch, Puttkammer 1976a], or wide guides [Saljé, Thomas 1976] are also being investigated as possible measures for vibration reduction and control.

8 Closing Remarks

While research has theoretically proposed and experimentally verified the essential features of circular saw vibration [Mote, Szymani 1977; Szymani, Mote 1977], the state of band saw vibration and stability research is less advanced. The effects of band axial velocity, band axial tension, parametric excitation, and normal edge loads on band saw vibration and stability have been investigated. However, many important and fundamental questions regarding the effects of guides, thermally induced and tensioning stresses, cutting forces, propagation of disturbances, and blade-workpiece lateral interactions still remain unanswered. Research into these areas, to determine stability criteria and relate these criteria to specific design and process parameters, is necessary to optimize band saw design and the band sawing process.

Die wichtigsten Forschungsarbeiten über Schwingungs- und Stabilitätsverhältnisse an Bandsägen

Die Autoren geben einen Überblick über Forschungsarbeiten, die sich mit den im Titel genannten Erscheinungen beim Bandsägen befassen.

Im einleitenden Abschnitt wird erklärt, nach welchen inhaltlichen Kriterien die besprochenen Arbeiten ausgewählt wurden. Im zweiten Abschnitt (Bedeutung von Schwingungen im Sägeprozeß) werden Arbeiten über die für den Bandsägeschnitt wichtigen Eigenschaften von Breitbandsägen (Block- bzw. Trennbandsägeblätter) genannt und der Zusammenhang mit Sägeblattschwingungen und den am häufigsten vorkommenden Instabilitäten von Maschine und Werkzeug erörtert. Der dritte Abschnitt (Darstellung des Bandsägens) nennt Veröffentlichungen über die Kräfte, geometrischen Größen und sonstigen Bedingungen, die das Wirken einer Bandsäge bestimmen, sowie die auf sie im negativen Sinne einwirkenden Größen (z.B. Rollenexzentrizität, geometrische Unregelmäßigkeiten des Sägebandes, Spannungsabweichungen, Wechselwirkungen zwischen Sägeband und Werkstück usw.).

Anliegen des vierten Abschnittes (Formulierung der mathematischen Beziehungen an Bandsägen) ist es, Publikationen über die auf Grund praktisch-experimenteller Beobachtungen bzw. Untersuchungen abgeleiteten mathematischen Beziehungen darzustellen und zu erläutern. Hierzu gehören u.a. das Modell des „Sägebandes als axial bewegter Stab", das Verfahren der „Charakteristik und Überlagerung der fortschreitenden Wellenbewegung von D'Alembert", ferner Lösungsansätze unter Verwendung von „Eigen- und Resonanzschwingungen angetriebener Sägebänder", sowie eine Reihe von Methoden zur Ermittlung der Parameter, die mit den statischen und dynamischen Eigenschaften des Systems Maschine/Werkzeug/Werkstück zusammenhängen. Auf rechnerische Probleme, die im Zusammenhang mit der mathematischen Behandlung von Bandgeschwindigkeit, Sägeblattführungen und den Schwingungen des Systems allgemein stehen, wird näher eingegangen.

Der fünfte Abschnitt (Mechanik der Sägeband-Schwingungserregung) ist eigens den Arbeiten gewidmet, die sich mit der Schwingungserregung des laufenden Sägebandes beschäftigen. Arbeiten über die wirkenden Kräfte und die Bedingungen, die zu Sägeblattschwingungen führen, werden hier erörtert.

Im sechsten Abschnitt (Kriterien der Sägeband-Instabilität) werden Arbeiten besprochen, die sich mit den wichtigsten Ursachen der statischen und dynamischen Instabilität des Sägebandes beim Schnitt beschäftigen. (Schneidenbelastung und Zahngeometrie; Material; Führungen; Bandspannung; Vorschubgeschwindigkeit; Blattdicke und Knicklast); Arbeiten über Geräuschentwicklung bzw. -minderung werden kurz erwähnt.

Der siebte Abschnitt (Schwingungsdämpfung) berichtet über Arbeiten, die sich mit der Dämpfung bzw. Vermeidung von Sägebandschwingungen befassen, in erster Linie durch mechanische oder thermische Spannung des Sägebandes, Beeinflussung der Vorschubgeschwindigkeit sowie Position, Größe und Art der Sägeblattführungen. In einer Schlußbetrachtung werden die noch offenen Probleme genannt. Das Literaturverzeichnis nennt 105 einschlägige Titel.

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