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THE NEW GRAVITY BASE NET 1976 OF THE FEDERAL REPUBLIC OF GERMANY (DSGN 76)

Abstract

A new gravity base net ("Schweregrundnetz 1976 der Bundesrepublik Deutschland", DSGN 76) has been established in the Federal Republic of Germany, to meet the increased requirements of geophysics, geology, metrology and geodesy. The net comprises 21 stations with three excenters each. The gravity values were determined using 4 absolute stations, 11 IGSN71–stations and about 3000 relative gravity meter observations with 4 gravity meters. Instrumental investigations and special treatment of local tidal and atmospheric effects improved the data for the least squares adjustment, which was performed by the method of observation equations following the use of condition equations. The final adjustment showed a point r.m.s. error of about 10 μGal [10^{-8} ms^{-2}]. Detailed results will be published in the "Veröffentlichungen der Deutschen Geodätischen Kommission".

1. Introduction

National gravity base nets were established in the 1930's and 1950's. The latter, one, DSN62 base net, was based on the fundamental station Bad Harzburg, which differed from the IGSN71–value by about 15 mGal (10^{-5} ms^{-2}), 14 mGal due to the error of Potsdam station and 1 mGal due to an erroneous connection Potsdam – Bad Harzburg. The difference in scale of DSN62 against IGSN71 reached $3 \cdot 10^{-4}$. Besides these systematic errors the DSN62 base net was reported to show random errors from 0.02 to 0.1 mGal. Most important, an estimated 30 % of the stations can not clearly be identified anymore. On the other hand, error limits of 0.01 mGal for global absolute level homogeneity and 0.1 mGal of random error are needed in order to achieve the decimeter geoid [Moritz, 1975]. For geodynamic investigations an error limit of 1 μGal for regional nets is desired. This figure corresponds to expected regional tectonic gravity changes in Europe within a few years, as may be concluded from the studies of recent height changes in the Alps (cf. Mueller / Lowrie 1980). Thus geodesy requires the highest accuracy, more stringent than the demands of geophysics and metrology.

These arguments and the establishment of the IGSN71 led to steps for the renewal of the base net. This was on behalf of the German Geodetic Commission guided by a working group [Torge, 1978] and performed by the "Deutsches Geodätisches Forschungsinstitut (DGFI), Abt. I, München" and the "Institut für Angewandte Geodäsie (Abt. II DGFI), Frankfurt". The work was supported by various state and university institutions.

2. Net Design, Measurements and Data Preprocessing

Net Design

The 21 stations of the DSGN76 *(Fig. 1)* are distributed to meet the requirements listed above and to be easily accessible both from within the FRG and for making connections to adjacent countries. Each station has 3 excenters in a distance of 500 m to 5 km. All points are situated in stable (public) buildings and marked by metal discs on a stable floor or on a pillar, where no mass changes and good measuring conditions can be expected. For all points the opinions of geological and the hydrological survey have been observed. They are linked to the first order levelling net by the land survey and carefully recorded through sketches, photographs and maps.

11 stations are either identical or closely linked to IGSN71 stations, and at 4 stations absolute gravity measurements have been made.

Once the station sites were chosen, the problem was to design an optimal observation scheme for the gravity meter measurements. For this purpose different approaches were tested. For the first approach a properly designed variance–covariance matrix of the unknown gravity values served as a criterion matrix, which was to be approximated by measurements with known apriori accuracy through linear programming techniques. Further investigations were made by model computations and by computations of non random error propagation by the MINIMAX–method [cf. Heindl/ Reinhart, 1976 and 1977].

The main result of these computations was, that it would be optimal for our net to measure mainly very long intervals. Furthermore it became clear, that the stations at the edges of the network have to be strengthened. The usefulness of these computations was limited, however, because of the following :

1. Uncertainties in the planned absolute measurements and the final adjustment model.
2. It was not feasible to perform many long range measurements, which take 20 hours a day.
3. Some influences could hardly be put into numerical values such as road conditions and timetable restrictions.

These arguments led to the final approach, a dynamic composition in man–computer dialogue. The program starts with a minimum configuration. In each loop it then provides a proposal for an optimal further connection according to a scalar objective function chosen beforehand. The user then decides whether to accept the proposal or to take another connection on the basis of the information shown on the screen and his personal background information. The resulting gravimetric connections are shown in *figure 1.*

Measurements

1975/76 the stations were selected and marked. In 1977 44 inter–station connections were measured back and forth, 38 of them repeatedly by an other party. Furthermore, the centers were linked to the excenters, to the absolute stations and to the IGSN71 stations. These measurements in total about 500, took 200 days for 3 people, who covered a distance of 80 000 km. All measurements followed a precise timetable, e.g., 0.5 hours transport of the meter before the first observation in the morning, 5 minutes waiting time after unclamping the La Coste–Romberg meters, etc.

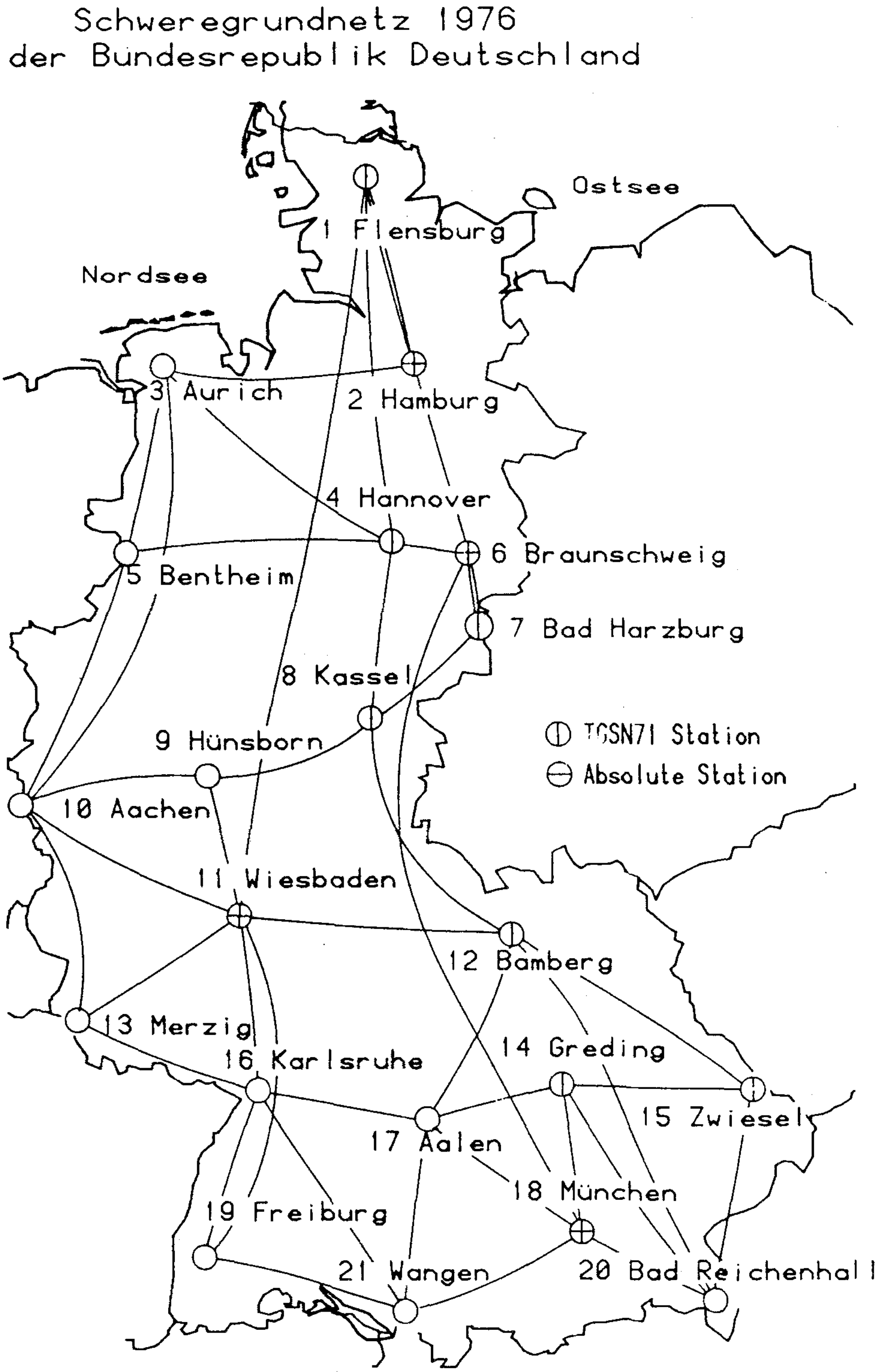

Fig. 1 – German Gravity Base Net DSGN76

At four stations absolute gravity measurements were carried out by an Italian party with a high precision transportable absolute gravity meter [Cannizzo, Cerutti, Marson, 1978]. This apparatus yields an accuracy of $10\ \mu Gal$ or better.

After preprocessing and correcting the data for instrumental, tidal and atmospheric effects, the final adjustment model was developed in accordance with data and residual analysis in 1979/80.

Tidal Corrections

For the calculations of earth–tides the complete Cartwright-Edden development (CARTWRIGHT/EDDEN 1973) with 505 waves was used, including the time independent term $M_0\ S_0$. In changing from the rigid to the elastic earth model it was necessary to multiply the amplitudes by the δ–factor and to consider the phase lag κ between the observed and theoretical earth–tides.

Based on harmonic analysis of observations at 22 earth–tide stations [Bonatz, unpublished data, 1978] distributed over the Federal Republic of Germany and the bordering areas the δ– and κ–factors of the wavegroups Q1, 01, P1S1K1, N2, M2, S2 were interpolated for the 21 gravity base net stations (cf. *fig. 2)*.

All other waves were multiplied by the δ–factors calculated from the Molodensky I earth model [Melchior, 1978].

Atmospheric Corrections

For high accuracy gravity measurements it is necessary to correct the data for mass changes in the atmosphere due to air pressure variations.

The two non–instrumental influences of air pressure variations are :

1. the attraction forces due to the air masses,
2. the deformation of the earth crust due to the air masses.

The relation between direct gravitational attraction, the deformation of the ground and also attraction due to the mass changes in the atmosphere have been calculated theoretically by several authors [Ecker/Mittermayer 1969]. Furthermore the comparison between earth tide– and air pressure registrations showed the same $-0.3\ \mu Gal/mbar$ factor [Brein 1969; Torge/Wenzel 1977].

To eliminate the height dependence [Möller 1962] on air pressure for each gravity base net station a "standard air pressure" was calculated.

The basis for these calculations is taken from the "Norm Atmosphäre" (DIN 5450, 1968) which has been adapted to the US Standard Atmosphere 1962. Within an atmospheric layer for which L' is a linear function of H, the hydrostatic equation and the perfect gas law yield the following expression for the pressure :

$$p(H) = p_0 \left(\frac{t_0 + L'H}{t_0} \right)^{\frac{g_0 M_0}{L' R^*}} \qquad (2.1)$$

With the constants

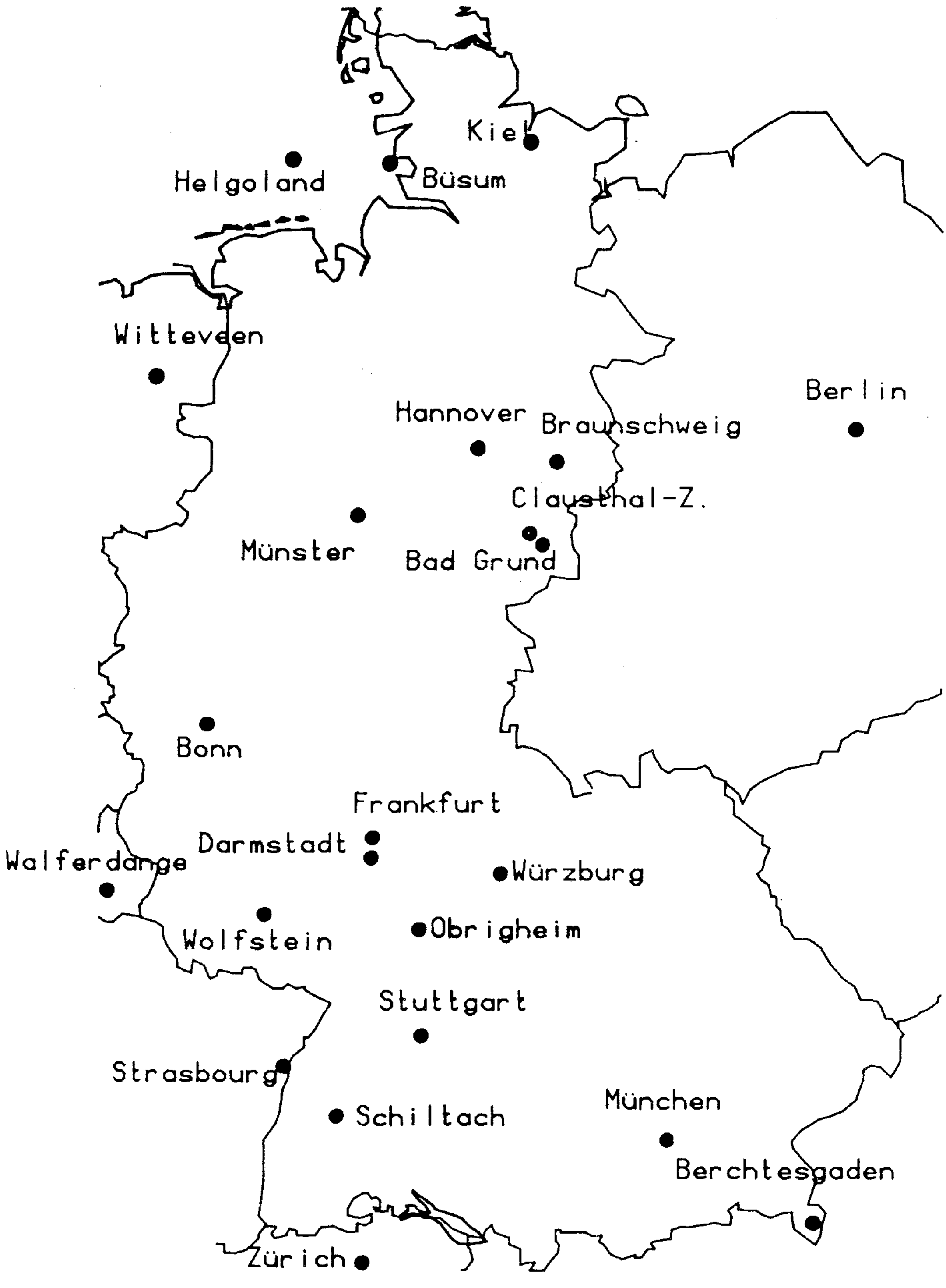

Fig. 2 — Earth Tide Stations

p_0	sea–level pressure	1013.250 mbar
t_0	sea–level temperature	288.15 K
L'	molecular scale temperature gradient	$-0.65 / 100$ m
H	normal height in units of 100 meters	
g_0	acceleration due to gravity at exactly 45° geographic latitude	9.80665 ms^{-2}
M_0	mean molecular weight of air	$28.9644 \text{ kg (kg–mol)}^{-1}$
R^*	universal gas constant	$8.31432 \cdot 10^3 \text{ J (kg–mol)}^{-1}$

$$p(H) = 1013.25 \left(\frac{288.15 - 0.65 \cdot H}{288.15}\right)^{5.2559} \tag{2.2}$$

A comparison between several mean annual air pressure values with calculated standard atmosphere values shows a constant difference of 4 mbar .

Instrumental Considerations and Corrections

The effects of environmental air pressure, temperature, magnetic field and changing battery voltage for the heater were studied in the laboratory by varying the parameters and reading the meter. Generally an abnormal drift could be observed particularly at the beginning of a series of tests when the environmental parameters changed. In addition to laboratory investigations the effect of air pressure and temperature were also studied by analyzing the measurements in the gravity net. For the effect of battery voltage we found typical coefficients of $\leq 10\,\mu Gal/V$. Air pressure effects were found to be about $2\,\mu Gal/100$ mb . The readings of one gravity meter varied by about $40\,\mu Gal$ when changing the magnetic azimuth.

For the effect of temperature for example ***fig. 3*** shows the laboratory test for LCR–G79, which gave a coefficient of $+2.3\,\mu Gal/\mathrm{K}$. In a second approach the residuals of an adjustment without any temperature correction were analyzed with a linear regression model, giving a coefficient of $1.0\,\mu Gal/\mathrm{K}$. In a third approach, the coefficient was included in the overall adjustment, now yielding $0.9\,\mu Gal/K$. From the theoretical point of view the last value should be the proper one. The laboratory conditions for this particular test obviously did not correspond to the real ones in the field.

In order to improve the environmental conditions the gravity meters were transported in a spring suspended air conditioned box. The voltage was regulated. Of course, the instruments were handled very carefully.

The structure of the calibration function of the LaCoste–Romberg gravity meters seems to show some small periodic components. In a first approach a gravity variation over a range of $15\,mGal$ was simulated by the tilt of the gravity meter, the tilt was measured very precisely with a laser interferometer. The results showed some evidence for periods of about $6\,mGal$. In a second approach the residuals of an adjustment were analyzed by a statistical spectral analysis. Despite the material for this kind of analysis being rather poor, periods of about 70 and $35\,mGal$ were found.

From the construction of the gravity meter periods of 35.47 and $70.94\,mGal$

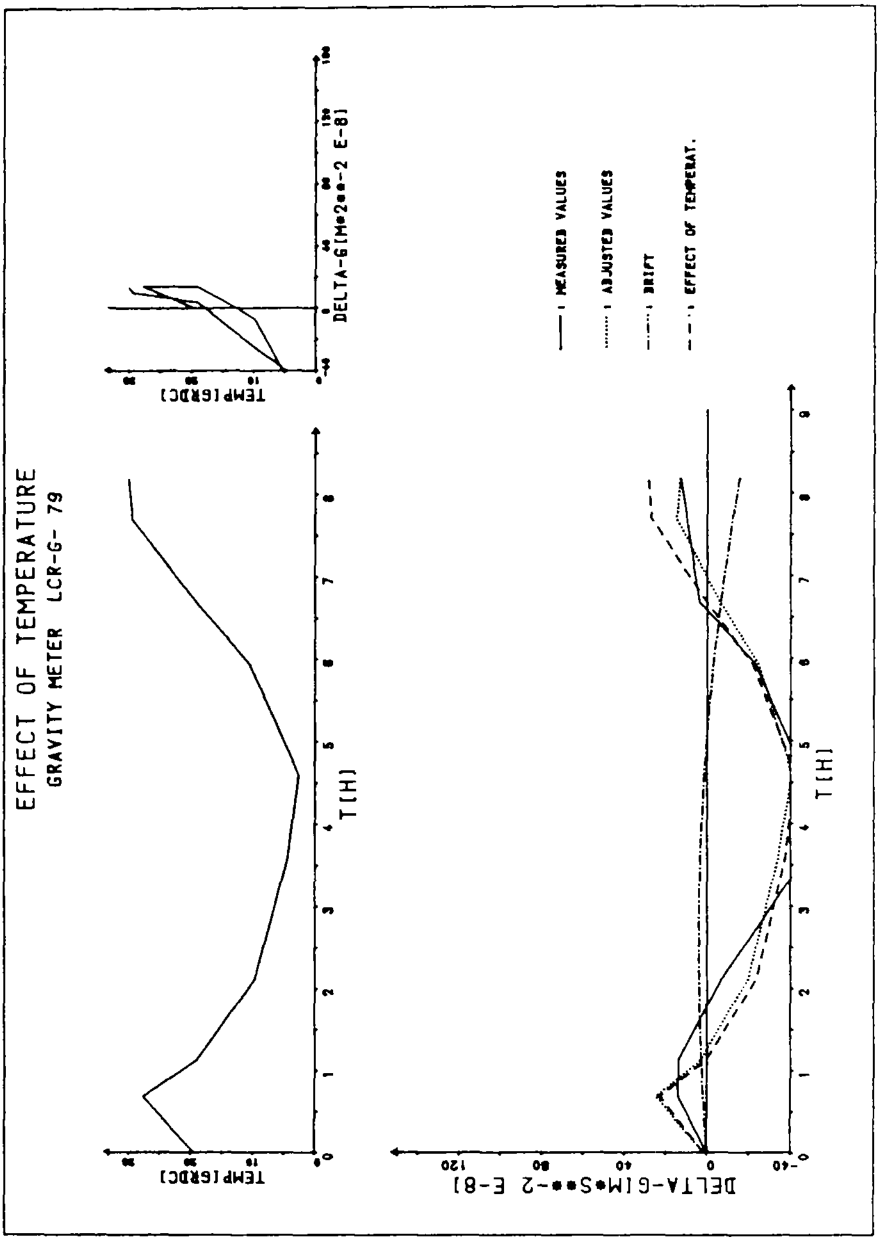

Fig. 3 -- Effect of Temperature

can be expected. When introducing these periods with unknown amplitudes and phases in the adjustment itself, we got amplitudes of $\leq 20\ \mu Gal$. These periodic errors seem to be very dangerous, because they can be detected only by employing several gravity meters at the same points, while the single gravity meters may show an apparently excellent "inner accuracy".

It was found, that the original calibration tables may cause rounding off errors of up to $10\ \mu Gal$. Therefore by adjustment new calibration tables were derived and applied, which maintain the piecewise linear approximation but avoid the rounding off effects.

3. Adjustment

Connection to the International Gravity Net and Absolute Gravity Measurements

For fixing level and scale of the DSGN76 there were two possibilities :

The International Gravity Standardization Net 1971 (IGSN71) assures global homogeneity for all subsequent measurements. According to a recommendation of a working group of the International Gravity Bureau national gravity nets should be connected to the IGSN71 considering the full dispersion matrix of the junction points (IGB 1977). 11 of the DSGN76–stations are either identical or closely linked to 16 IGSN71–stations, the main part of which belonging to the European Calibration Line.

At the DSGN76–stations Hamburg, Braunschweig, Wiesbaden and München, absolute gravity measurements [Cannizzo / Cerutti / Marson 1978] have been carried out in 1977. The apparatus applies the free rise and fall method, where the length is measured similarly to a Michelson interferometer by counting the interference fringes produced by the motion of one cube corner in the gravity field with respect to a second, fixed, one. The mean of (usually) 100 single measurements (within about 3 days) gave a r.m.s. error of less than $10\ \mu\ Gal$.

The accuracy of the absolute stations is superior to the IGSN71 stations. Therefore the DSGN76 is based on the absolute gravity measurements. At the 1978 meeting of the International Gravity Commission it was stated, that the DSGN76 nevertheless may be regarded as "in the system of IGSN71", because the differences are very small.

For the reason of an independent control, the adjustment of the data was performed by two different methods and two different working groups.

Adjustment by Condition Equations

The gravity differences were measured from point A to point B and back again on the same day. So it was possible to obtain an impression of the drift–rate by comparison of the two readings at the starting point. The question arose whether the drift was a random process or, if not, how it could be calculated.

For this reason the daily drifts for the two measuring campaigns were determined, normalized and added together. In *fig. 4* the result for one gravity meter throughout the two campaigns is shown. It is clearly seen that there is no random process and one can say that the drift could be described by a linear function for a single period of observations. However, this instrument displayed a very high drift rate. Turning to another instrument the situation was not so clear, even during the two campaigns *(fig. 5)* In this case it was not advisable to approximate the drift by a single linear

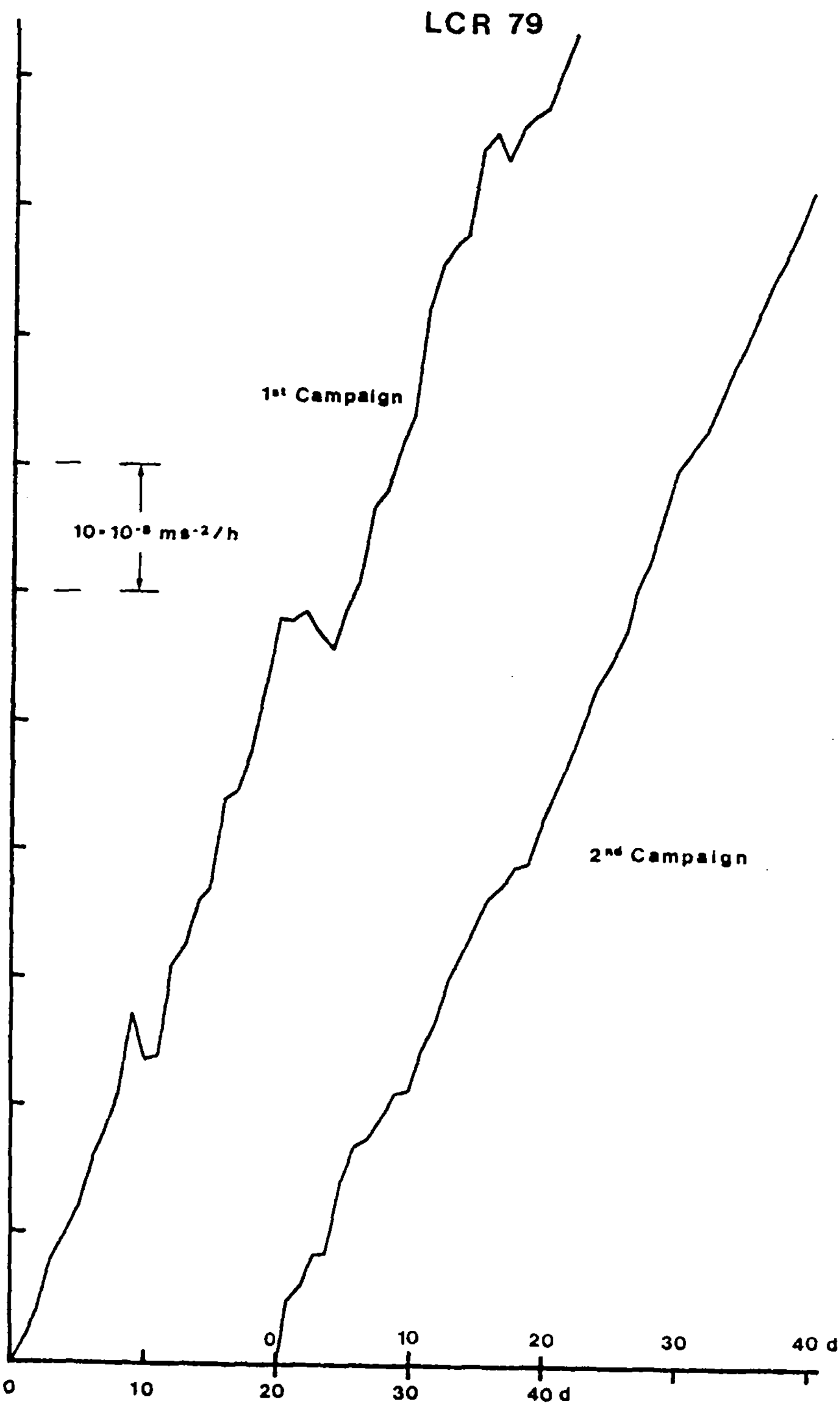

Fig. 4 – Drift behaviour during two campaigns

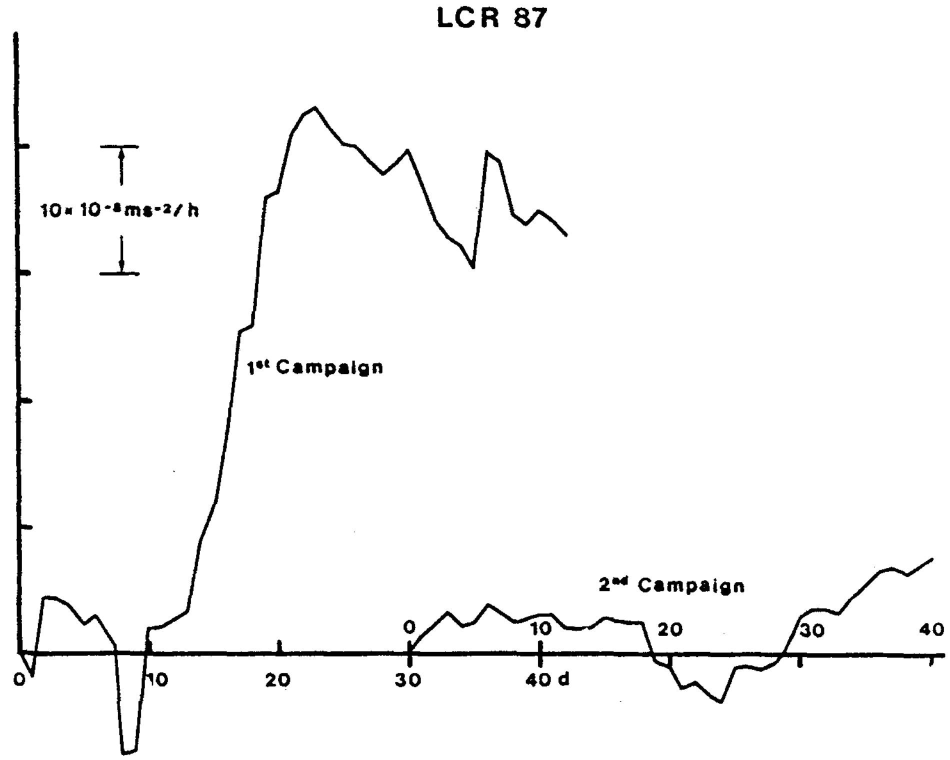

Fig. 5 — Drift behaviour during two campaigns

function but perhaps by piecewise linear or higher polynomial functions.

Nevertheless it can be seen that a significant drift behaviour over a longer period was present. Taking these facts into account the conclusion was, that the drift may not be neglected and due to the complicated function a daily drift parameter has to be introduced. The disadvantage of this method was the risk of interpreting some real errors as drift. The daily drift was then eliminated using a linear function, thereby obtaining one drift free "observed" gravity difference L between point A and B .

During the spring campaign 42 gravity differences were measured followed by 40 in the autumn. (For this step, the connections of excenters to the centers were not included). Four instruments were used for the observations.

For each instrument it was necessary to develop 22 (20) condition equations to satisfy the closure of the loops of the kind,

$$(L_1 + v_1) + \ldots + (L_n + v_n) = 0 \tag{3.1}$$

For combining one instrument with another, connection condition equations were developed in which a scale unknown is included. These equations have the form,

$$(L_{i_I} + v_{i_I}) - (L_{i_{II}} + v_{i_{II}}) \times (1 - k) = 0 \tag{3.2}$$

with

L_{i_I} observation of instrument I

$L_{i_{II}}$ observation of instrument II

k scalefactor

This implies that following adjustment (e.g. Wolf 1975) a gravity difference $L_{1\,I}$ measured by one instrument must be the same as the difference $L_{1\,II}$ observed with another.

For the net 20 connection equations were derived for each instrument and the four absolute stations were included as observations in the system. Therefore three more condition equations and a further scale unknown were introduced. In all 311 condition equations with 8 unknowns were adjusted.

To obtain an error estimate it is unnecessary to invert the complete matrix of 319 equations as we are primarily interested in obtaining the r.m.s. errors of the point values and not of the adjusted gravity differences. The absolute values, together with the observed gravity differences, were used to derive the 21 point gravity values, the resulting 21 functions being added to the system of equations. Only for these error functions was it necessary to operate upon the complete functional matrix to obtain the r.m.s. errors of the point values and, applying the correlation coefficients the r.m.s. errors of the adjusted gravity differences. This procedure saves considerable computer time as the system contained 348 equations which could be reduced 319 times using the Gauss algorithm rather than performing a complete inversion of the 319 equations.

Now we turn to a problem encountered during the adjustment procedure, namely the periodic error in the calibration function. First point values were calculated for the total net in an adjustment using the observations for a single instrument. The weights for all observations were set to one and the results gave a standard error of an observation of weight one $6 < m_0 < 11\ \mu Gal$. Taking then all measurements together, and again setting the weights to one for each instrument a m_0 of $18\ \mu Gal$ was obtained. This led to the conclusion that there was an unmodelled systematic effect involved in the process. As an example in *fig. 6a* the adjustment point values for two instruments during the two campaigns with respect to a mean value are shown. Following the lines the deflections from the median line are quite similar throughout the two measuring epochs. Corresponding curves were given for the other instrument. Consider now the periodic errors due to the gear with periods of approximately 35 , 70 , 603 and 1206 $mGal$. Analysing these residuals with respect to the predicted periods one saw the amplitudes and the phases for all periods. Most significant were the 35 and 70 $mGal$ periods. The amplitudes ranged up to 25 μGal . Taking the periodic errors into account, repeating the procedure of single adjustment the picture shown in *fig. 6b* was obtained. The results of the point values were closer together and no systematic effect could be shown. The total adjustment using all instruments showed that, weighting all observations equal the m_0 ($11\ \mu Gal$) was the same as that computed for the single adjustments. With this procedure it was shown that periodic errors in the calibration function exist (see sec. 2 Instrumental Considerations and Corrections).

Adjustment by Observation Equations

Applying this method we can use two different ways, introducing either the (corrected) readings of the gravity meters as observations or the differences of subsequent readings. If one applies the last technique, one must regard also the off–diagonal elements

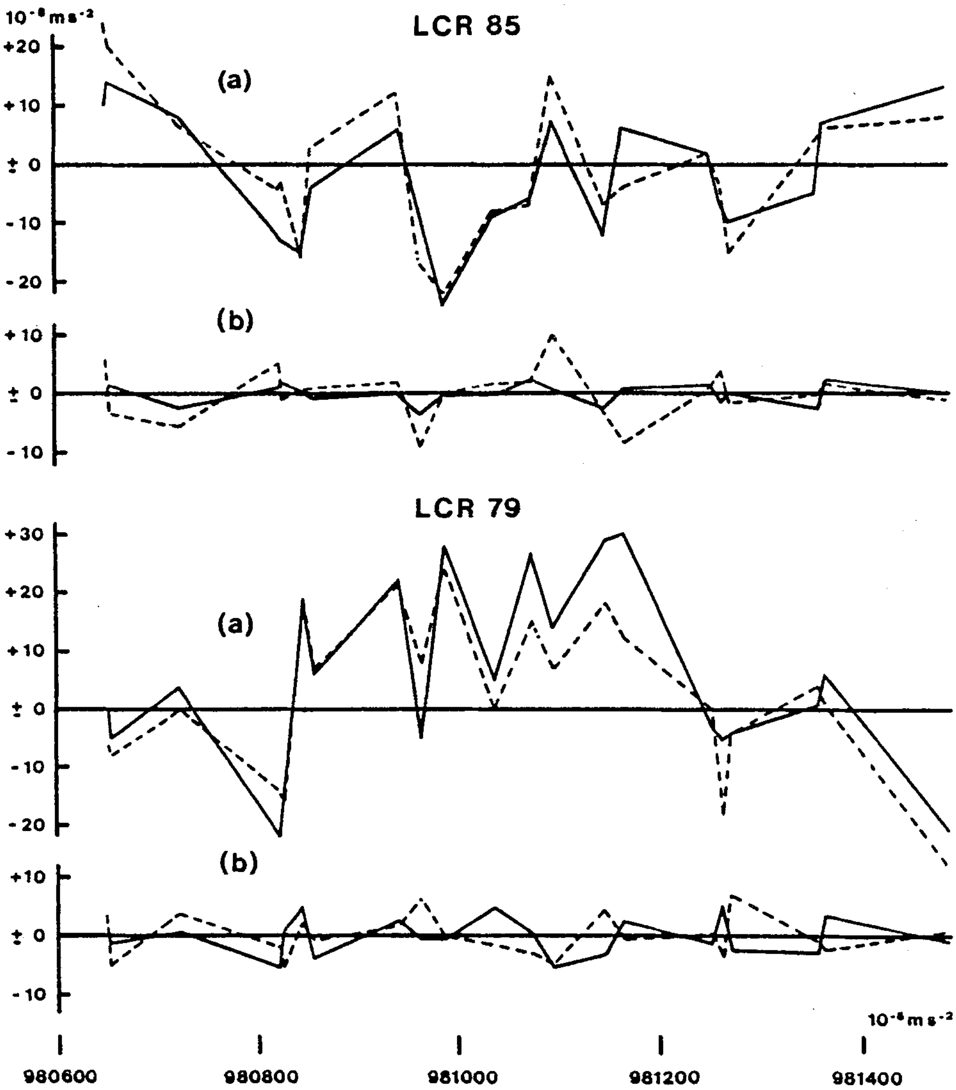

Fig. 6 – Differences between the mean point values and the point values

(—— First campaign – – – Second campaign)

(a) First adjustment

(b) Second adjustment taking the periodic errors into account of the variance–covariance matrix for the differences, which is often neglected.

Using the first method, we have an observation equation of the kind :

$$v_i = g_j - r_i + o_\ell + \sum_{k=1}^{m_\ell} z_i^k \cdot e_{g,k} + \sum_{k=1}^{m_d} t_i^k \cdot d_{g,k} + s_g \cdot T_i \tag{3.3}$$

$$+ \sum_{k=1}^{m_p} (p_{1,g,k} \cdot \sin \nu_{g,k} z_i + p_{2,g,k} \cdot \cos \nu_{g,k} z_i)$$

with

v_i residual of observation number i

g_j gravity value of station number j

r_i gravity meter reading, corrected for tidal, atmospheric and air pressure effects

o_ℓ orientation parameter of measurement series ℓ

$e_{g,k}$ coefficient of polynomial for the calibration function of gravity meter number g

z_i^k raw gravity meter reading

m_ℓ degree of polynomial for calibration function (in practice, specific components of the polynomial can be chosen)

$d_{g,k}$ coefficient of drift polynomial

t_i (relative) time of measurement

s_g coefficient of environmental temperature effect

T_i temperature

$p_{1,g,k}$ auxiliary parameter $p_{1,g,k} = a_{g,k} \cdot \cos \varphi_{g,k}$

where

$a_{g,k}$ amplitude of periodic error number k of instrument number g

$\varphi_{g,k}$ phase of periodic error

$p_{2,g,k}$ auxiliary parameter $p_{2,g,k} = - a_{g,k} \cdot \sin \varphi_{g,k}$

m_p number of periods

$\nu_{g,k}$ frequency

In our case, $n = 2868$ observations $(i = 1 \ldots n)$ were introduced, the number of the gravity stations (centers and excenters) amounts to $m_g = 84$ $(j = 1 \ldots m_g)$ and the number of measurement series was $\ell = 507$. Normally for the calibration function and for the drift function linear components sufficed. Periodic errors of the calibration function with periods 35.47 and 70.94 *mGal* were considered. In order to simplify computations, the orientation parameter o_ℓ and – by choice – a linear daily drift are reduced from the normal equations.

For the four absolute stations the observation equations read as

$$v_i = g_j - g_i \text{ (Abs.)} + \sum_{k=0}^{m_f} f_k \cdot \overset{o}{g}{}_j^k \tag{3.4}$$

with

g_i (Abs.)	absolute gravity measurement
f_k	transformation coefficient
$\overset{o}{g}{}_j^k$	approximate gravity value
m_f	degree of transformation polynomial

For the 16 IGSN71 stations included the observation equations are

$$v_i = g_j - g_i \text{ (IGSN71)} + \sum_{k=0}^{m_h} h_k \cdot \overset{o}{g}{}_j^k \tag{3.5}$$

with

g_i (IGSN71)	gravity value from IGSN71
h_k	transformation parameters
m_h	degree of transformation polynomial

With this system, the whole net may either be based on the absolute or on the IGSN71–values. As shown in section 3, we chose the absolute stations as a reference. In this case the series in (3 . 4) has to be omitted. For the series in (3 . 5) a constant and a linear parameter was applied.

Weighting all observations, the absolute values were given $\sigma = \pm 10\ \mu Gal$. For the IGSN71 values, the proper variance–covariance matrix was introduced. The weights of the gravity meter observations were determined for 16 groups of different quality in an iteration process giving mean square errors for one observation (mean of three readings) $\sigma = \pm 5 \ldots 15\ \mu Gal$.

4. Final Results

The results of the adjustment according to the two different methods show consistently maximum r.m.s. error of less than $\pm 11\ \mu Gal$ for the gravity values and $\pm 15\ \mu Gal$ for the r.m.s. of the maximum gravity difference between the stations 1 and 20 (cf. *fig. 1*). Some stations have an r.m.s. of $\pm 6\ \mu Gal$ from the adjustment. This small number, however, is not of any value, because groundwater variations or uncontrollable instrumental effects may exceed this number. In any case the DSGN76 will be a sound basis for subsequent users of geodesy, geophysics and metrology. It may even be a means for the detection of secular variations of gravity. The final gravity values of the 21 stations together with their r.m.s. errors and the variance–covariance matrix are given in the appendices.

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REFERENCES

R. BREIN : Report on some investigations concerning gravimeter recordings in Frankfurt/M, VI. International Symposium on Earth Tides, Strasbourg 1969.

L. CANNIZZO, G. CERUTTI, I. MARSON : Absolute–Gravity Measurements in Europe. IL Nouvo Cimento, Vol. 1C, N. 1, 1978.

D.E. CARTWRIGHT, A.C. EDDEN : Corrected tables of tidal harmonics. The Geophysical Journal, Vol. 33, No. 3, 253–264, Oxford 1973.

W. DOERGE, E. REINHART, G. BOEDECKER : Das "International Gravity Standardization Net 1971 (IGSN71)" in der Bundesrepublik Deutschland. Deutsche Geod. Komm., R.B. Nr. 225, München 1977.

E. ECKER, E. MITTERMAYER : Gravity Corrections for the Influence of the Atmosphere. Bolletino d. Geofisica Teorica, Vol. XI, 70–80, 1969.

G. HEINDL, E. REINHART : Eine allgemeine Methode zur Berechnung von MINIMAX–Fehlern Teil 2 : Vor der Durchführung von Messungen. ZfV 101, 238–241, 1976.

G. HEINDL, E. REINHART : A Priori Computation of Minimax–Errors Considering Both Given and Planned Observations. Paper pres. IAG–Symp. on Optimization of Design and Computation of Control Networks, July 3–9, Sopron 1977.

IGB (Ed.) : Guidelines for the Adjustment of National and Continental Gravity Base Networks within the Framework of the International Gravity Standardization Net 1971 (IGSN71).

P. MELCHIOR : The tides of the planet earth. Pergamon Press, Oxford 1978.

D. MÖLLER : Beiträge zur barometrischen Höhenmessung. Deutsche Geod. Komm., R.C. Nr. 52, München 1962.

C. MORELLI et *al.* : The International Gravity Standardization Net 1971 (I.G.S.N. 71). I.A.G. Paris 1974.

H. MORITZ : Die Zukunft der gravimetrischen Geodäsie. Zeitschrift f. Vermessungswesen 100 (1975), 1, 15–21.

St. MUELLER, W. LOWRIE : Die geodynamische Entwicklung des westlichen Mittelmeerraumes und der Alpen. Vermessung, Photogrammetrie, Kulturtechnik 12/1980.

W. TORGE : Das neue Deutsche Schweregrundnetz. Deutsche Geod. Komm., R. E., Nr. 17, 49–59, München 1978.

W. TORGE, H.–G. WENZEL : Comparison of earth tide observations with nine different gravimeters at Hannover. Proceed. of the 8th Intern. Symp. on Earth Tides, M. Bonatz and P. Melchior (eds.), 632–640, Bonn 1977.

U.S. Standard Atmosphere 1962. U.S. Government Printing Office, Washington 25, D.C., 1962.

H.–G. WENZEL : Einfluss von Luftdruck und Lufttemperatur auf gravimetrische Erdgezeiten–beobachtungen. Rundgespräch Geodäsie / Geophysik, Hannover 1974.

H. WOLF : Ausgleichungsrechnung. Dümmler Verlag, Bonn 1975.

Appendix A

Final gravity station values of the DSGN76

Station No	Name	g–value mGal	r.m.s. mGal
1	Flensburg	981 485.580	0.011
2	Hamburg	981 363.679	0.009
3	Aurich	981 357 250	0.009
4	Hannover	981 262.404	0.008
5	Bentheim	981 270.640	0.008
6	Braunschweig	981 252.943	0.007
7	Bad Harzburg	981 165.520	0.007
8	Kassel	981 146.704	0.006
9	Hünsborn	981 073.920	0.006
10	Aachen	981 094.951	0.006
11	Wiesbaden	981 036.864	0.006
12	Bamberg	980 986.584	0.006
13	Merzig	980 963.716	0.007
14	Greding	980 856.339	0.008
15	Zwiesel	980 822.120	0.008
16	Karlsruhe	980 941.458	0.007
17	Aalen	980 845.332	0.008
18	München	980 723.129	0.009
19	Freiburg	980 826.469	0.008
20	Bad Reichenhall	980 650.409	0.011
21	Wangen	980 653.728	0.011

Appendix B

Variance–covariance matrix of the station gravity values in $[\mu Gal^2]$
(upper right of symmetric matrix)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	131	99	99	77	80	73	55	51	36	39	26	16	10	-10	-19	5	-14	-41	-19	-57	-57
2		81	78	63	64	60	47	45	33	37	28	20	16	0	-5	12	-2	-20	-5	-32	-31
3			85	65	69	61	49	46	36	38	27	20	17	3	-4	13	0	-19	-3	-29	-30
4				56	55	51	43	42	34	36	29	24	21	10	6	19	9	-3	7	-11	-11
5					63	52	44	42	35	37	29	23	22	11	6	19	8	-5	6	-11	-12
6						52	43	40	32	34	27	57	21	11	7	18	9	-1	7	-8	-9
7							44	37	33	32	28	27	26	22	19	24	19	12	18	10	9
8								40	34	34	30	29	27	22	21	26	22	15	21	12	12
9									38	33	32	31	32	31	30	31	30	27	30	29	28
10										37	32	30	32	26	26	30	27	24	27	22	23
11											34	32	34	32	33	34	33	33	35	34	35
12												38	35	39	41	36	40	42	40	46	46
13													42	41	42	39	42	46	44	51	51
14														59	58	43	54	65	55	77	73
15															66	45	58	70	59	83	80
16																43	45	50	47	55	56
17																	58	66	57	76	76
18																		88	69	100	99
19																			65	80	81
20																				123	116
21																					120

Received : 20.04.1981
Accepted : 20.06.1981