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HOW FAST CAN A CRACK GO ?

K. B. Broberg

The assumption of a material specific relation between the energy dissipation at the edge of a crack propagating under small scale yielding conditions leads to upper limits of the crack velocity: the Rayleigh wave velocity for modes I and II, and the S wave velocity for mode III. If a mode II crack can pass the "forbidden" subsonic super-Rayleigh region, then the upper limit would be the P wave velocity. Experiments invariably show lower maximum speeds, typically less than 60% of the S wave velocity, but they also frequently show accelerations to a constant velocity, depending on the acceleration history, and increasing surface roughness, even during the constant velocity phase. Other experiments show that the energy dissipation at the crack edge usually is several times larger at very high than at very low velocities. These results indicate a considerable growth of the process region, so much that the intrinsic length parameter which determines its height at very low crack speeds becomes insignificant, and then the rationale for a material specific relation between energy dissipation and velocity disappears. This disappearance can contribute to the explanation of the experimental results.

Introduction

Determination of crack speeds might be of greater importance than satisfaction of one's curiosity. Would a crack in a pipeline run so fast that the gas does not flow out quickly enough to relieve the load needed to propagate the crack? The higher the velocity, the higher the kinetic energy in the structure, implying greater difficulty to arrest the running crack. In an earthquake, more damage is caused the larger the kinetic energy is.

It has been known for a long time (e.g., Schardin [1, 2]) that cracks tend to accelerate to a constant velocity. The highest velocities recorded in homogeneous bodies are of the order of 60 % of the S wave velocity.

The first solutions for rapidly propagating mode I cracks indicated an upper crack speed limit at the Rayleigh wave velocity. This is most clearly seen by calculating the energy flux into the crack edge region at small scale yielding (Broberg [3, 4]). The result for mode I is shown in Fig. 1.

For mode II, the picture is more complicated, showing a positive energy flux both for sub-Rayleigh and intersonic crack velocities, with a negative flux in the subsonic super-Rayleigh region, see Fig. 2.

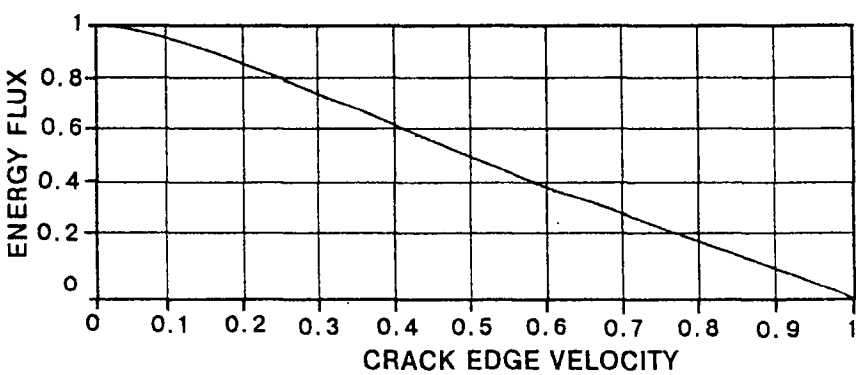

Fig. 1. Energy flux into the dissipative region of an expanding mode I crack at small scale yielding as a function of the crack velocity. The energy flux is normalized with respect to the energy flux at slow crack expansion, and the velocity is normalized with respect to the Rayleigh wave velocity. Poisson's ratio equals 1/4 if plane strain prevails, or 1/3, in the plane stress approximation for a thin plate.

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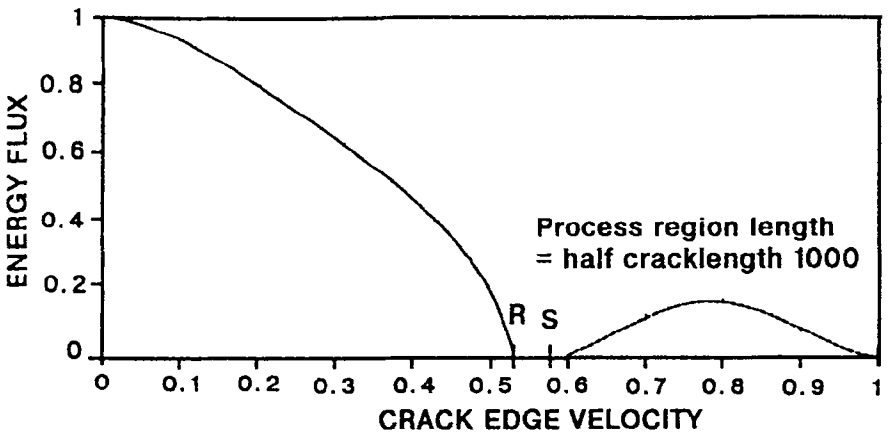

Fig. 2. Energy flux into the dissipative region of an expanding mode II crack as a function of the crack velocity. The energy flux is normalized with respect to the energy flux at slow crack expansion, and the velocity is normalized with respect to the P wave velocity. Poisson's ratio equals 1/4 if plane strain prevails, or 1/3, in the plane stress approximation for a thin plate. Labels R and S show the positions of the Rayleigh and S wave velocities. Formally, the flux is negative between R and S.

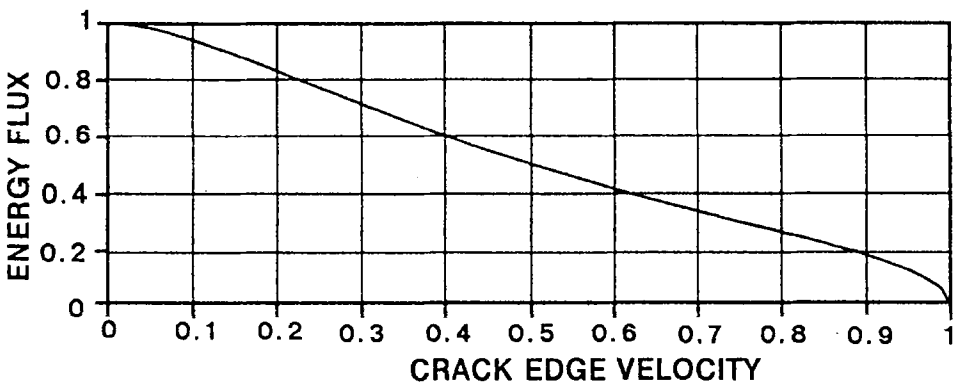

Fig. 3. Energy flux into the dissipative region of an expanding mode III crack at small scale yielding as a function of the crack velocity. The energy flux is normalized with respect to the energy flux at slow crack expansion, and the velocity is normalized with respect to the S wave velocity.

For sub-Rayleigh velocities, the energy flux is fairly independent of the scale of yielding, provided it is small. In the intersonic region, its dependence on the scale of yielding cannot be neglected (Broberg [5])—it vanishes at a vanishing scale of yielding, except at the curious velocity $\sqrt{2}c_S$, where c_S is the S wave velocity. The dashed curve in Fig. 2 is drawn under the assumption of the Barenblatt–Leonov–Panasyuk–Dugdale model of a dissipative region (Barenblatt [6], Leonov and Panasyuk [7], Panasyuk [8], Dugdale [9]), with length equal to 0.001 of the half crack length.

For mode III, the picture is again simple, with positive energy flow in the whole velocity region (the maximum sound velocity for mode III is, of course, the S wave speed), see Fig. 3.

The acceleration of an expanding crack can be estimated by establishing an equation for the energy balance: the energy flux into the dissipative region, G, should be equal to the energy dissipation Γ. Approximately, the energy flux can be taken as the energy flux calculated as if the current velocity had been prevailing from the onset of crack growth, i.e., the result shown by Figs. 1–3 can be used. For Γ, the general form $\Gamma(\sigma_\infty, V, a)$, where σ_∞ is the remote load, V is the crack edge velocity, and a is the half length of the crack, was suggested by Broberg [4], but the assumption that only events at the crack edge and not the history are influential leads to the simpler form $\Gamma(V)$. For an expanding crack,

$$G = Caw(V), \tag{1}$$

where C is a constant and $w(V)$ is the energy flux normalized with respect to the energy flux at a very low crack velocity, i.e., the function shown in Figs. 1–3. Thus,

$$w(V) = \frac{\Gamma(V)}{Ca} \to 0 \quad \text{as} \quad a \to \infty. \tag{2}$$

Since $V = da/dt$, where t is time, this equation (which was later called "the crack tip equation of motion") is a differential equation for approximate determination of the crack motion. It leads to the result that an expanding crack would eventually reach the neighborhood of the Rayleigh wave velocity at modes I and II (or, if the region between the Rayleigh wave velocity and S wave velocity can be bypassed, the P wave velocity) and the neighborhood of the S wave velocity at mode III.

Modes II and III are extremely difficult to realize experimentally, although they are the common modes at earthquakes. Some observations from the Imperial Valley Earthquake, 1979 (Archuleta [10], Scholtz [11]) indicate mode II intersonic velocities. There are two possible mechanisms by which the "forbidden region" can be bypassed. One consists of slip initiation ahead of an expanding crack due to the fat shear stress peak traveling with the S wave velocity. The other is the convergence of crack edges traveling in somewhat different direction, as just after the passage of a crack edge around an obstacle.

The Cell Model of Materials

The discrepancy between theoretical estimates and experimental results has for a long time been a challenge for research workers. Could it be that local Rayleigh wave velocities near the crack edge were setting the limit at mode I ? Due to high tensile strains, these velocities could be substantially lower than the normal Rayleigh wave velocity. I assumed that so could be the case (Broberg [3]), but a closer study (Broberg [12]) revealed that it was not. This study was presented at a Symposium on Dynamic Crack Propagation at Bethlehem, Pennsylvania, USA, 1972. At the same symposium, Paxson and Lucas [13] showed results of great significance for the understanding of high-speed crack propagation, namely, that the energy dissipation at very high crack velocities could be as high as 50 times the dissipation at low velocities. They used PMMA, but numerous later results have shown similar tendencies, though not always quite so pronounced.

It seemed out of question that a 50 times increase of the energy dissipation could be caused by higher energy dissipation in a process region of about the same height as at slow crack growth. At slow crack growth, the height of the process region is given by the distance between kernels of microseparation. These kernels are often particles in the material, and the distance between them provides an intrinsic length parameter.

Obviously, Paxson's and Lucas' results could only be interpreted so that the process region height increased, perhaps up to a factor 50. What could be the possible mechanism for such a growth ? It appeared plausible that the difference between low and high velocity cases had to do with the time needed for communications between microseparations straight ahead of the crack and those at offside locations. Opening of a microseparation is accompanied by lowering of the forces in its neighborhood. This is evident from the cohesion–decohesion curve, which depicts loading of one material cell containing one kernel of microseparation. For cells belonging to the process region, the height increase continues until complete failure of the load carrying capacity, whereas the load carrying capacity of offside cells remains intact, see Fig. 4.

The process region, as characterized here, consists of cells which have reached the decohesive state. Energy dissipation generally also occurs in offside cells as "ordinary" plasticity. However, at small scale yielding, the development of the plastic region can be expected to follow the development of the process region: its current size ought to be essentially determined by the current size of the process region and the current velocity (cf. Broberg [14]).

This cell model has been described previously in greater detail by the author (see, e.g., [15]).

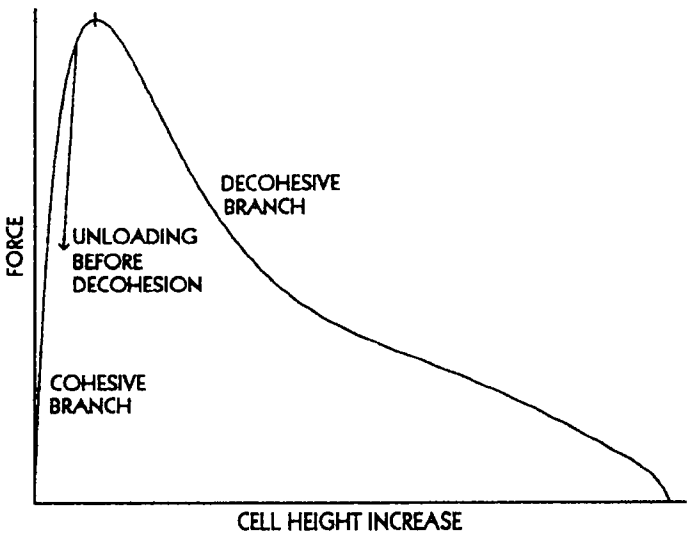

Fig. 4. Force on a material cell containing one kernel of microseparation. The ascending part represents stable cohesive response. The descending part represents decohesion, which would be unstable under load controlled conditions. Cells in the process region belong to the decohesive branch, and cells outside to the cohesive branch. Unloading of cells follows the decohesive branch for cells which have reached the decohesive state, and the line of unloading indicated in the figure for other cells.

Study a cell straight ahead of the crack. When the crack edge moves towards the cell, the load increases until the maximum load carrying capacity of the cell is reached. This generally happens when the crack edge still is several cell distances away. Then the cell load decreases whereas the cell height continues to increase, following the decohesive part of the cohesion–decohesion curve. For a slowly moving crack, this load decrease is transferred to the neighboring offside cells, which therefore never reach their maximum load carrying capacity, but start to unload under decreasing cell height, following the unloading line in Fig. 4. In this way, essentially only one row of cells enters the decohesive state, i.e., the process region height will be about the same as the cell height, which equals approximately the intrinsic length parameter of material, in many cases, the distance between particles.

Assume now that the crack edge is moving very fast. Load decrease at offside cells is then transferred from cells straight ahead of the crack edge by means of stress waves. Consequently, a delay time is involved, and the load decrease might arrive too late to prevent offside cells to enter the decohesive state. Then the process region height might be larger than the intrinsic length parameter. At very high crack speeds, it might be much larger. But then the significance of this parameter disappears. This implies that there is no material specific relation between Γ and V at very high crack velocities, since the dimension of Γ is force/length, so it cannot depend only on stresses (like the yield stress and the decohesive stress), elastic constants, hardening parameters, and the ratio V/c_S. A length parameter is obviously needed for a material-specific function $\Gamma(V)$.

This discussion was developed to greater detail by Broberg [14]. The picture appeared rather disturbing: it seemed that the absence of a unique relation between Γ and V would lead to anarchy: crack propagation at very high velocities would not be obliged to follow a strict law.

Experimental Evidence and the Cell Model

Later experimental work, particularly, by Kalthoff [16], Ravi-Chandar [17], Ravi-Chandar and Knauss [18], Takahashi and Arakawa [19], and by Arakawa and Takahashi [20] shed new light on the behavior of cracks at very high velocities. Thus, they showed that, under certain conditions, in the absence of or before crack branching,

(i) the crack accelerates to a constant velocity;

(ii) this constant velocity is different for different experimental conditions, although the material is the same;

(iii) the stress intensity factor and surface roughness increase during the constant velocity phase.

These results are obviously not consistent with the balance equation which relates energy flux from the stress-strain field to a required energy dissipation, which is a function of the crack velocity, only, for each given material. They are, however, consistent with the loss of an intrinsic material parameter and, thereby, also of a material specific relation between Γ and V. But instead of anarchy, certain law seems to compel the crack to propagate with a constant velocity.

There have been several attempts, perhaps for philosophical reasons, to defend the existence of a unique relation between energy dissipation and crack velocity. Some of these attempts focus on possible experimental errors related to velocity measurements, the influence of three-dimensional state of stress at the crack edge, effects of nonsingular stresses at methods unable to reach a sufficiently close vicinity of the crack edge (such as caustics), etc. (Freund [21], Dally et al. [22], Yang and Freund [23], Rosakis and Ravi-Chandar [24], Freund and Rosakis [25], Liu, Rosakis, and Freund [26]). However, while these experimental errors would necessitate some corrections of the experimental data, such corrections appear to be rather marginal and not at all sufficient for invalidating the conclusions listed here as items (i)–(iii). Neither are, as it seems, attempts to explain why the highest crack speeds recorded in homogeneous bodies are limited to about 70 % of the Rayleigh wave speed, by oscillations of the crack propagation direction (Gao [27], Slepyan [28]). Such attempts seem to be unable to explain why the stress intensity factor and surface roughness increase during constant velocity phases, and do so to a very substantial degree.

Further evidence in favor of a theory involving implications of an intrinsic parameter came from experiments by Washabaugh and Knauss [29]. They merged two PMMA plates together to form one large plate with a very thin connecting layer in which the material strength was considerably reduced. In this way, they effectively introduced a new length parameter, the layer thickness. Although this parameter is not intrinsic to the material but to the body geometry, it serves the purpose of limiting the height of the process region, since the decohesive force (the maximum force in the cohesion–decohesion curve) is considerably higher in the base material than in the layer. Thus, cells outside the weak layer never reach the decohesive state. As a result, very high velocities were obtained, about 90% of the Rayleigh wave velocity.

Numerical Evidence and the Cell Model

The cell model was used in numerical simulations by Johnson [30, 31, 32]. He assumed a behavior of cells such that a cohesion–decohesion curve with a character similar to the one shown in Fig. 4 was followed at uniaxial loading under grip control. However, the need to incorporate a more general type of loading leads to a more complicated description of the behavior of a single cell in situ.

The result showed that, in the absence of or before crack branching,

(i) the crack accelerates to a constant velocity;

(ii) this constant velocity is different for different acceleration histories, although the material parameters are the same;

(iii) the stress intensity factor and the process region height increase during the constant velocity phase.

These findings are obviously in direct agreement with experimental results. In addition, somewhat surprisingly, another rather common phenomenon was experienced in certain simulations, namely, attempted and successful branching.

Johnson [32] also simulated crack propagation allowing only one row of cells straight ahead of the crack. This corresponds to the experiment by Washabaugh and Knauss [29], since it confines the process region to a thin layer and, in this way, introduces a significant length parameter, the cell height. The result was in full accordance with what was expected and later confirmed by the experiments by Washabaugh and Knauss, although the numerical capacity did not allow the simulations to continue towards the neighborhood of the Rayleigh speed. However, the crack accelerated during the whole simulation—no constant terminal velocity was obtained.

In one set of simulations, Johnson [31] investigated the role of statistical distribution of cell properties. The results were essentially the same as for identical cell properties, although, as expected, with a less smooth development of the process region and a somewhat increased tendency towards branching.

CONCLUSION

The cell model of materials seems to be able to explain most of the experimental results about very fast crack propagation. However, even if one accepts that there is no unique material specific relation between energy dissipation at the crack edge velocity, a still unanswered question is why just a constant velocity is the preferred choice. Thus, the crack seems to respond to loading by increasing the energy dissipation at the crack edge and keeping the edge velocity constant rather than keeping the energy dissipation at the crack edge constant and increasing the velocity, or something in between. It is difficult to imagine a thermodynamic reason for such a behavior, but, obviously, there is a tendency towards the preference of energy dissipation rather than of kinetic energy.

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