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I_δ-INDEX, A MEASURE OF DISPERSION OF INDIVIDUALS*

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INTRODUCTION

The I_δ-index has been proposed by the present author as a measure of dispersion of individuals in a population (MORISITA, 1959).

The index value is computed as :

$$I_\delta = q\frac{\sum_{i=1}^{q} x_i(x_i-1)}{T(T-1)} \qquad (1)$$

where x_i is the number of individuals in the ith sample unit ($i=1, 2, 3, \cdots, q$), q is the number of sample units, and $T=\sum_{i=1}^{q} x_i$. The most important property of the index is that it is uninfluenced by T provided that each sample unit is randomly taken from each of q groups into which an infinite population is divided with respective proportions. Utilizing this index, methods for analyzing the pattern of distribution of individuals and estimating the clump sizes on an area have also been developed (MORISITA, 1959). However, it may further be desirable that the relations between I_δ and the parameters of the mathematical distributions commonly applied to the natural and experimental populations are clarified. In this paper, some of these relations shall be described.

POISSON DISTRIBUTION AND VARIANCE-MEAN RATIO

When the distribution of individuals follows a Poisson series, we have

$$E(I_\delta)=1 \qquad (2)$$

If the distribution is contagious or uniform (regular), $E(I_\delta)$ will be larger or smaller than unity, respectively (MORISITA, 1959).

The relation between the variance-mean ratio and I_δ value is given as :

$$\frac{V}{\bar{x}} = \frac{q}{q-1}\left\{(I_\delta-1)\bar{x}+(1-\delta)\right\}, \qquad (3)$$

where V is the unbiased estimate of variance, $\bar{x}=\frac{1}{q}\sum_{i=1}^{q} x_i$, and $\delta=\frac{\sum_{i=1}^{q} x_i(x_i-1)}{T(T-1)}$.

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If $q \gg I_\delta \geqq 1$, we have

$$\frac{V}{\bar{x}} \doteqdot (I_\delta - 1)\bar{x} + 1. \qquad (4)$$

As the I_δ value is not influenced by the mean, it is clear from (3) or (4) that the $V/\bar{x}$ value changes with the change of the mean even if the proportions of the groups in the population be unchanged. Therefore, as pointed out by several investigators (EVANS, 1952 ; MORISITA, 1959 ; *etc.*), the $V/\bar{x}$ should not be recommended as an index of dispersion representing the degree of aggregation of individuals though it is useful for the test of significance of the departure from randomness.

Example :

The results of censuses carried out by UTIDA *et al.* (1952) on the eggs, larvae and pupae of the common cabbage butterfly, *Pieris rapae*, on cabbage plants are given in Table 1.

Table 1. The dispersion of the common cabbage butterfly, *Pieris rapae*, on cabbage plants (UTIDA *et al.*, 1952)

Stage of insect	Census I q=129 (18/V, 1949)			Census II q=91 (10/VI, 1949)			Census III q=190 (15/VI, 1949)		
	$\bar{x}$	$s^2/\bar{x}$	I_δ*	$\bar{x}$	$s^2/\bar{x}$	I_δ*	$\bar{x}$	$s^2/\bar{x}$	I_δ*
eggs	1.434	3.59	2.82	12.89	9.94	1.80	11.86	4.02	1.26
1st instar larvae	1.94	2.28	1.65	2.66	4.58	2.36	10.08	3.51	1.25
2nd instar larvae	1.40	1.52	1.38	1.34	2.77	2.34	7.74	3.41	1.31
3rd instar larvae	1.94	1.82	1.43	0.17	1.24	2.60	8.64	3.96	1.34
4th instar larvae	2.92	1.76	1.26	0.10	2.24	15.17	3.30	3.21	1.67
5th instar larvae	2.45	1.86	1.36	0.16	2.97	13.88	0.66	2.39	3.14
pupae	0.07	1.15	3.58	0.91	2.09	2.41	0.16	1.09	1.63

$\bar{x}$ = mean number of individuals on a cabbage plant.
q = number of cabbage plants.
* I_δ values were computed and added to the table by the present author.

It was concluded by UTIDA *et al.* through the examination of the data by square root transformation that the density-dependent death or movement between plants occurred in the successive larval instars and pupae (UTIDA *et al.*, 1952). However, the I_δ values computed from $\bar{x}$ and $s^2/\bar{x}$ values in the table do not show any systematic change of increase or decrease in successive stages, except for the large values in the 4th and 5th instar larvae at the census II. Thus, it is better to consider that the larval death or their movement between plants might occur rather density-independently in the observed field.

Negative Binomial Distribution

If the distribution of x is given by a negative binomial distribution as :

$$p(x)=\frac{(k+x-1)!}{(k-1)!\,x!}\cdot\frac{m^x}{(1+m)^{k+x}} \quad , \qquad (5)$$

the following relation is obtained (Simpson, 1949).

$$E(\delta)=\frac{k+1}{qk+1} \quad . \qquad (6)$$

Then we get

$$\frac{1}{k}=\frac{E(I_\delta)-1}{1-E(\delta)} \quad . \qquad (7)$$

When $q \gg I_\delta$, we have

$$\frac{1}{k}\fallingdotseq E(I_\delta)-1 \quad . \qquad (8)$$

If the distribution approaches a Poisson distribution, k tends to infinity, and accordingly $E(I_\delta)$ tends to unity.

Binomial Distribution

If the distribution of x follows a binomial distribution as :

$$p(x)=\frac{N!}{x!\,(N-x)!}\,p^x(1-p)^{N-x} \quad , \qquad (9)$$

we have

$$E(I_\delta)=\frac{N-1}{N-\frac{1}{q}} \quad \text{(see Appendix).} \qquad (10)$$

When $q \gg 1$

$$E(I_\delta)\fallingdotseq 1-\frac{1}{N} \quad . \qquad (11)$$

The significance of the departure from randomness of the distribution may be tested by comparing the following F_o value with the value of $F_{\infty}^{q-1}(\alpha)$:

$$F_o=\frac{q}{q-1}\,\frac{N}{N-\bar{x}}\left\{(I_\delta-1)\bar{x}+(1-\delta)\right\} \quad . \qquad (12)$$

I_B-index

The binomial distribution is associated with N mutually independent observations of an event which occurs with constant probability p. However, the cases may happen that either the probability in occurrence differs among q groups or among N observations in each group. Then, the I_B-index defined as :

$$I_B = I_\delta \frac{N - \frac{1}{q}}{N - 1} \tag{13}$$

may be used for measuring the dispersion among $x_1, x_2, x_3, \cdots, x_q$ which are the observed numbers of occurrence of an event in corresponding q groups occurring in each N observations with the probabilities $p_1, p_2, p_3, \cdots, p_q$. The regularity, randomness ($p_1 = p_2 = p_3 = \cdots = p_q = p$) and contagiousness of the distributions will be indicated by the values of I_B less than unity, equal to unity and greater than unity, respectively. The first case may represent the dependence (repulsive relation) of an occurrence or occurrences on the others in each group, while the last may represent either difference in probability of occurrence among groups or the dependence (attractive relation) of an occurrence or occurrences on the others in each group.

Example :

A frequency distribution of the days of capturing the vole, *Chlethrionomys rufocanus bedfordiae*, by 81 live traps in successive six days is given in Table 2a. The I_B value computed from total 6 days ($N=6$) is 2.15, indicating that there are

Table 2. Frequency distributions of the days of capturing the vole, *Chlethrionomys rufocanus bedfordiae*, by live traps.

Days of capturing vole	Number of traps						
	a	b		c			d
		1st period	2nd period	1st period	2nd period	3rd period	
	1st–6th days	1st–3rd days	4th–6th days	1st–2nd days	3rd–4th days	5th–6th days	3rd–6th days
0	39	58	45	65	54	52	41
1	17	13	23	10	19	20	20
2	9	4	6	6	8	9	11
3	8	6	7	—	—	—	5
4	6	—	—	—	—	—	4
5	0	—	—	—	—	—	—
6	2	—	—	—	—	—	—
Total ($=q$)	81	81	81	81	81	81	81
T	95	39	56	22	35	38	73
I_δ	1.796	2.405	1.420	2.104	1.089	1.037	1.546
N	6	3	3	2	2	2	4
I_B	2.151	3.593	2.121	4.182	2.165	2.081	2.055

a. Total of six days.
b. Total days were divided into two periods.
c. Total days were divided into three periods.
d. The data in the period from 3rd to 6th day were treated.

The data were collected by Dr. R. TANAKA who carried out a census in Hokkaido for estimating the population density and home range of the vole. As to the detail of the method, see TANAKA (1961).

some differences in probability of capturing voles among the traps. These differences are much larger in the first 3-day period (I_B=3.59) than in the second 3-day period (I_B=2.12) (Table 2b). The results of examination dividing the total days into three of 2-day periods (N=2) indicate that the differences in catch probability among traps are extremely large in the first 2-day period, and after this period, the differences become small and stable. These figures suggest that there might be a change in the vole behavior against the traps from the period of 2 days immediately after setting of the traps to the periods afterwards.

It is noteworthy that the I_B values obtained from the second and third periods in Table 2c are almost the same as the value in Table 2d, the latter having been computed putting the data used for the former two together. This result shows that the I_B-index is useful enough for comparing the dispersions with each other regardless of difference in N.

Conclusion and Summary

As the I_δ-index is neither affected by the mean per sample unit except for regular distribution nor standing on the assumption of any definite type of contagious distribution, it may have the most wide range of application among the ones hitherto deviced for measuring the dispersion of individuals in a population. The relations of the I_δ-index to $V/\bar{x}$, k of negative binomial distribution and N of binomial distribution as well as the new dispersion index, I_B, given in this paper may serve, if necessary, for the analysis of data in the ecological works.

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Appendix by Hiroshi Mito

In the binomial distribution (9),

$$E(I_\delta)=\frac{N-1}{N-\frac{1}{q}} .$$

Proof :

$$E(I_\delta)=q\sum_{x_1,x_2,\cdots,x_q=0}^{N}\frac{\sum_{i=0}^{q}x_i(x_i-1)}{T(T-1)}\binom{N}{x_1}p^{x_1}(1-p)^{N-x_1}\cdots\cdots\binom{N}{x_q}p^{x_q}(1-p)^{N-x_q}$$

$$=q\sum_{T=0}^{Nq}\frac{1}{T(T-1)}\sum_{i=1}^{q}\sum_{T=x_1+\cdots+x_q}x_i(x_i-1)\binom{N}{x_1}p^{x_1}(1-p)^{N-x_1}\times\cdots\binom{N}{x_q}p^{x_q}(1-p)^{N-x_q} .$$

Here the following relations hold :

$$x_i(x_i-1)\binom{N}{x_i}p^{x_i}(1-p)^{x_i}=N(N-1)p^2\cdot\binom{N-2}{x_i-2}p^{x_i-2}(1-p)^{N-x_i},\ \ i=1,\ 2,\ \cdots,\ q .$$

Therefore

$$E(I_\delta)=q\sum_{T=0}^{Nq}\frac{N(N-1)p^2}{T(T-1)}\sum_{i=1}^{q}\sum_{T=x_1+\cdots+x_q}\binom{N}{x_1}p^{x_1}(1-p)^{N-x_1}\cdots\binom{N}{x_{i-1}}p^{x_{i-1}}(1-p)^{N-x_{i-1}}$$

$$\times\binom{N-2}{x_i-2}p^{x_i-2}(1-p)^{N-x_i}\binom{N}{x_{i+1}}p^{x_{i+1}}(1-p)^{N-x_{i+1}}\cdots\times\binom{N}{x_q}p^{x_q}(1-p)^{N-x_q} .$$

Since, by the reproductive property of the binomial distribution,

$$\sum_{T=x_1+\cdots+x_q}=\binom{N_q-2}{T-2}p^{T-2}(1-p)^{N_q-T} ,$$

we have

$$E(I_\delta)=q\sum_{T=0}^{N_q}\frac{N(N-1)p^2}{T(T-1)}q\binom{N_q-2}{T-2}p^{T-2}(1-p)^{N_q-T} .$$

Here the following relation holds :

$$\frac{1}{T(T-1)}\binom{N_q-2}{T-2}=\frac{1}{N_q(N_q-1)}\binom{N_q}{T} .$$

Therefore

$$E(I_\delta)=\frac{q^2N(N-1)}{N_q(N_q-1)}\sum_{T=0}^{N_q}\binom{N_q}{T}p^T(1-p)^{N_q-T}$$

$$=\frac{N-1}{N-\frac{1}{q}} .$$

個体の集合度の指標としての I_δ-指数

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著者はさきに一定空間内における個体の集合状態を示す指標として I_δ-指数を提唱したが（Morisita, 1959），本報においては，

1. I_δ とポアソン散布指数（$V/\bar{x}$）との関係
2. I_δ と負の二項分布の k との関係
3. I_δ と二項分布の N との関係

を記述するとともに，有限の観察回数 N の中でのある事象の生起回数 x の間の集中度（その事象の起る確率がグループ間で異なる場合，またはその確率が N 中の各回の観察毎に異なる場合の x 間の集中度）の指標として新らしく I_B-指数を提唱した。なお I_δ-指数ならびに I_B-指数の適用例として，圃場におけるモンシロチョウ卵，幼虫，蛹の分布（内田その他，1952の資料による）およびネズミ捕獲率のワナ間の相違（田中，1961の原資料による）の問題をそれぞれ取扱い，その有効性を示した.