

MAXIMUM DEGREE AND FRACTIONAL MATCHINGS IN UNIFORM HYPERGRAPHS

Zoltán FÜREDI

Mathematical Institute of the
Hungarian Academy of Sciences
Budapest, Hungary H—1053

Received 11 January 1980

Let \mathcal{H} be a family of r -subsets of a finite set X . Set $D(\mathcal{H}) = \max_{x \in X} |\{E: x \in E \in \mathcal{H}\}|$, (maximum degree). We say that \mathcal{H} is intersecting if for any $H, H' \in \mathcal{H}$ we have $H \cap H' \neq \emptyset$. In this case, obviously, $D(\mathcal{H}) \cong |\mathcal{H}|/r$. According to a well-known conjecture $D(\mathcal{H}) \cong |\mathcal{H}|/(r-1+1/r)$. We prove a slightly stronger result. Let \mathcal{H} be an r -uniform, intersecting hypergraph. Then either it is a projective plane of order $r-1$, consequently $D(\mathcal{H}) = |\mathcal{H}|/(r-1+1/r)$, or $D(\mathcal{H}) \cong |\mathcal{H}|/(r-1)$. This is a corollary to a more general theorem on not necessarily intersecting hypergraphs.

1. Introduction, definitions

1.1 Some well-known definitions

We list the basic definitions and notation to be used throughout:

hypergraph \mathcal{H} — a finite collection of non-empty finite sets (edges);
vertex set of \mathcal{H} — $V(\mathcal{H}) = \cup \{E: E \in \mathcal{H}\}$;
rank of \mathcal{H} — $r(\mathcal{H}) = \max \{|E|: E \in \mathcal{H}\}$;
 \mathcal{H} is *r -uniform* if the cardinality of every $E \in \mathcal{H}$ is r ;
degree of a vertex x (in \mathcal{H}) — $d_x(x) = |\{E: x \in E \in \mathcal{H}\}|$;
 $D(\mathcal{H}) = \max \{d_x(x): x \in V(\mathcal{H})\}$;
 \mathcal{H} is *D -regular* if the degree of every vertex x is D ;
partial hypergraph — $\mathcal{H}' \subset \mathcal{H}$;
matching — partial hypergraph of \mathcal{H} whose edges are pairwise disjoint;
 $\nu(\mathcal{H})$ — *matching number* — maximum number of edges in a matching;
intersecting hypergraph — $\nu(\mathcal{H}) = 1$;
transversal (or *cover*) — a set $T \subset V(\mathcal{H})$ which meets all the edges;
 $\tau(\mathcal{H})$ — *transversal number* — minimum cardinality of a transversal.

1.2 Fractional transversals and matchings

A survey with applications on fractional hypergraph theory can be found e.g. in Berge [1] or Lovász [12], [16]. Here we shall define only the most important concepts of this theory which appeared in first papers published on this topic. (Berge and Simonovits [2], Lovász [11].)

A *fractional transversal* of a hypergraph \mathcal{H} is a weight function $t: V(\mathcal{H}) \rightarrow \mathbf{R}$ satisfying

$$t(x) \geq 0 \text{ for every } x \in V(\mathcal{H}),$$

and

$$\sum_{x \in E} t(x) \geq 1 \text{ for every edge } E \in \mathcal{H}.$$

The *value* of a fractional transversal t is

$$|t| = \sum_{x \in V(\mathcal{H})} t(x).$$

The minimum of $|t|$ when t ranges over all fractional transversals is called the *fractional transversal number* and is denoted by

$$\tau^*(\mathcal{H}) = \min \{|t| : t \text{ is a fractional transversal of } \mathcal{H}\}.$$

Similarly the *fractional matching number* is the maximum value of the fractional matchings of \mathcal{H} , i.e.

$$v^*(\mathcal{H}) = \max \left\{ \sum_{E \in \mathcal{M}} w(E) \mid w: \mathcal{H} \rightarrow \mathbf{R}, w(E) \geq 0, \forall x \in V(\mathcal{H}) \text{ we have } \sum_{E \ni x} w(E) \leq 1 \right\}.$$

Clearly, to determine the fractional transversal number and the fractional matching number is a problem of linear programming. This is a dual pair so by the duality principle of linear programming we have $\tau^*(\mathcal{H}) = v^*(\mathcal{H})$ for every hypergraph \mathcal{H} . Thus

$$1 \leq v \leq v^* = \tau^* \leq \tau \leq rv.$$

In view of the fact that $w(E) \equiv 1/D$ and $t(x) \equiv 1/\min |E|$ are a fractional matching resp. fractional transversal we have

$$(1) \quad \frac{|\mathcal{H}|}{D(\mathcal{H})} \leq \tau^*(\mathcal{H}) \leq \frac{|V(\mathcal{H})|}{\min \{|E| : E \in \mathcal{H}\}}.$$

1.3 An important example

If \mathcal{H} is D -regular and r -uniform then (1) yields

$$(2) \quad |\mathcal{H}|/D = |V(\mathcal{H})|/r = \tau^*(\mathcal{H}).$$

For $r \geq 3$ write \mathcal{P}_r for the hypergraph consisting of the lines of the r -uniform finite projective plane (if there exists) further let \mathcal{P}_2 consist of the 2-tuples of a 3-element set (i.e. \mathcal{P}_2 is a triangle) and let \mathcal{P}_1 be the hypergraph having only 1 point. It is well-known that \mathcal{P}_r exists provided $r = P + 1$, where P is a prime power.

It is evident that every line of the projective plane \mathcal{P}_r is a minimal transversal of \mathcal{P}_r . For $r = 1, 2, 3$ there is no other minimal transversal. For the projective

plane \mathcal{P}_r , with $r > 3$, J. Pelikán [18] proved that the only transversals of cardinality r are the edges, and

$$(3) \quad \text{all other transversal sets have size } \cong r+2.$$

Summing up: $|\mathcal{P}_r| = |V(\mathcal{P}_r)| = r^2 - r + 1$, \mathcal{P}_r is r -uniform and r -regular, $v(\mathcal{P}_r) = 1$, $\tau(\mathcal{P}_r) = r$, $\tau^*(\mathcal{P}_r) = r - 1 + 1/r$.

2. Results

Considering all the r -tuples of an underlying set with $rv + r - 1$ elements it can be seen that the inequality $\tau \cong rv$ cannot be improved in general. Nevertheless, as L. Lovász observed, the inequality $\tau^* \cong rv$ is not sharp. He showed (see [14], [15]), that, for any hypergraph \mathcal{H} , $\tau^*(\mathcal{H}) < r(\mathcal{H})v(\mathcal{H})$, furthermore

$$\tau^*(r, v) = \sup \{ \tau^*(\mathcal{H}) : r(\mathcal{H}) \cong r, v(\mathcal{H}) \cong v \} < rv.$$

For $v = 1$ he proved that $\tau^*(r, 1) \cong r - 1 + 2/(r + 1)$ and he conjectured that $\tau^*(r, 1) \cong r - 1 + 1/r$. In this paper we shall prove a bit more.

Theorem. *Let \mathcal{H} be a hypergraph of rank $r \cong 3$, $v(\mathcal{H}) = v$. Suppose further that \mathcal{H} does not contain a partial hypergraph which consists of $p + 1$ copies of pairwise disjoint r -uniform projective planes. Then*

$$\tau^*(\mathcal{H}) \cong (r - 1)v + p/r.$$

(The proof of the Theorem is in § 5.) We mention that the inequality of the Theorem is sharp. To see this consider the hypergraph \mathcal{P}'_r which we get from \mathcal{P}_r by omitting a line. ($\tau^*(\mathcal{P}'_r) = r - 1$.)

The case $r = 1$ is of no importance. For $r = 2$ the Theorem does not hold true, because for the odd circuits C_{2n+1} one has $v(C_{2n+1}) = n$, $p = 0$ but $\tau^*(C_{2n+1}) = n + 1/2$. L. Lovász [13] proved that for an ordinary graph G

$$\tau^*(G) \cong \frac{1}{2}(\tau + v) \cong \frac{3}{2}v.$$

The following corollaries are true even if $r < 3$.

Corollary 1. *If \mathcal{H} is the union of v pairwise disjoint copies of \mathcal{P}_r , then $\tau^*(\mathcal{H}) = (r - 1 + 1/r)v$ otherwise $\tau^*(\mathcal{H}) \cong (r - 1 + 1/r)v - 1/r$.*

Proof. The inequality $\tau^*(\mathcal{H}) > (r - 1 + 1/r)v - 1/r$ implies that \mathcal{H} has a partial hypergraph \mathcal{H}_v which is the disjoint union of v copies of \mathcal{P}_r . That is $\mathcal{H}_v \subset \mathcal{H}$. Then it follows from (3) that $\mathcal{H}_v \cong \mathcal{H}$. ■
(The case $r = 2$ is left to the reader).

Corollary 2. *Let r be a positive integer for which \mathcal{P}_r does exist. Then $\tau^*(r, v) = (r - 1 + 1/r)v$. If \mathcal{P}_r does not exist then $\tau^*(r, v) \cong (r - 1)v$. ■*

I think that for the time being the determination of the exact value of $\tau^*(r, v)$ for other r 's is hopelessly difficult, because to solve this problem one probably has to decide whether or not the projective plane \mathcal{P}_r does exist for a given r .

3. Applications

3.1 The maximum degree of hypergraphs

Let us consider an intersecting, r -uniform hypergraph \mathcal{H} . Obviously $D(\mathcal{H}) \cong |\mathcal{H}|/r$. It is a well-known conjecture [6, 7] that $D \cong |\mathcal{H}|/(r-1+1/r)$. From the Theorem and from (1) a slightly stronger result follows.

Corollary 3. *Let \mathcal{H} be an r -uniform, intersecting hypergraph. Then either it is a projective plane of order $r-1$ and consequently $D(\mathcal{H}) = |\mathcal{H}|/(r-1+1/r)$ or $D(\mathcal{H}) \cong |\mathcal{H}|/(r-1)$. ■*

In general $D(\mathcal{H}) \cong |\mathcal{H}|/(r-1+1/r)v$. This was proved for $r=2$ by B. Bollobás [4] in a slightly different form. This Corollary is sharp because $D(\mathcal{P}'_r) = |\mathcal{P}'_r|/(r-1)$.

3.2 The number of vertices of regular hypergraphs

Using his result $(\tau^*(r, 1) \cong r-1+2(r+1))$ mentioned above L. Lovász [14], [15] proved the following conjecture of P. Erdős [6] and B. Bollobás [4]:

If \mathcal{H} is an intersecting, r -uniform and regular hypergraph, then $|V(\mathcal{H})| \cong r^2-r+1$.

By the Theorem and (2) we generalize this result as follows. (For $r=2$ see Bollobás—Eldridge [5]).

Corollary 4. *If \mathcal{H} is r -uniform and regular then $|V(\mathcal{H})| \cong (r^2-r+1)v$. Moreover equality holds if and only if \mathcal{H} is the disjoint union of projective planes of order $r-1$. Furthermore if there is no such r -uniform plane then $|V(\mathcal{H})| \cong (r^2-r)v$. ■*

By omitting from \mathcal{P}_r a point together with all the lines containing it we get the hypergraph \mathcal{P}'_r which is $(r-1)$ -regular, r -uniform and intersecting. It has r^2-r points. This example shows that Corollary 4 is sharp, too.

3.3 Fractional transversal number of r -partite hypergraphs

The hypergraph \mathcal{H} is said to be r -partite if $V(\mathcal{H})$ is the disjoint union of X_1, \dots, X_r , and for each $E \in \mathcal{H}$: $|E \cap X_i| = 1$ holds ($i=1, 2, \dots, r$).

A well-known conjecture of H. J. Ryser states that for an r -partite hypergraph $\tau \cong (r-1)v$. (In particular for $r=2$ this is simply König's Theorem (see [12]). For some small values of r and v this conjecture has recently been proved by Zs. Tuza [19]). A. Gyárfás [10] proved an easier version of Ryser's conjecture. His result follows from our Theorem because \mathcal{P}'_r is not r -partite ($r \cong 2$).

Corollary 5. *If the hypergraph \mathcal{H} is r -partite then $\tau^*(\mathcal{H}) \cong (r-1)v(\mathcal{H})$. ■*

This Corollary is sharp for \mathcal{P}'_r is r -partite and $\tau^*(\mathcal{P}'_r) = r-1$.

3.4 Some further applications of this Theorem to extremal graphs and to extremal set-systems can be found in J. Pach—L. Surányi [17], Z. Füredi [9] and P. Frankl—Z. Füredi [8], respectively.

4. The reduction lemma

We denote by $(\mathbf{a}_i, b_i, \mathbf{c}, I)$ the following linear program with minimum value M .

$$(4) \quad \begin{cases} \mathbf{x} \geq \mathbf{0} \\ \mathbf{a}_i \mathbf{x} \leq b_i \text{ for all } i \in I. \\ \min \mathbf{c} \mathbf{x} = M(\mathbf{a}_i, b_i, \mathbf{c}, I) \end{cases}$$

Here \mathbf{a}_i, \mathbf{x} and \mathbf{c} are n -dimensional vectors, b_i 's are real numbers and $|I|$ is the number of conditions of the program ($|I| < \infty$).

The following Proposition is well known in the theory of linear programming. For the sake of completeness we give its short proof in the Appendix.

Proposition. *If the linear program $(\mathbf{a}_i, b_i, \mathbf{c}, I)$ with n variables has a finite optimum, then there exists a $J \subset I$ such that $M(\mathbf{a}_i, b_i, \mathbf{c}, I) = M(\mathbf{a}_i, b_i, \mathbf{c}, J)$ and $|J| \leq n$.*

In other words this Proposition states that the number of conditions of a linear program can be reduced to n without changing the optimum value.

The proof of the Theorem is based on the following Lemma which may help to determine τ^* in some other cases as well.

Lemma. *For any hypergraph \mathcal{H} there exists a partial hypergraph $\mathcal{H}' \subset \mathcal{H}$ such that $\tau^*(\mathcal{H}') = \tau^*(\mathcal{H})$ and $|\mathcal{H}'| \leq |V(\mathcal{H}')|$.*

Proof. To determine τ^* one has to solve a linear program of dimension $|V(\mathcal{H})|$, with index set I , where $|I| = |\mathcal{H}|$. Of course, this program always has a finite optimum. So by applying the Proposition (possibly several times) one can find a suitable $\mathcal{H}' \subset \mathcal{H}$. ■

5. Proof of the Theorem

Let \mathcal{H} be an r -uniform hypergraph which does not contain $p+1$ disjoint copies of the projective plane \mathcal{P}_r and $v(\mathcal{H}) = v$. Suppose $r \geq 3$. (Our proof can be applied for $r=2$, too, but the details are left to the reader.)

It is sufficient to give a suitable fractional transversal t of \mathcal{H} . We shall give it by induction on v , while r is fixed. The proof in the case $v=1$ is similar to that one for $v>1$ and that is why we do not separate them, but sometimes we mention the differences.

For $\mathcal{H} = \emptyset$ put $\tau^*(\mathcal{H}) = 0$. By the Lemma we may suppose that $|\mathcal{H}| \leq |V(\mathcal{H})|$. Consequently,

$$(5) \quad \min_{x \in V(\mathcal{H})} d(x) \leq \frac{\sum \{d(x) : x \in V(\mathcal{H})\}}{|V(\mathcal{H})|} = \frac{r|\mathcal{H}|}{|V(\mathcal{H})|} \leq r.$$

Case 1. There exists $x_0 \in V(\mathcal{H})$ with $d_{\mathcal{H}}(x_0) = k < r$. Put $\mathcal{H}_0 = \{E \in \mathcal{H} : x_0 \in E\} = \{E_1, \dots, E_k\}$, and $\mathcal{H}_i = \{E \in \mathcal{H} : E \cap E_i = \emptyset\}$ for $1 \leq i \leq k$.

For the hypergraph \mathcal{H}_i the induction hypothesis can be applied, because $v(\mathcal{H}_i) \leq v-1$ and of course \mathcal{H}_i does not contain more than p disjoint \mathcal{P}_r as partial

hypergraphs. Hence there exists a fractional transversal $t_i: V(\mathcal{H}_i) \rightarrow \mathbf{R}$ of \mathcal{H}_i , such that $|t_i| \cong (r-1)(v-1) + p/r$. (If here $v=1$ then $\mathcal{H}_i = \emptyset$, $t_i \equiv 0$.)

Put

$$t(x) = \begin{cases} 0 & \text{if } x = x_0 \\ \frac{1}{k} \left(d_0(x) + \sum_{i=1}^k t_i(x) \right) & \text{if } x \in V(\mathcal{H}) - \{x_0\}, \end{cases}$$

where $d_0(x)$ is the degree of x in the hypergraph \mathcal{H}_0 . We claim that this is a fractional transversal of \mathcal{H} . Indeed, $t(x) \geq 0$, and for any $E \in \mathcal{H}_0$ we have

$$\sum_{x \in E} t(x) \geq \frac{1}{k} \left(\sum_{x \in E - \{x_0\}} d_0(x) \right) \geq \frac{1}{k} (r-1) \geq 1.$$

If $E \in \mathcal{H} - \mathcal{H}_0$ then

$$\begin{aligned} \sum_{x \in E} t(x) &= \frac{1}{k} \left(\sum_{x \in E} d_0(x) + \sum_{i=1}^k \left[\sum_{x \in E} t_i(x) \right] \right) \\ &\geq \frac{1}{k} \left(\sum_{\substack{i=1 \\ E \cap E_i \neq \emptyset}}^k 1 + \sum_{\substack{i=1 \\ E \cap E_i = \emptyset}}^k 1 \right) = 1. \end{aligned}$$

Finally

$$\begin{aligned} \tau^*(\mathcal{H}) &\leq |t| = \frac{1}{k} \left(\sum_{x \in V(\mathcal{H}) - \{x_0\}} d_0(x) + \sum_{i=1}^k \left[\sum_{x \in V(\mathcal{H})} t_i(x) \right] \right) \\ &= \frac{1}{k} \left((r-1)k + \sum_{i=1}^k |t_i| \right) \leq (r-1)v + p/r. \end{aligned}$$

(For $v=1$ we get $\tau^*(\mathcal{H}) \leq r-1$.)

Case 2. $\min_{x \in V(\mathcal{H})} d_{\mathcal{H}}(x) \geq r$.

Then, by (5), \mathcal{H} is r -regular, so $|\mathcal{H}| = |V(\mathcal{H})|$. We shall show that $|\mathcal{H}| \geq (r^2-r)v + p$ from here, by (2), the Theorem follows.

Suppose on the contrary that $|\mathcal{H}| \leq (r^2-r)v + p + 1$. Let E_1 be an arbitrary edge of \mathcal{H} and put $\mathcal{H}_1 = \{E \in \mathcal{H} : E \cap E_1 = \emptyset\}$. Applying the induction hypothesis to \mathcal{H}_1 and using (1) we get that

$$|\mathcal{H}| = |\{E : E \cap E_1 \neq \emptyset\}| + |\mathcal{H}_1| \leq 1 + r(r-1) + \tau^*(\mathcal{H}_1)D(\mathcal{H}_1) \leq (r^2-r)v + p + 1.$$

Here the right side is at most $(r^2-r)v + p$ if there is an edge E with $|E \cap E_1| \geq 2$. Consequently it is enough to consider the following case.

(6) $|\mathcal{H}| = |V(\mathcal{H})| = v(r^2-r) + p + 1$

$$|\mathcal{H} - \mathcal{H}_1| = r^2 - r + 1$$

(7) $|E \cap E_1| = 0$ or 1 , for any edges E, E_1 .

(If $v=1$ and $p=1$ then (6) yields a contradiction, because in this case $\mathcal{H}_1 = \emptyset$. Similarly, for $v=1, p=0$ we have by (6) and (7) that \mathcal{H} is an r -uniform, r -regular, intersecting system of sets on (r^2-r+1) points with the same number of edges.

This in turn implies that $\mathcal{H} \equiv \mathcal{P}_r$, contradicting to $p=0$. From now on we suppose that $v \geq 2$.)

Let E_1, E_2, \dots, E_v any fixed matching. We call an edge $E \in \mathcal{H}$ *crossing* if it intersects more than one E_i ($1 \leq i \leq v$). We count the number of edges of $\mathcal{H} - \{E_1 \dots E_v\}$ with multiplicities in the points of $\bigcup_{i=1}^v E_i$. We get $vr(r-1)$. From the other hand $|\mathcal{H} - \{E_1, \dots, E_v\}| = v(r^2-r) + p + 1 - v$, and so there are at most $v-1-p$ crossing edges. This means that there is an edge, say E_1 , in the matching $\{E_1 \dots E_v\}$ which intersects at most one crossing edge.

Case 2a. E_1 is not intersected by a crossing edge.

In other words, the system $\{E_2, \dots, E_v\}$ has no common point with edges intersecting E_1 . Since $v(\mathcal{H}) = v$ we get that $\mathcal{H} - \mathcal{H}_1$ is an intersecting family. Further $D(\mathcal{H} - \mathcal{H}_1) = r$, and this together with (6) and (7) implies that $\mathcal{H} - \mathcal{H}_1 \equiv \mathcal{P}_r$. Moreover, the underlying sets $V(\mathcal{H}_1)$ and $V(\mathcal{H} - \mathcal{H}_1)$ are disjoint. Applying the induction hypothesis to \mathcal{H}_1 with parameters $v-1$ and $p-1$ we get by (1)

$$|\mathcal{H} - \mathcal{H}_1| \leq \tau^*(\mathcal{H} - \mathcal{H}_1)r \leq (r^2-r)(v-1) + (p-1),$$

contradicting to (6).

Case 2b. There is a unique crossing edge E' intersecting E_1 .

It remains true that $\mathcal{H} - \mathcal{H}_1 - \{E'\}$ is an intersecting family, $|\mathcal{H} - \mathcal{H}_1 - \{E'\}| = r^2 - r$, $D(\mathcal{H} - \mathcal{H}_1 - \{E'\}) = r$. We claim that in this case $\mathcal{H} - \mathcal{H}_1 - \{E'\} \equiv \mathcal{P}'_r$. Indeed, by (7), every edge of $\mathcal{H} - \mathcal{H}_1 - \{E'\}$ contains a point of degree $r-1$. There are exactly r points of this type, they form a set T . It is easy to check that $(\mathcal{H} - \mathcal{H}_1 - \{E'\}) \cup \{T\}$ is a finite projective plane. $E' \neq T$ for E' is crossing. So there is an edge $E'' \in \mathcal{H} - \mathcal{H}_1$ such that $E' \cap E'' = \emptyset$, and

(8) *at least $r-1$ edges E of $\mathcal{H} - \mathcal{H}_1$ have the property that $E \cap E' \neq \emptyset$ and $E \cap E'' \neq \emptyset$.*

Let $\mathcal{H}^1 = \{E \in \mathcal{H} : E \cap E' = \emptyset \text{ and } E'' \cap E = \emptyset\}$ and $\mathcal{H}^2 = \mathcal{H} - \mathcal{H}^1$. Applying the induction hypothesis to \mathcal{H}^1 with parameters $v-2$ and p , and using that upper bound for $|\mathcal{H}^2|$ which follows from (8), we get

$$|\mathcal{H}| = |\mathcal{H}^1| + |\mathcal{H}^2| \leq (v-2)(r^2-r) + p + 2(r^2-r) + 2 - (r-1).$$

This again contradicts to (6), provided $r \geq 3$. ■

6. Appendix: Proof of the Proposition

Dropping some of the inequalities of (4) the minimal value of the program can only decrease. Hence we have to prove that there is a $J \subset I, |J|=n$ such that $M(\mathbf{a}_i, b_i, \mathbf{c}, J) \equiv M(\mathbf{a}_i, b_i, \mathbf{c}, I) = M$.

Suppose, on the contrary, that for every $J \subset I, |J|=n$, we have $M(\mathbf{a}_i, b_i, \mathbf{c}, J) < M$. This means that any n of the halfspaces $\{\mathbf{y} : \mathbf{a}_i \mathbf{y} \geq b_i\}$ have a point in common with the open convex polytope $\{\mathbf{x} \in \mathbf{R}^n : \mathbf{c} \mathbf{x} < M, \mathbf{x} \geq 0\}$. The system (4) has a solution, hence any $n+1$ of the halfspaces $\{\mathbf{y} : \mathbf{a}_i \mathbf{y} \geq b_i\}$ have a point

in common. Now Helly's Theorem in \mathbf{R}^n implies that the intersection of the $|I|+1$ convex sets $\{\mathbf{y}: \mathbf{a}_i \mathbf{y} \cong b_i\}$ and $\{\mathbf{x}: \mathbf{c} \mathbf{x} < M, \mathbf{x} \cong 0\}$ is not empty, i.e. it contains a point \mathbf{x}_0 . This point \mathbf{x}_0 is feasible for the program (4) and $\mathbf{c} \mathbf{x}_0 < M = M(\mathbf{a}_i, b_i, \mathbf{c}, I)$. This contradiction proves the existence of the appropriate J . ■

Acknowledgment. I would like to express my thanks to P. Frankl and I. Bárány for their help.

References

- [1] C. BERGE, Packing problems and hypergraph theory: a survey, *Annals of Discrete Math.* **4** (1979), 3—37.
- [2] C. BERGE and M. SIMONOVITS, The coloring number of the direct product of two hypergraphs, in [3], 21—33.
- [3] C. BERGE and D. K. RAY-CHAUDHURI eds., *Hypergraph Seminar 1972, Lecture Notes* **411** (Springer-Verlag, Berlin, 1972).
- [4] B. BOLLOBÁS, Disjoint triples in a 3-graph with given maximal degree, *Quart. J. Math. Oxford, Sec. Ser.* **28** (1977), 81—87.
- [5] B. BOLLOBÁS and S. E. ELDRIDGE, Maximal matchings in graphs with given minimal and maximal degrees, *Math. Proc. Cambridge Phil. Soc.* **79** (1976), 221—234.
- [6] P. ERDŐS, *private communication*.
- [7] P. FRANKL, On intersecting families of finite sets, *J. Combinatorial Th. (A)* **24** (1978), 146—161.
- [8] P. FRANKL and Z. FÜREDI, Disjoint r -tuples in an r -graph with given maximal degree, submitted to *Quart. J. Math. Oxford*.
- [9] Z. FÜREDI, Erdős—Ko—Rado type theorems with upper bounds on the maximal degree, in: *Algebraic Methods in Graph Theory*, (Proc. Conf. Szeged 1978, L. Lovász et al. eds.) *Coll. Math. Soc. J. Bolyai* **25**, North-Holland 1981, to appear.
- [10] A. GYÁRFÁS, *to appear*.
- [11] L. LOVÁSZ, Normal hypergraphs and the perfect graph conjecture, *Discrete Math.* **2** (1972), 253—267.
- [12] L. LOVÁSZ, *Combinatorial problems and exercises*, Akadémiai Kiadó, Budapest—North-Holland, Amsterdam 1979.
- [13] L. LOVÁSZ, Minimax theorems for hypergraphs, in [3], 111—126.
- [14] L. LOVÁSZ, Doctoral thesis, Szeged 1977.
- [15] L. LOVÁSZ, On minimax theorems of combinatorics (in Hungarian) *Matematikai Lapok* **26** (1975), 209—264.
- [16] L. LOVÁSZ, Graph theory and integer programming, *Annals of Discrete Math.*, **2** (1978)
- [17] J. PACH and L. SURÁNYI, Graphs of diameter 2 and linear programming, in *Algebraic Methods in Graph Theory*, (Proc. Conf. Szeged 1978, L. Lovász et al. eds.) *Coll. Math. Soc. J. Bolyai* **25**, North-Holland 1981, to appear.
- [18] J. PELIKÁN, Properties of balanced incomplete block designs, in: *Combinatorial Theory and its Applications* (P. Erdős, A. Rényi and V. T. Sós, eds.) *Coll. Math. Soc. J. Bolyai* **4**, North-Holland 1971), 869—890.
- [19] Zs. TUZA, Some special cases of Ryser's conjecture, to appear.