Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction

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An exact solution to the flow due to impulsive motion of an infinite vertical plate in its own plane in the presence of i) species concentration ii) constant heat flux at the plate iii) chemical reaction of first order, has been derived by the Laplace-transform technique. Velocity and concentration profiles are shown on graphs. It is observed that due to the presence of first order chemical reaction, the velocity decreases but the skin-friction being positive at large values of the chemical reaction parameter, there may not occur separation of the flow near the plate.

1. Introduction

The flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate was first studied by *Stokes* (1851), who gave the first exact solution to the Navier-Stokes equation. Instead of a horizontal plate, if an infinite vertical isothermal plate is given an impulsive motion in its own plane, how the flow is affected by the free convection currents which exists due to the temperature difference between the plate temperature and that of the fluid far away from the plate? This was first studied by *Soundalgekar* (1977) who again presented an exact solution. The effects of heating or cooling of the plate on the flow were discussed. Another physical situation in the industrial applications of such a problem is the supply of heat to the impulsively started infinite vertical plate. This phenomenon is usually studied in the literature by assuming constant heat flux to the plate. The effect of constant heat flux on the flow past an impulsively started infinite vertical plate was again studied by *Soundalgekar* and *Patil* (1980) in case of air or water.

But in nature, the presence of pure air or water is rather impossible. It is always possible that some other foreign mass is either present naturally in air or water or foreign masses are mixed with air or water. Simple example is the naturally available water-vapour in nature which causes the flow of air. The flow is also caused by the differences in concentration or material constitution. The equations of motion for flow of gases or water, taking into account the presence of foreign mass of low level were derived by *Gebhart* (1971) and the effect of the presence of foreign mass on the free-convection flow past a semi-infinite vertical plate was also studied by *Gebhart* and *Pera* (1971).

The effects of mass transfer on the flow past an impulsively started infinite vertical plate at constant temperature or under constant heat flux conditions were studied by *Soundalgekar* (1979) and by *Soudalgekar* et al. (1984) respectively. Exact solutions were derived by the Laplace-transform technique.

But the presence of foreign mass in air or water causes, many times, some kind of chemical reaction, e.g. ammonia, benzene, ethyl alcohol etc. react with air when they come in contact under certain conditions. During chemical reaction between two species concentration, heat is also generated. But here we assume the level of species concentration to be very low and hence heat generated during chemical reaction can be neglected. Hence these results will be found useful in chemical technology where ammonia, ethyl alcohol are used.

Under these conditions, we now study the flow of a fluid in the presence of chemical reaction with a species concentration due to impulsive motion of an infinite vertical plate in its own plane. In Chap. 2, the mathematical analysis is presented, solutions to the non-dimensional coupled partial differential equations are derived by the Laplace-transform technique and in Chap. 3, the conclusions are set out.

2. Mathematical analysis

Consider the plate, the fluid and the species concentration to be initially at rest and at the same temperature and same level everywhere. At time t', the infinite plate is given an impulsive motion such that it attains a uniform velocity U_0 and heat is supplied to the plate at a constant rate and the concentration level near the plate is changed from C'_∞ to C'_w where C'_∞ is the species concentration at the initial stage everywhere. It is also assumed that there exists first-order chemical reaction between the fluid and the species concentration. Under these conditions, the flow can be shown to be governed by the following system of coupled linear partial differential equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial y'^2} \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - \kappa_1 C' \tag{3}$$

and the initial boundary conditions are

$$\begin{aligned} &u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \qquad \text{for all } y', \quad t' \le 0 \\ &u' = U_0, \quad \frac{\partial T'}{\partial y'} = -\frac{q}{K}, \quad C' = C'_W \qquad \text{at } y' = 0 \\ &u' = 0, \quad T' \to T'_\infty, \quad C' \to C'_\infty \qquad \text{as } y' \to \infty \end{aligned} \tag{4}$$

The x'-axis is assumed to be taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate.

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On introducing the following non-dimensional quantities

$$U = u'/U_0,$$

$$y = y'U_0/\nu,$$

$$t = t'U_0^2/\nu,$$

$$\theta = (T' - T'_\infty)/(q\nu/kU_0),$$

$$Gr = \frac{g\beta q \nu^2}{KU_0^4}, \tag{5}$$

$$Gm = \frac{\nu g \beta^*(C'_w - C'_\infty)}{U_0^3},$$

$$Pr = \mu C_p/K,$$

$$Sc = \nu/D$$

$$\gamma = \mu k_1/U_0^2$$

in Eqs. (1) to (4), we have

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial y^2} \tag{6}$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}. \tag{7}$$

$$Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} - \gamma Sc C \tag{8}$$

with following initial and boundary conditions:

$$U = 0, \quad \theta = 0, \quad C = 0 \qquad \text{all } y, \quad t \le 0$$

$$U = 1, \quad \frac{d\theta}{dy} = -1, \quad C = 1 \qquad \text{at } y = 0 \tag{9}$$

$$U = 0, \quad \theta \to 0, \quad C \to 0 \qquad \text{as } y \to \infty$$

Here, Pr is the Prandtl number, Sc the Schmidt number, γ the chemical reaction parameter, Gr the Grashof number and Gc is the modified Grashof number.

These equations are solved by the usual Laplace-transform technique and the solutions are as follows:

$$\theta = \frac{2\sqrt{t}}{\sqrt{Pr}}\left[\frac{\exp(-\eta Pr)}{\sqrt{\pi}} - \eta \operatorname{erfc}(\eta\sqrt{Pr})\right] \tag{10}$$

$$C = \frac{1}{2}[\exp(2\eta\sqrt{\gamma t Sc}) \cdot \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{\gamma t}) + \\ + \exp(-2\eta\sqrt{\gamma t Sc}) \cdot \operatorname{erfc} \cdot (\eta\sqrt{Sc} - \sqrt{\gamma t}] \tag{11}$$

$$U = \left(1 + \frac{Gc}{\gamma Sc}\right)\operatorname{erfc}(\eta) + \frac{Gr \cdot t^{3/2}}{3(Pr-1)/\sqrt{Pr}} \cdot \\ \cdot \left\{\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2) - \eta(6+4\eta^2)\operatorname{erfc}(\eta)\right\} - \\ - \frac{Gc}{\gamma Sc} \cdot \frac{\exp(at)}{2}\{\exp(2\eta\sqrt{at}) \cdot \operatorname{erfc}(\eta + \sqrt{at}) + \\ + \exp(-2\eta\sqrt{at})\operatorname{erfc}(\eta - \sqrt{at})\} - \\ - \frac{Grt^{3/2}}{3(Pr-1)\sqrt{Pr}}\left\{\frac{4}{\sqrt{\pi}}(1+\eta^2 Pr) \cdot \\ \cdot \exp(-\eta^2 Pr) - \eta\sqrt{Pr}(6 + 4\eta^2 Pr)\operatorname{erfc}(\eta\sqrt{Pr})\right\} + \\ + \frac{Gc}{\gamma Sc}\frac{\exp(at)}{2}\{\exp(2\eta\sqrt{ctSc})\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{ct}) + \\ + \exp(-2\eta\sqrt{ctSc})\operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{ct})\} - \\ - \frac{Gc}{2\gamma Sc}\{\exp(2\eta\sqrt{\gamma t Sc})\operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{\gamma t}) + \\ + \exp(-2\eta\sqrt{\gamma Sct}) \cdot \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{\gamma t})\}. \tag{12}$$

where

$$a = \frac{\gamma Sc}{1 - Sc}, \quad C = \frac{\gamma(1 + Sc)}{1 - Sc} \quad \text{and} \quad \eta = y/2\sqrt{t}$$

In order to get an insight into the physical situation of the problem, we have computed the numerical values of the velocity and the concentration. As the values of the Schmidt number ≈ 1, we have chosen the values of Sc as 0.7 or 0.9.

The concentration profiles are plotted on Fig. 1. We observe from this figure that an increase in the value of the Schmidt number or the reaction parameter leads to a decrease in the concentration profiles.

The velocity profiles for air and water are shown on Fig. 2 and 3 respectively. We observe from the Fig. 1 and 2 that there is a fall in the velocity of air or water due to an increase in the value of the Schmidt number or the chemical reaction parameter. But an increase in the Grashof number Gr leads to a rise in the velocity of air, but very little change is observed in case of water. But an increase in the modified Grashof number leads to a decrease in the velocity of air or water. The velocity also decreases with increasing time.

From the velocity field, it is interesting to study the effects of the chemical reaction on the skin-friction. It is given by

$$\tau = -\tau'/\rho U_0^2 = -\left.\frac{\partial U}{\partial y}\right|_{y=0} \tag{13}$$

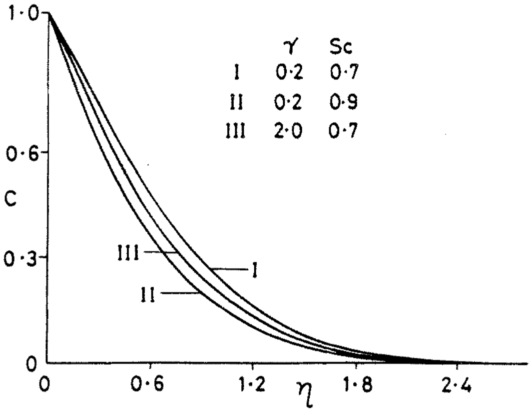

Fig. 1. Concentration profiles

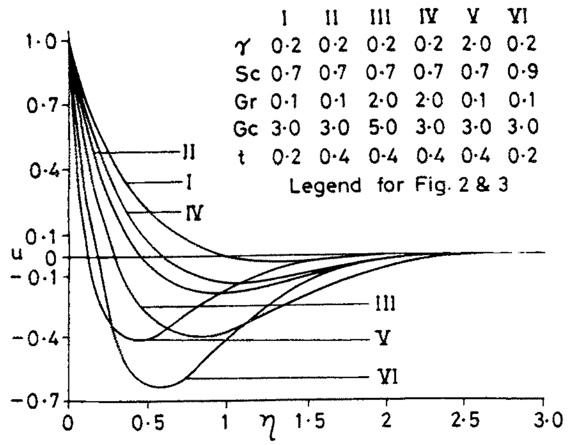

Fig. 2. Velocity profiles, $Pr = 0.71$

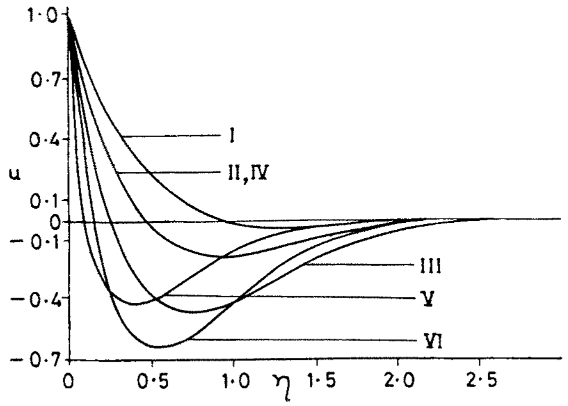

Fig. 3. Velocity profiles, $Pr = 7.0$

From Eqs. (12) and (13), we have

$$\tau = -\frac{1}{2\sqrt{t}}\left[\left(1+\frac{Gc}{\gamma Sc}\right)+\frac{2Gc}{\gamma \cdot Sc}\exp\left(at\right)(\sqrt{at}-\sqrt{ctSc})+\right.$$

$$+\frac{2Gc}{\gamma Sc \cdot \sqrt{\pi}}(1-\sqrt{Sc}\cdot\exp\left((a-c)t\right))+\frac{2Gc}{\gamma \cdot Sc}\cdot$$

$$\left.\cdot\left(\frac{\sqrt{Sc}}{\pi}\exp\left(-\gamma t\right)\right)+\frac{2Grt^{3/2}}{\sqrt{Pr}\cdot(Pr-1)}(\sqrt{Pr}-1)\right] \quad (14)$$

The numerical values of τ are calculated and these are listed in the Table.

Values of the skin-friction

Pr	γ	Sc	Gr	Gc/t	0.2	0.4
0.71	0.2	0.7	0.1	3	−3.82273	−4.30492
		0.9	0.1	3	0.51155	0.44959
		0.7	2.0	3	−4.06747	−4.79442
		0.7	2.0	5	−7.44840	−8.24190
	2.0	0.7	0.1	3	1.37001	6.81881
7.0	0.2	0.7	0.1	3	−3.81190	−4.28330
	0.2	0.9	0.1	3	0.52230	0.45170
	0.2	0.7	2.0	5	−7.23220	−7.80950
	0.2	0.7	2.0	3	−3.85510	−4.36210
	2.0	0.7	0.1	3	1.38080	6.84040

We observe from this table that at small values of the Schmidt number Sc and the chemical reaction parameter γ, the skin-friction-values are negative i.e. there may occur separation near the plate, but at large values of Sc or γ, the skin-friction values are positive indicating that there may not occur separation of the flow at the plate.

3. Conclusions

- The velocity decreases with increasing the Schmidt number and the chemical reaction parameter.
- When the Schmidt number Sc or the reaction parameter γ is high, there may not occur separation of the flow from the plate as then the skin-friction is positive.

Nomenclature

C'	species concentration in the fluid
C'_∞	species concentration in the fluid far away from the plate
C'_w	species concentration near the plate
C_p	specific heat at constant pressure
D	chemical molecular diffusivity
g	acceleration due to gravity
Gr	Grashof number
Gm	modified Grashof number
K	theremal conductivity
K_l	chemical reaction parameter
Pr	Prandtl number
q	heat supplied at constant rate
Sc	Schmidt number
t'	time
T'	temperature of the fluid near the plate
T'_∞	temperature of the fluid far away from the plate
T'_w	temperature of the plate
u'	axial velocity
U_0	velocity of the plate
x'	axial coordinate axis
y'	normal coordinate axis
β	coefficient of volume expansion
β^*	coefficient of expansion with concentration
ν	kinematic viscosity
ρ	density of fluid
γ	non-dimensional chemical reaction parameter.

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Received August 23 1993 F 4168

BÜCHER

Darstellung von Acrylsäure aus Propen. Die chemische Kinetik im technischen Prozeß. Von *Rüdiger Recknagel.* Fortschr.-Ber. VDI Reihe 3, Nr. 370. Düsseldorf: VDI-Verlag 1994. 106 S., 60 B., 19 Tab., Preis: 69,–DM.

In der Arbeit wurde Acrolein an einem nach der Patentliteratur hergestellten Katalysator in einem Zapfstellenreaktor, an dem in axialer Richtung seitliche Ausgänge angebracht sind, zu Acrylsäure umgesetzt. Mit einem mathematischen Modell wurden die im Reaktor ablaufenden Parallel- und Folgereaktionen und der Einfluß der Temperatur auf die Reaktionskinetik beschrieben. Die im technischen Prozeß im Reaktionsgut bei der Oxidation von Acrolein zu Acrylsäure (2. Stufe der technischen Acrylsäureherstellung aus Propen) enthaltenen geringen Mengen an CO, CO_2, Acrylsäure, Wasser sowie nicht umgesetztem Propen wurden dem Reaktor einzeln zugeführt und ihr Einfluß im mathematischen Modell erfaßt. Die Anwesenheit von Wasserdampf in der Gasphase führt dabei zu einer selektiven Beschleunigung der Reaktionsgeschwindigkeit von Acrolein zu Acrylsäure, verbunden mit einer 5% höheren Acrylsäureausbeute. Mit dem mathematischen Modell konnten die für eine hohe Acrylsäureausbeute des Gesamtverfahrens günstigsten Betriebsbedingungen ermittelt und im Experiment verifiziert werden. F 10004

PIV-Messungen instationärer Geschwindigkeitsfelder an einem schwingenden Rotorprofil. Von *Markus Raffel.* DLR-Forschungsbericht 93-50, 1994, 93 S., 106 B., 1 Tab., Preis: 12,–DM zzgl. MWSt.

Das instationäre Geschwindigkeitsfeld über einem schwingendem NACA 0012 Profil wurde erstmalig im Windkanal mit Hilfe der Particle Image Velocimetry ermittelt. Durch die kurze Meßzeit von nur wenigen Mikrosekunden war die Bestimmung kompletter, momentaner Geschwindigkeitsfelder möglich. Zunächst wurde bei Messungen im Hochgeschwindigkeitswindkanal Göttingen die Strömungsablösung bei geringer Anströmungeschwindigkeit und hohen Anstellwinkeln ($Ma = 0{,}25$, $\alpha = 5° - 25°$) und bei transonischer Anströmgeschwindigkeit und kleinen Anstellwinkeln ($Ma = 0{,}75$, $\alpha = 0° - 5°$) mit Verdichtungsstößen über dem Profil untersucht. Während einer Meßkampagne im Niedergeschwindigkeitswindkanal des Deutsch-Französischen Forschungsinstitutes Saint Louis (ISL) wurden dann auch die instationären Geoschwindigkeitsfelder der dynamischen Strömungsablösung an einem mit einer Frequenz von $f = 6{,}7$ Hz schwingendem Profil ermittelt. Die Ergebnisse zeigen das stark instationäre Verhalten des Strömungsfeldes über dem Profil insbesondere im Bereich abnehmender Anstellwinkel, und ermöglichen so neue Einblicke in die zugrundeliegenden Strömungsphänomene. Die experimentell bestimmten Geschwindigkeitsfelder und daraus abgelneitete Größen wie die „Vorticity" sind von hoher Qualität, so daß sie mit den Ergebnissen numerischer Berechnungen verglichen werden können. F 10006

Optische Auswertung von Particle-Image-Velocimetry-Messungen im Nachlauf eines quer angeströmten Kreiszylinders. Von *Andreas Vogt.* DLR-Forschungsbericht 93-49, 1993, 121 S., 149 B., 6 Tab., Preis: 15,50 DM zzgl, MWSt.

Es wird ein neues Verfahren vorgestellt, mit dem die Auswertung von Particle-Image-Velocimetry-Messungen größtenteils auf optischem Wege realisiert werden kann. Mit dem Verfahren werden PIV-Messungen aus dem Nachlauf eines quer angeströmten Kreiszylinders bei Reynoldszahlen von $2 \cdot 10^4$ ausgewertet. Anhand der so gewonnenen momentanen Geschwindigkeitsvektorfelder sowie der auf diese Vektorfelder angewendeten Vektoroperatoren Divergenz und Rotation werden Aussagen über die Dreidimensionalität des Nachlaufs und die hier entstehenden Wirbelstraßen gemacht. Insbesondere ist die einfache Bestimmung der Wirbeltransportgeschwindigkeit und der Periodizität der Wirbelstraße möglich. F 10007

Modellierung des Tropfenverdunstungsprozesses bei überkritischem Umgebungsdruck. Von *Peter Olthoff.* DLR-Forschungsbericht 93-54, 1993, 174 S., 94 B., 20 Tab., 65,–DM zzgl. MWSt.

Die vorliegende Arbeit untersucht die Tropfenverdunstung in einer ruhenden Hochdruck- und Hochtemperaturatmosphäre. Zur theoretischen Simulation des Tropfenverdunstungsprozesses wird eine Modellierung unter Berücksichtigung der wesentlichen Einflußfaktoren, insbesondere derer des Hochdrucks, weiterentwickelt. Theoretische Entwicklungen und Untersuchungen haben gezeigt, daß Einflüsse des realen Hochdruckverhaltens für die Berechnung des Mol- bzw. Masseanteils für das Gas-Flüssigkeits-Gleichgewicht an der Tropfengrenzfläche sowie die entsprechende reale Enthalpiedifferenz berücksichtigt werden müssen. Involviert ist auch die damit einhergehende Absorption des Umgebungsfluids in die Tropfenoberfläche. Stoffdaten der betrachteten Fluide müssen als Funktion von Temperatur und Druck approximiert werden. Zur experimentellen Verifikation wurde ein Hochdruck-Hochtemperatur-Versuchsstand entwickelt, in dem die Verdunstung von Heptantropfen in einer Stickstoffumgebung beobachtet und durch Tropfengrößen/Zeit- und Tropfentemperatur/Zeit-Charakteristika vermessen wurde. Der Systemdruck variierte im Bereich von $p_{\mathrm{Umg}} = 2 - 100$ bar und die Systemtemperatur im Bereich von $T_{\mathrm{Umg}} = 296 - 773$ K. Vergleiche zwischen theoretischer Modellierung und den eigenen experimentell gewonnenen Daten sowie auch denen von früheren Experimentatoren zeigen nunmehr eine exzellente Übereinstimmung. F 10008