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A COST–TOLERANCE ANALYTICAL APPROACH FOR DESIGN AND MANUFACTURING

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Tolerance assignment in mechanical engineering product design and manufacturing is critical both for product quality and performance as well as for its manufacturing cost. Simple design rules, industry available comparative experimental data, tables, charts, and personal expertise are currently used for cost optimum tolerancing. Such a methodology is not very practical, requires considerable time and effort and is not always suitable for a CAD/CAM environment. To face these limitations, a new model for the manufacturing cost–tolerance function has been developed and is presented in the paper. It may well be used for direct-tolerance optimisation applications and it is shown that it produces results that are in satisfactory agreement with existing cost–tolerance data. Its use is further analysed and demonstrated in solving linear manufacturing tolerance transfer problems.

Keywords: Manufacturing cost; Tolerancing; Tolerance optimisation; Tolerance transfer

1. Introduction

Tolerance assignment in mechanical engineering product design and manufacturing is critical both for product quality and performance as well as for its manufacturing cost. Dimensional tolerances are in general described as functional, design or manufacturing tolerances. Functional and/or design

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tolerances are related to the operational requirements of a mechanical assembly or of a component, whereas manufacturing tolerances are mainly connected with the process or processes which are considered for component manufacturing. Manufacturing tolerances must obviously respect and, in any case, satisfy the functional tolerances. On the other hand, however, they play a major role in the development of the manufacturing cost and every effort is therefore justified for their optimum designation. Numerous parameters affect the manufacturing accuracy of a dimension and the manufacturing cost related to this accuracy: machine tool capabilities, tooling, inspection equipment, workpiece material (size, shape), operator skill, lot size and scrap allowance. The consequent derivation of any detailed analytical expression relating the manufacturing cost of a dimension with its specified tolerance zone has proved extremely difficult if not impossible. Simple design rules of the type "the lower the tolerance the higher the cost of manufacturing" or "do not specify higher accuracy than is really needed", industry available comparative experimental data, tables, charts and personal expertise are commonly therefore used instead for cost optimum tolerancing. Such a methodology is not however very practical, requires considerable time and effort and is not always suitable for a CAD/CAM environment.

Tolerance transfer of linear dimensions is of particular importance in manufacturing because it is directly associated with the establishment of the locating surfaces for machining and inspection. Practice has nevertheless shown that functional/design datums and manufacturing datums of a part do not usually coincide. As a consequence of this, new manufacturing tolerances have to be determined. Similar cases are also met in the CAM coordinate dimensioning. For these cases the single condition that the transferred new manufacturing tolerances must observe the initial functional/design tolerances is not sufficient for their precise evaluation. Additional conditions, related to the manufacturing costs and the capabilities of the available machine tools, are desirable for an optimum and systematic assignment of the new tolerances. Additional conditions are also required for optimum tolerance designation in the linear and non-linear assembly tolerance systems [1,2]. For the solution of this kind of problem a general and uncomplicated mathematical model of the manufacturing cost–tolerance function offers definite advantages in comparison with the previously mentioned methods, as it leads to a more clear and time saving approach.

The problem of tolerancing the components of an assembly or the dimensions of a dimensional chain for minimum manufacturing cost has been faced with statistical or single-value point estimates for each tolerance by Peters [3] and Ostwald [4] respectively. Weil [5] has proposed a unidirectional tolerance transfer strategy aiming to optimise the tolerances for minimum manufacturing cost, given the capabilities of the available machinery. Relationships for the cost–tolerance function have been introduced by Speckhart [6] and Spotts [1], nevertheless with certain limitations regarding their direct use.

In this paper the derivation of a new model for the manufacturing cost–tolerance function is presented which may serve designers and process planners as a simple and efficient tool for a cost optimum tolerancing. It is shown that the model produces results which lie in satisfactory agreement with currently approved cost–tolerance data [7–10]. Its use is further analysed and demonstrated in solving linear tolerance transfer problems.

2. Theoretical Aspects

Research and industry wide data have established a more or less typical trend for the manufacturing cost–tolerance relationship as that shown in Fig. 1 (e.g. [6, 8–10]). From the total production cost of a dimension D with a tolerance $\pm t$,$C(D,t)$, the cost $C'(D)$ corresponds apparently to the upper threshold of the usual commercial accuracy. This level of accuracy does not require any particular machining and inspection effort, equipment, skill etc., i.e. $C'(D)$ is the normal manufacturing cost of the dimension D.

Speckhart [6] and Spotts [1] have proposed cost–tolerance relationships of the form

$$C(D,t) = A + B.e^{-F.t}, \quad A, B, F > 0 \tag{1}$$

and

$$C(D,t) = \frac{K}{t^2}, \quad K > 0 \tag{2}$$

respectively. In equations (1), (2), $C(D,t)$ is the overall manufacturing cost for producing the dimension D with a given tolerance zone $\pm$ t. A, B, F are constants determined by a non-linear least squares curve fit procedure of experimental results. K is also a constant to be obtained experimentally for the considered application. These expressions do not also cope well with the limit cases of the "absolute accuracy" $t = 0$, equation (1) and of the "commercial accuracy" $t \gg$, equation (2).

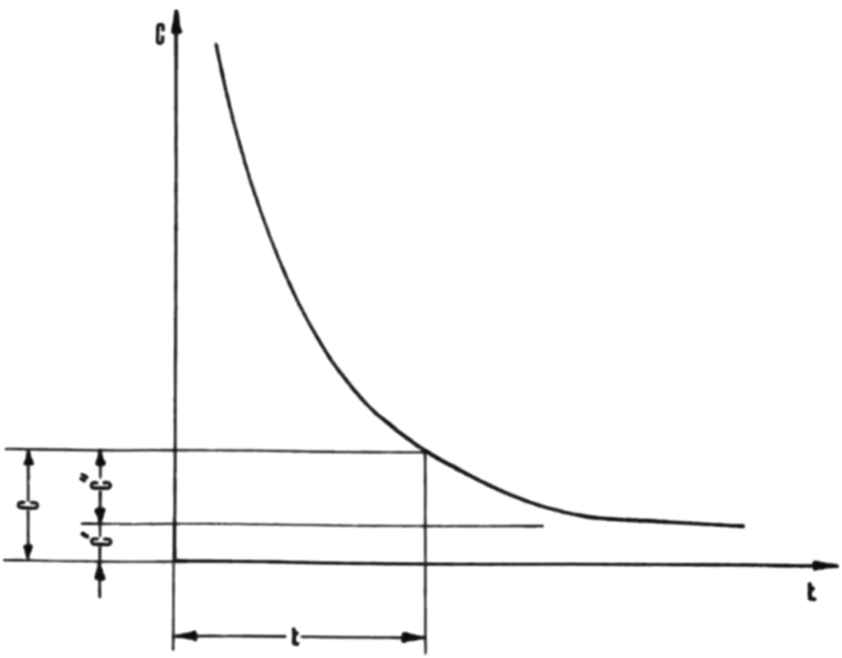

Fig. 1. Typical manufacturing cost–tolerance relationship.

Cost–tolerance functions may be generally modelled as (Fig. 1)

$$C(D,t) = C'(D) + C''(D,t) \tag{3}$$

where $dC'(D)/dt = 0$ and $C''(D,t)$ is the "accuracy cost" which is required in order to achieve for the dimension D a tolerance zone 2t ($\pm t$). For this additional cost manufacturing practice suggests that for a relative tolerance variation dt/t a multiple relative variation of the manufacturing cost is caused, in a way – usual for most technical relationships – which can be here formulated as

$$\frac{dC''(D,t)}{C''(D,t)} = -r.\frac{dt}{t}, r > 0 \tag{4}$$

where r is the "cost sensitivity" to tolerance of the manufacturing process. From equations (3) and (4),

$$C(D,t) = C'(D) + \frac{C_0''(D)}{t^r} \tag{5}$$

and $C_0''(D)$ is a dependent on the size of the dimension D constant.

Equation (5) lies much closer – as a general mathematical relationship – to the manufacturing practice than equation (1). This is demonstrated through the comparison of the accuracy costs $C''(D,t_1)$ and $C''(D,t_2)$ of a dimension D alternatively produced in two tolerance zones $\pm t_1$, $\pm t_2$, with $t_2 = t_1 + 1$ and $t_1 \gg 1$. From equation (1),

$$\frac{C(D,t_2)-A}{C(D,t_1)-A} = \frac{1}{e^F} \tag{6}$$

and from equation (5),

$$\frac{C''(D,t_2)}{C''(D,t_1)} = \left(\frac{t_1}{t_2}\right)^r = \left(\frac{t_1}{t_1+1}\right)^r \simeq 1 \tag{7}$$

From the above ratios, the result (7) is that which is obviously in full agreement with practice.

The accuracy cost $C''(D,t)$ equations (3) and (5) can now be further analysed, if two dimensions D_1, D_2 of the same workpiece are considered, which are produced by the same manufacturing set-up with tolerances t_1, t_2 of the same ISO standard tolerance grade. Because of this, and taking into account the industrial evidence reflected in the ISO standard tolerance grade methodology [11], the ratio of the accuracy costs $C''(D_1,t_1)$, $C''(D_2,t_2)$ may be taken,

$$\frac{C''(D_1,t_1)}{C''(D_2,t_2)} = \frac{i(D_1)}{i(D_2)} \tag{8}$$

where $i(D_1)$, $i(D_2)$ are the corresponding to D_1, D_2 ISO standard tolerance factors. From equations (5) and (8),

$$\frac{C''(D_1,t_1)}{C''(D_2,t_2)} = \frac{C_0''(D_1)}{C_0''(D_2)} \cdot \left(\frac{t_2}{t_1}\right)^r = \frac{C_0''(D_1)}{C_0''(D_2)} \cdot \left[\frac{i(D_2)}{i(D_1)}\right]^r = \frac{i(D_1)}{i(D_2)}$$

or,

$$C_0''(D_1) = C''(D_2) \cdot \left[\frac{i(D_1)}{i(D_2)}\right]^{r+1} \tag{9}$$

given that for the same tolerance grade (IT5–IT18), $t_2/t_1 = i(D_2)/i(D_1)$. Combining equations (5) and (9) a final model of the cost–tolerance function is obtained as

$$C(D,t) = C'(D) + C''_0 \cdot \frac{i(D)^{r+1}}{t^r} \tag{10}$$

where the constant C''_0 in this case depends on other than the size of the dimension cost parameters of the specific application.

Equation (10), as it is, can certainly be used for tolerance optimisation with reference to costs attributed to the manufacturing accuracy. It permits optimum tolerancing to be based on an explicit relationship between the size of the particular dimension, its specified tolerance zone and the related manufacturing cost. The cost sensitivity to tolerance r, an exponent characteristic of the manufacturing process, available through recent extensive research [1, 6–10], can be evaluated as lying, for different applications, inside a range of $1/2 < r < 2$ (Fig. 2). Nevertheless, lower r-values ($1/2 < r < 1$), appear to be more reasonable, a conclusion supported by the relatively "less expensive accuracy" that modern machine tools and inspection equipment offer. In Fig. 2 these lower r-values demonstrate the actual industrial fact that lower ISO standard tolerance grades are now more readily accessible and less costly.

3. Transfer of Tolerances

A linear dimensional chain of a part with n dimensions D_j, $j = 1,2,\ldots,n$, and functional/design tolerances $\pm t_j$ which are transferred to a reference or locating

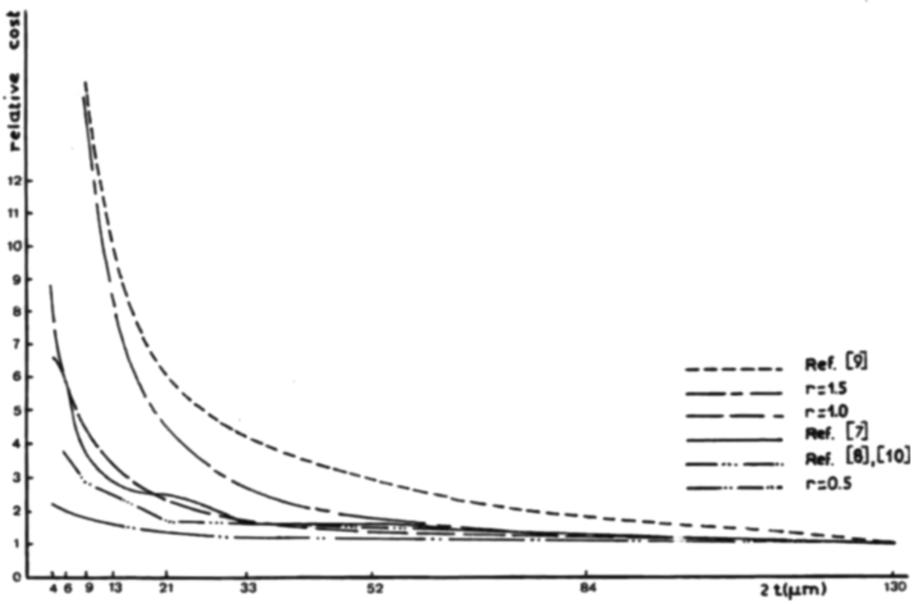

Fig. 2. Cost sensitivity to tolerance. D = 18. . .30 mm (hole), ISO IT4 – IT11.

surface LS, is shown in Fig. 3. The tolerances for convenience are taken symmetrically relative to the nominal size; however, alternative positioning of the tolerance zone is possible through arithmetical adjustment of the nominal dimension. For the new manufacturing tolerances $\pm t'_j$ of the new dimensions D'_j, the tolerance transfer principle which has to apply is expressed through the relationships

$$t'_{m-1} + t'_m \leqq t_m, \; t_m, t'_m > 0, \; 1 < m \leqq n \tag{11}$$

The inequalities (11) alone obviously allow for an infinite number of correct (from the tolerancing point of view) solutions for the new tolerances. The appropriate solution has nevertheless to be conceived by the manufacturing engineer or the process planner. In any case the manufacturing costs induced by the new tolerances will also have to be considered in some way, on the basis of the existing data and rules. If equation (10) is now introduced for linear dimensions and the resulting total manufacturing cost of the dimensions D'_j is desired to be a minimum, this additional condition may be used for the evaluation of the new tolerances. For the same manufacturing process and set-up,

$$\sum_{j=1}^{n} C(D'_j, t'_j) = \sum_{j=1}^{n} C'(D'_j) + C''_0 \cdot \sum_{j=1}^{n} \frac{i(D'_j)^{r+1}}{t'^{r}_j} \tag{12}$$

from which the condition for cost optimum tolerance transfer is obtained,

$$\sum_{j=1}^{n} \frac{i(D'_j)^{r+1}}{t'^{r}_j} \rightarrow \min \tag{13}$$

The new tolerances t'_j can finally be generated through the minimisation of (13) with the tolerance transfer principle relationships (11) acting as constraints. For this task a Fortran computer program OPTOL has been developed, which

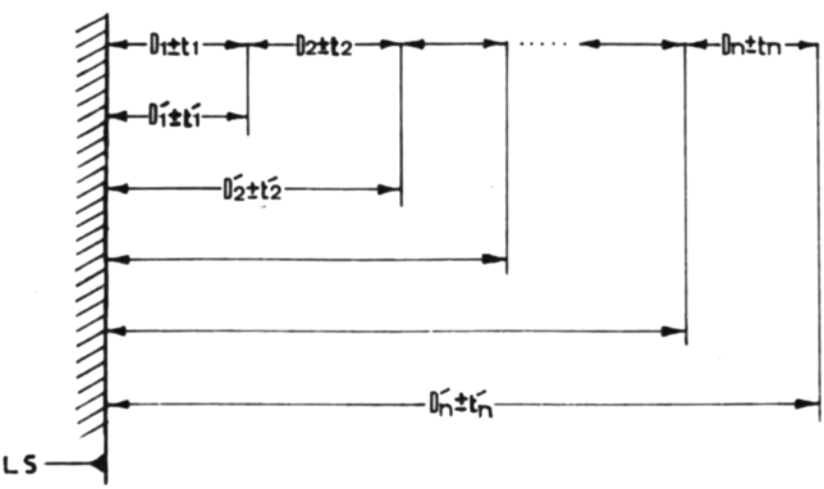

Fig. 3. Transfer of tolerances.

makes use of the Box's COMPLEX optimisation algorithm [12]. It provides for the considered part reference or locating surfaces the manufacturing dimensions and tolerances (D'_j, t'_i). Depending on the specific design and/or process planning facilities, the program may be further linked to the production of the part working drawing or for the allocation of the suitable machine tool, from the accuracy point of view, (Fig. 4). Its extension to two dimensional tolerance problems is also under development. An application example of OPTOL for a geared shaft is shown in Fig. 5, whereas, for this case, the cost sensitivity to tolerance has been taken r = 0.8.

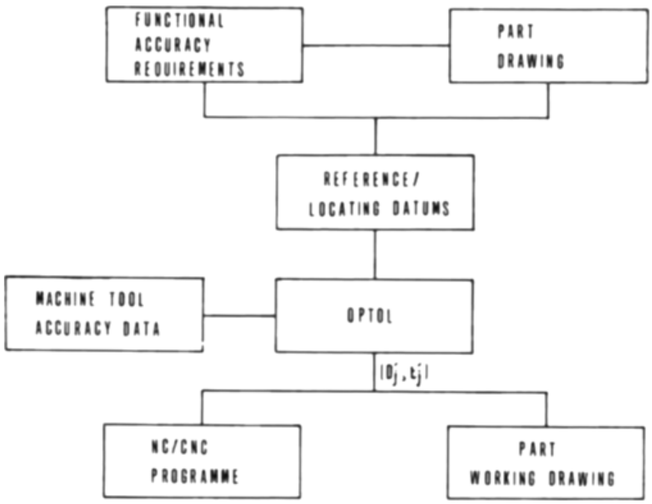

Fig. 4. Part reference/manufacturing datums and optimum tolerancing.

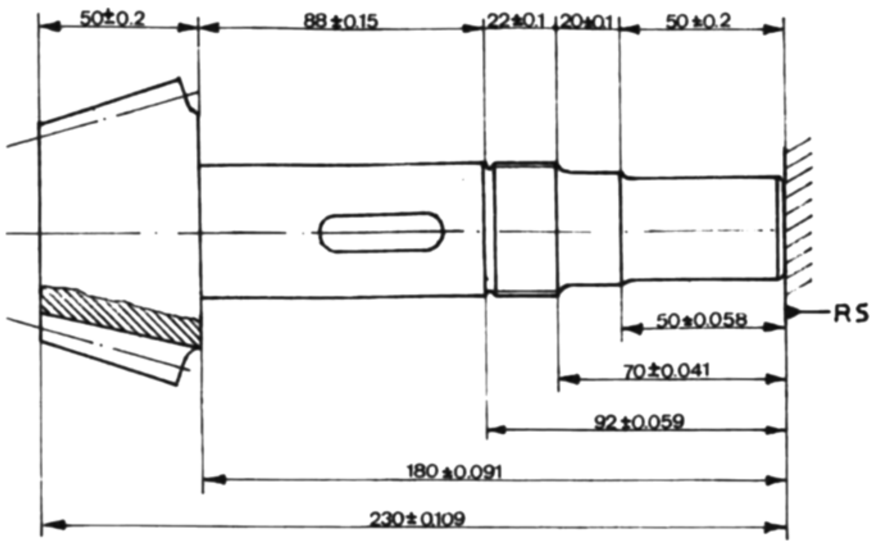

Fig. 5. Application example.

5. Conclusions

Relationships between the tolerance of a dimension and the manufacturing cost for this dimension are more readily applicable for tolerance optimisation than comparative cost data and/or design rules currently in use. They are particularly useful for a CAD/CAM environment and they may be applied for cost-optimum tolerancing in linear and non-linear problems both in the design and in the process planning stages. The presented model for the cost–tolerance function lies in satisfactory agreement with the existing data. It enables tolerance optimisation to be based on an explicit relationship between the size of the considered dimension, its tolerance zone and the related manufacturing cost. The cost sensitivity to tolerance r, an exponent characteristic of the manufacturing process, varies inside a rather narrow band of values and has to be known for the specific application. For tolerance transferring to new reference or locating surfaces, the developed cost–tolerance model in conjunction with the tolerance transfer principle provides a simple tool towards a new systematic tolerance designation.

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Appendix A. Nomenclature

A,B,F,K	manufacturing cost–tolerance constants
C(D,t)	manufacturing cost of dimension D with tolerance zone ± t
C'(D)	manufacturing cost of dimension D with commercial accuracy

$C''(D,t)$	additional accuracy cost for producing dimension D with tolerance $\pm$ t
C''_0	accuracy cost constant
D, D_j, D'_j	linear dimensions, mm
$i(D)$	ISO standard tolerance factor, μm
r	cost sensitivity to tolerance exponent
t, t_j, t'_j	tolerances, μm

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