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ERROR BOUNDS FOR MONOTONE LINEAR COMPLEMENTARITY PROBLEMS

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We give a bound on the distance between an arbitrary point and the solution set of a monotone linear complementarity problem in terms of a condition constant that depends on the problem data only and a residual function of the violations of the complementarity problem conditions by the point considered. When the point satisfies the linear inequalities of the complementarity problem, the residual consists of the complementarity condition plus its square root. This latter term is essential and without it the error bound cannot hold. We also show that another natural residual that has been employed to bound errors for strictly monotone linear complementarity problems fails to bound errors for the monotone case considered here.

Key words: Linear complementarity problems, condition number, error bounds, convex programming.

1. Introduction

Consider the monotone linear complementarity problem [2] of finding an x in the n-dimensional real space R^n such that

$$Mx+q\geqslant 0, \quad x\geqslant 0, \quad x(Mx+q)=0, \tag{1.1}$$

where M is an $n\times n$ positive semidefinite real matrix and q is in R^n. Suppose that the problem has a nonempty solution set $\bar{S}$. The question we wish to address in this work is the following. Given an arbitrary point x in R^n that violates one or all three conditions of (1.1), how close is x to $\bar{S}$ in terms of its violations of the conditions (1.1)? More specifically we are interested in a measure of the distance between x and $\bar{S}$ in terms of the residual vector

$$((-Mx-q)_+, (-x)_+, |x(Mx+q)|) \tag{1.2}$$

where $((-x)_+)_i=\max\{0, -x_i\}$, $i=1,\ldots,n$. Note that the residual vector vanishes if and only if x is in the solution set $\bar{S}$. A principal result, Theorem 2.7 below, shows

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that for each x in R^n there exists an $\bar{x}(x)$ in $\bar{S}$ such that the ∞-norm distance $\|x-\bar{x}(x)\|_\infty$ is bounded by a *condition constant* $\tau_2(M, q)$(dependent on M and q only) times a positive function of the residual vector (1.2) which vanishes if and only if the residual vector (1.2) is zero. The condition constant $\tau_2(M, q)$ plays the same role for the monotone linear complementarity problem (1.1) as does $\|A^{-1}\|$ in bounding the distance $\|x-\bar{x}\|$, between an arbitrary point x in R^n and the exact solution $\bar{x}=A^{-1}b$ to $A\bar{x}=b$, by the residual vector $Ax-b$ as follows:

$$\|x-\bar{x}\| \leq \|A^{-1}\| \; \|Ax-b\|.$$

Theorem 2.7 simplifies considerably to Corollary 2.8 if the point x in R^n is feasible, that is, it satisfies the first two inequalities of (1.1), in which case the error $\|x-\bar{x}(x)\|_\infty$ is bounded by $\tau_2(M, q)$ times the residual $x(Mx+q)+(x(Mx+q))^{1/2}$. Example 2.9 shows that the term $(x(Mx+q))^{1/2}$ is an essential part of the residual, without which the error $\|x-\bar{x}(x)\|_\infty$ cannot be bounded. Theorem 2.11 and Corollary 2.12 give bounds on the *relative* error $\|x-\bar{x}(x)\|_\infty/\|\bar{x}(x)\|_\infty$ in terms of the *condition number* $\tau_2(M, q)\cdot\|M\|_\infty$ times a *relative* residual function.

Pang has given error bounds for nonlinear complementarity problems [8] and linearly constrained variational inequalities [9]. When applied to the linear complementarity problem (1.1), Pang's result requires in effect [8, Lemma 2] that the matrix M be positive definite, whereas our results merely require that M be positive semidefinite. Although Pang's natural residual [8, Lemma 2]

$$\left(\sum_{i=1}^{n} (\min\{x_i, M_ix+q_i\})^2\right)^{1/2} \tag{1.3}$$

is simpler than ours, we show by means of Example 2.10 that this residual cannot be used as an error measure for the positive semidefinite case under consideration in this paper.

A brief word about notation and some basic concepts employed. For a vector x in the n-dimensional real space R^n, $|x|$ and x_+ will denote the vectors in R^n with components $|x|_i := |x_i|$ and $(x_+)_i := \max\{x_i, 0\}$, $i=1,\ldots,n$, respectively. For a norm $\|x\|_\beta$ on R^n, $\|x\|_{\beta^*}$ will denote the dual norm [3, 7] on R^n, that is $\|x\|_{\beta^*} := \max_{\|y\|_\beta=1} xy$, where xy denotes the scalar product $\sum_{i=1}^n x_iy_i$. The generalized Cauchy-Schwarz inequality $|xy| \leq \|x\|_\beta \cdot \|y\|_{\beta^*}$, for x and y in R^n, follows immediately from this definition of the dual norm. For $1 \leq p$, $q \leq \infty$, and $1/p+1/q=1$, the p-norm $(\sum_{i=1}^n |x_i|^p)^{1/p}$ and the q-norm are dual norms on R^n [7]. If $\|\cdot\|_\beta$ is a norm on R^n, we shall, with a slight abuse of notation, let $\|\cdot\|_\beta$ also denote the corresponding norm on R^m for $m \neq n$. For an $m\times n$ real matrix A signified by $A \in R^{m\times n}$, A_i denotes the ith row, $A_{\cdot j}$ denotes the jth column, $A_I := A_{i\in I}$, and $A_{\cdot J} := A_{\cdot j\in J}$, where $I \subset \{1,\ldots,m\}$ and $J \subset \{1,\ldots,n\}$. $\|A\|_\beta$ denotes the matrix norm [7] subordinate to the vector norm $\|\cdot\|_\beta$, that is $\|A\|_\beta = \max_{\|x\|_\beta=1}\|Ax\|_\beta$. The consistency condition $\|Ax\|_\beta \leq \|A\|_\beta\|x\|_\beta$ follows immediately from this definition of a matrix norm. A monotonic norm on R^n is any norm $\|\cdot\|$ on R^n such that for a, b in R^n, $\|a\| \leqq \|b\|$ whenever $|a| \leq |b|$ or equivalently if $\|a\| = \||a|\|$ [3, p 47]. The p-norm for $p \geq 1$ is monotonic [7]. A vector of ones in any real space will be denoted by e. The identity matrix of any order will be denoted by I. The nonnegative orthant in R^n will be denoted by R^n_+.

2. Principal results

Throughout this paper M will denote an $n\times n$ real matrix, q a point in R^n, (M, q) will denote the linear complementarity problem (1.1), and

$$\hat{M} := \tfrac{1}{2}(M+M^{\mathrm{T}}), \tag{2.1}$$

$$S = S(M,q) := \{x \mid Mx+q \geq 0,\ x\geq 0\}, \tag{2.2}$$

$$\bar{S} = \bar{S}(M,q) := \{x \mid Mx+q\geq 0,\ x\geq 0,\ x(Mx+q)=0\}. \tag{2.3}$$

It is well known [2] that the solution set $\bar{S}$ is nonempty if and only if the feasible set S is nonempty, provided that M is positive semidefinite. We begin with some preliminary results.

Lemma 2.1 (Adler and Gale [1]; polyhedrality of the solution set of (M, q)). *Let M be positive semidefinite and let $\bar{x}\in\bar{S}$. Then*

$$\bar{S} = \{x \mid Mx+q\geq 0,\ x\geq 0,\ x(2\hat{M}\bar{x}+q)+q\bar{x}\leq 0,\ \hat{M}(x-\bar{x})=0\}.$$

Proof. $(x-\bar{x})(Mx+q-(M\bar{x}+q)) = (x-\bar{x})M(x-\bar{x})$.
Hence

$$x(Mx+q) = \bar{x}(Mx+q)+x(M\bar{x}+q)+(x-\bar{x})M(x-\bar{x}).$$

Since for $x\in S$, each quantity on the right-hand side of the last equation is nonnegative, it follows that

$$\begin{aligned}\bar{S} &= \{x\in S \mid x(Mx+q)=0\}\\ &= \{x \mid Mx+q\geq 0,\ x\geq 0,\ \bar{x}(Mx+q)+x(M\bar{x}+q)\leq 0,\ (x-\bar{x})M(x-\bar{x})=0\}\\ &= \{x \mid Mx+q\geq 0,\ x\geq 0,\ x(2\hat{M}\bar{x}+q)+q\bar{x}\leq 0,\ \hat{M}(x-\bar{x})=0\}.\end{aligned}$$

The last equality follows from the equivalence of $zMz=0$ and $\hat{M}z=0$ for a positive semidefinite matrix, since $2\hat{M}z$, the gradient of zMz, must vanish when $zMz=0$. □

Lemma 2.2. *Let $\hat{M}\in R^{n\times n}$ be symmetric positive semidefinite. Then*

$$\hat{M}x=0 \Leftrightarrow \hat{M}^{1/2}x=0.$$

Proof. $\hat{M}^{1/2}x=0 \Rightarrow \hat{M}x=0 \Rightarrow \hat{M}^{1/2}\hat{M}^{1/2}x=0 \Rightarrow x\hat{M}^{1/2}\hat{M}^{1/2}x=0 \Rightarrow \|\hat{M}^{1/2}x\|_2^2=0 \Rightarrow \hat{M}^{1/2}x=0$. □

Lemmas 2.1 and 2.2 combined give the following.

Lemma 2.3. *Let M be positive semidefinite and let $\bar{x} \in \bar{S}$. Then*

$$\bar{S} = \{x \mid Mx + q \geq 0,\ x \geq 0,\ x(2\hat{M}\bar{x} + q) + q\bar{x} \leq 0,\ \hat{M}^{1/2}(x - \bar{x}) = 0\}. \tag{2.4}$$

By using the polyhedral characterization (2.4) and the condition number result for linear inequalities and equalities of either [4] or [6], we are able to obtain a preliminary bound on the distance between any point in R^n and the solution set $\bar{S}$ of (M, q).

Proposition 2.4. *Let M be positive semidefinite and let $\bar{x} \in \bar{S}$. For each x in R^n there exists an $\bar{x}(x)$ in $\bar{S}$ that is independent of the choice of $\bar{x}$, such that*

$$\|x - \bar{x}(x)\|_\infty \leq \tau_\beta(M, q) \|((-Mx - q, -x, x(2\hat{M}\bar{x} + q) + q\bar{x})_+, \hat{M}^{1/2}(x - \bar{x}))\|_\beta$$

where $\|\cdot\|_\beta$ is some norm on R^{3n+1}, $\|\cdot\|_{\beta^*}$ is its dual norm and

$$\tau_\beta(M, q) := \sup_{(u,v,z,\xi) \in R^{3n+1}} \left\{ \|u, v, z, \xi\|_{\beta^*} \;\middle|\; \begin{array}{l} \|uM + v + z\hat{M}^{1/2} - \xi(2\bar{x}\hat{M} + q)\|_1 = 1, \\ (u, v, \xi) \geq 0 \\ \text{Rows of} \begin{pmatrix} M \\ I \\ \hat{M}^{1/2} \\ 2\hat{M}\bar{x} + q \end{pmatrix} \text{corresponding} \\ \text{to nonzero elements of } (u, v, z, \xi) \\ \text{are lin. indep.} \end{array} \right\} \tag{2.5}$$

Proof. Follows by the application of Theorem 2.2′ of [6] (or the essentially equivalent Theorem 1 of [4]) to $\bar{S}$ as defined by (2.4), and by noting from Lemma 2.1 that $\hat{M}\bar{x}$ is constant for all $\bar{x}$ in $\bar{S}$. □

Note that in view of Lemma 2.1 we have that $\hat{M}^{1/2}\bar{x}$ is constant for all $\bar{x}$ in $\bar{S}$. In addition, since for $\bar{x}$ and $\hat{x}$ in $\bar{S}$, $q(\hat{x} - \bar{x}) = 2\hat{x}\hat{M}(\bar{x} - \hat{x}) = 0$, it follows that $q\bar{x}$ is also constant for all $\bar{x}$ in $\bar{S}$. Thus the bound on $\|x - \bar{x}(x)\|_\infty$ of Proposition 2.4 is independent of the choice of $\bar{x}$ in $\bar{S}$.

We need two more lemmas before stating our principal results.

Lemma 2.5. *Let $\bar{x} \in \bar{S}$. Then, for each $x \in R^n$,*

$$\|\hat{M}^{1/2}(x - \bar{x})\|_2^2 \leq x(Mx + q) + \|(-Mx - q, -x)_+\|_\beta \cdot \|(\bar{x}, M\bar{x} + q)\|_{\beta^*}$$

where $\|\cdot\|_\beta$ is some norm on R^{2n} and $\|\cdot\|_{\beta^*}$ is its dual norm.

Proof

$$\begin{aligned} \|\hat{M}^{1/2}(x - \bar{x})\|_2^2 &= (x - \bar{x})\hat{M}(x - \bar{x}) \\ &= x(Mx + q) + \bar{x}(-Mx - q) + (-x)(M\bar{x} + q) \\ &\leq x(Mx + q) + \bar{x}(-Mx - q)_+ + (-x)_+(M\bar{x} + q) \\ &\leq x(Mx + q) + \|(-Mx - q, -x)_+\|_\beta \cdot \|(\bar{x}, M\bar{x} + q)\|_{\beta^*}. \end{aligned}$$

□

Lemma 2.6. *Let $\bar{x} \in \bar{S}$ and let M be positive semidefinite. Then, for any $x \in R^n$,*

$$(\bar{x}(Mx+q)+x(M\bar{x}+q))_+ \leq (x(Mx+q))_+.$$

Proof

$$(\bar{x}(Mx+q)+x(M\bar{x}+q))_+ = (x(Mx+q)-(x-\bar{x})\hat{M}(x-\bar{x}))_+$$
$$\leq (x(Mx+q))_+. \qquad \square$$

We are now ready for our principal results.

Theorem 2.7 (Absolute error bound for approximate solutions of monotone linear complementarity problems). *Let M be positive semidefinite and let $S \neq \emptyset$. For each x in R^n there exists an $\bar{x}(x)$ in $\bar{S}$ such that*

$$\|x-\bar{x}(x)\|_\infty \leq \tau_2(M,q)[\|(x(Mx+q), -Mx-q, -x)_+\|_2 + (x(Mx+q)+\sigma_\beta\|(-Mx-q,-x)_+\|_\beta)^{1/2}] \tag{2.6}$$

where $\tau_2(M,q)$ is defined by (2.5), $\|\cdot\|_\beta$ *is some norm on R^{2n} and σ_β is defined by*

$$\sigma_\beta = \sigma_\beta(M,q) := \min_{\bar{x}\in\bar{S}} \|\bar{x}, M\bar{x}+q\|_{\beta^*}. \tag{2.7}$$

Proof. By [2], $\bar{S} \neq \emptyset$ since $S \neq \emptyset$. Let $\bar{x} \in \bar{S}$. Then by Proposition 2.4 above, for each x in R^n there exists an $\bar{x}(x)$ in $\bar{S}$ that is independent of the choice of $\bar{x}$ such that

$$\|x-\bar{x}(x)\|_\infty \leq \tau_2(M,q)\|(-Mx-q, -x, x(M\bar{x}+q) + \bar{x}(Mx+q))_+, \hat{M}^{1/2}(x-\bar{x})\|_2$$
$$\leq \tau_2(M,q)[\|(-Mx-q,-x,x(Mx+q))_+\|_2 + (x(Mx+q)+\|(-Mx-q,-x)_+\|_\beta \cdot \|(\bar{x}, M\bar{x}+q)\|_{\beta^*})^{1/2}]$$

(by Lemmas 2.6, 2.5 and monotonicity of the 2-norm [3]). Hence taking the infimum of the right side over all $\bar{x}$ in $\bar{S}$, we get

$$\|x-\bar{x}(x)\|_\infty \leq \tau_2(M,q)[\|(x(Mx+q), -Mx-q, -x)_+\|_2 + (x(Mx+q)+\sigma_\beta\|(-Mx-q,-x)_+\|_\beta)^{1/2}]. \qquad \square$$

When x is in S, Theorem 2.7 simplifies to the following.

Corollary 2.8 (Absolute error bounds for *feasible* approximate solutions of monotone complementarity problems). *Let M be positive semidefinite and let $S \neq \emptyset$. For each x in S there exists an $\bar{x}(x)$ in $\bar{S}$ such that*

$$\|x-\bar{x}(x)\|_\infty \leq \tau_2(M,q)[(x(Mx+q))+(x(Mx+q))^{1/2}] \tag{2.8}$$

where $\tau_2(M,q)$ is defined by (2.5).

The following example shows that the residual term $(x(Mx+q))^{1/2}$ in (2.8) is essential and cannot be dispensed with.

Example 2.9. Suppose that

$$M=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix},\qquad q=\begin{bmatrix}0\\ 1\end{bmatrix},$$

so that $\bar{S}=\{0\}$ and $S=\{x\in R^2 \mid x_2\geqslant 0,\ x_2\leqslant x_1\}$. Let

$$x(\varepsilon):=\begin{bmatrix}\varepsilon\\ \varepsilon^2\end{bmatrix}\in S \text{ for } 0\leqslant \varepsilon\leqslant 1.$$

Then

$$\frac{\|x(\varepsilon)-0\|_\infty}{x(\varepsilon)(Mx(\varepsilon)+q)}=\frac{\varepsilon}{2\varepsilon^2+\varepsilon^4}\to\infty \quad \text{as } \varepsilon\to 0.$$

However,

$$\frac{\|x(\varepsilon)-0\|_\infty}{x(\varepsilon)(Mx(\varepsilon)+q)+(x(\varepsilon)(Mx(\varepsilon)+q))^{1/2}}=\frac{\varepsilon}{2\varepsilon^2+\varepsilon^4+(2\varepsilon^2+\varepsilon^4)^{1/2}}$$

$$\to\frac{\sqrt{2}}{2} \quad \text{as } \varepsilon\to 0.$$

In [8, Lemma 2] Pang uses the natural residual (1.3) as an error measure for the positive definite linear complementarity problem. It is easy to show, by considering for each component i the two cases of $x_i\geqslant M_ix+q_i$ and $x_i<M_ix+q_i$, that the residual (1.3) is equivalent to $\|x-(x-(Mx+q))_+\|_2$. The following example shows that (1.3) cannot be used as a measure of error for the positive semidefinite case under consideration in this work.

Example 2.10. Let

$$M=\begin{pmatrix}0 & -1\\ 1 & 1\end{pmatrix},\qquad q=\begin{pmatrix}1\\ -2\end{pmatrix}.$$

The unique solution of this problem is $\bar{x}=\binom{1}{1}$. Letting $x(t):=\binom{t}{1}$, we get that, for $t\geqslant 2$,

$$x(t)-(x(t)-(Mx(t)+q))_+=\begin{pmatrix}0\\ 1-(2-t)_+\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix},\ x(t)-\bar{x}=\begin{pmatrix}t-1\\ 0\end{pmatrix}.$$

Hence

$$\frac{\|x(t)-\bar{x}\|_2}{\|x(t)-(x(t)-(Mx(t)+q))_+\|_2}=t-1\to\infty \quad \text{as } t\to\infty.$$

Consequently, the residual (1.3) cannot be used as an error bound for the positive semidefinite case.

By noting that for $\bar{x}(x) \in \bar{S}$

$$(-q)_+ \leq (M\bar{x}(x))_+ \leq |M\bar{x}(x)|, \; \|(-q)_+\|_\infty \leq \|M\bar{x}(x)\|_\infty \leq \|M\|_\infty \|\bar{x}(x)\|_\infty,$$

the following theorem and corollary follow directly from Theorem 2.7 and Corollary 2.8, respectively, thus giving rise to a bound on the relative error $\|x-\bar{x}(x)\|_\infty / \|\bar{x}(x)\|_\infty$ in x in terms of the *condition number* $\tau_2(M, q)\|M\|_\infty$ and the corresponding relative residual.

Theorem 2.11 (Relative error bound for approximate solutions of monotone linear complementarity problems). *Let M be positive semidefinite, let $q \not\geq 0$ and let $S \neq \emptyset$. For each x in R^n there exists an $\bar{x}(x)$ in $\bar{S}$ such that*

$$\frac{\|x-\bar{x}(x)\|_\infty}{\|\bar{x}(x)\|_\infty} \leq \tau_2(M, q)\|M\|_\infty [\|(x(Mx+q), -Mx-q, -x)_+\|_2 + (x(Mx+q) + \sigma_\beta \|(-Mx-q, -x)_+\|_\beta)^{1/2}] / \|(-q)_+\|_\infty \qquad (2.9)$$

where $\tau_2(M, q)$ is defined by (2.5) and σ_β by (2.7).

Corollary 2.12 (Relative error bound for approximate *feasible* solutions of monotone linear complementarity problems). *Let M be positive semidefinite, let $q \not\geq 0$, and let $S \neq \emptyset$. For each x in S there exists an $\bar{x}(x)$ in $\bar{S}$ such that*

$$\frac{\|x-\bar{x}(x)\|_\infty}{\|\bar{x}(x)\|_\infty} \leq \tau_2(M, q)\|M\|_\infty [x(Mx+q) + (x(Mx+q))^{1/2}] / \|(-q)_+\|_\infty. \qquad (2.10)$$

Remark 2.13. When the solution set $\bar{S}$ of (1.1) is bounded, σ_∞ of Theorems 2.7 and 2.11 can be bounded above by a single program as follows. $\bar{S}$ is bounded if and only if there exists an $\hat{x} > 0$ such that $\hat{w} := M\hat{x} + q > 0$ [5]. Hence [5] for *any* $\bar{x} \in \bar{S}$ and $\bar{w} := M\bar{x} + q$ it follows that

$$\hat{x}\hat{w} - \bar{x}\hat{w} - \hat{x}\bar{w} = (\hat{x}-\bar{x})(\hat{w}-\bar{w}) = (\hat{x}-\bar{x})M(\hat{x}-\bar{x}) \geq 0.$$

Consequently,

$$\|(\bar{x}, \bar{w})\|_1 \cdot \min_{1\leq i\leq n} \{\hat{w}_i, \hat{x}_i\} \leq \bar{x}\hat{w} + \hat{x}\bar{w} \leq \hat{x}\hat{w}$$

and from the definition (2.7) of σ_β we have that

$$\sigma_\infty = \min_{\bar{x}\in\bar{S}} \|\bar{x}, \bar{w}\|_1 \leq \hat{x}\hat{w} \Big/ \min_{1\leq i\leq n} \{\hat{w}_i, \hat{x}_i\}. \qquad (2.11)$$

Hence given a monotone linear complementarity problem (1.1), one can first solve the linear program

$$\max_{(x,\varepsilon) R^{n+1}} \{\varepsilon \mid Mx+q \geq e\varepsilon, \; x \geq e\varepsilon, \; 1 \geq \varepsilon \geq 0\}. \qquad (2.12)$$

The maximum $\hat{\varepsilon}$, achieved at $(\hat{x}, \hat{\varepsilon})$, is positive if and only if $\bar{S}$ is nonempty and bounded, in which case σ_∞ is bounded above by $\hat{x}\hat{w}/\hat{\varepsilon}$ where $\hat{w} = M\hat{x} + q$.

We conclude by giving an interesting application of Corollary 2.8 to the dual quadratic programs

$$\min_{z} \tfrac{1}{2}zQz + cz \quad \text{subject to} \quad Az \geq b, \ z \geq 0 \tag{2.13}$$

$$\max_{z,u} -\tfrac{1}{2}zQz + bu \quad \text{subject to} \quad -Qz + A^{\mathrm{T}}u \leq c, \ u \geq 0, \tag{2.14}$$

where $Q \in R^{n \times n}$ is symmetric positive semidefinite, $A \in R^{m \times n}$, $c \in R^n$ and $b \in R^m$. If the point $(z, u) \in R^{n+m}$ is feasible for both (2.13) and (2.14), then there exists a solution $(\bar{z}(z, u), \bar{u}(z, u))$ to the dual pair (2.13)-(2.14) such that

$$\begin{aligned} &\|(z, u) - (\bar{z}(z, u), \bar{u}(z, u))\|_\infty \\ &\quad \leq \tau_2(M, q)[(zQz + cz - bu) + (zQz + cz - bu)^{1/2}] \end{aligned} \tag{2.15}$$

where $\tau_2(M, q)$ is defined by (2.5) and

$$M := \begin{bmatrix} Q & -A^T \\ A & 0 \end{bmatrix}, \qquad q := \begin{bmatrix} c \\ -b \end{bmatrix}, \qquad x := \begin{bmatrix} z \\ u \end{bmatrix} \in R^{n+m}.$$

Note that the linear programming case is included as the special case of $Q = 0$. For this case, a stronger result not involving the square root term has been given in Theorem 4 of [4].

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