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# DETERMINATION OF THE ANGULAR VALUE OF A GRADUATION INTERVAL OF HANGING LEVEL TUBE ON A TRANSIT INSTRUMENT BY OBSERVATIONS OF STARS

## I

The method of determining the angular value of a graduation interval of level tube by means of a level trier gives reliable results in the case only when the trier is carefully examined, the influence of temperature defined and faults of the screw, which should be of perfect construction, eliminated. Moreover, the position of level tube, its loading and fastening or suspending should be the same during whole examination as in the time of observations when it is used to define the axis inclination of a transit instrument. All these difficulties fall off when a more direct method is used for the level tube examination, consisting in the determination of the value of intervals from star observations during their transit across the meridian of a place.

## II

The suggested method consists in observation of passage of stars across the meridian of a place, very near of zenith point, the inclination of horizontal axis of the instrument being changed during observations by means of a setting screw, position of the bubble read and transit moments of a star registered for both tilts of the axis.

## III

In spite of imperceptible changes in azimuth which occur as a result of the revolutions of setting screw - the program of star observations shall be limited to the stars, whose azimuth coefficient of the Mayer's formula

$$K = \frac{\sin(\varphi - \delta)}{\cos \delta}$$

does not exceed $0^s,1$.

## IV

Investigations of the level tube from the star observations were performed with a Zeiss transit instrument with an object lens of 100 mm diameter and 1000 mm focal length, a photoelectric contrivance for transit observations and an electronic chronograph of Belin, registering the electronic impulses. The movable vertical hair of the impersonal micrometer (or a fixed reticule) is replaced by a diaphragm, which has 15 slots, symmetric to the optical centre of the telescope.

The passage of each star is observed in four groups :

for first telescope tilt - on the eastern side with N° 5 to 8 slots,

for second telescope tilt - on the eastern side with N° 11 to 14 slots and on the western side with N° 2 to 5 slots,

for third telescope tilt - on the farther western side - with N° 8 to 11 slots.

A following order of observations was composed :

1. By revolving the setting screw placed in the direction of level tube axis a tilt of about 5 intervals from the level tube centre is caused; the level bubble shall be of normal length - about 40 intervals.

2. Both bubble ends are read out.

3. Extreme four eastern slots are destined for a choice of appropriate strength of light which comes from the star to the photoelectric cell and which depends from the star size, spectrum type, atmosphere transparency and extinction and following four slots (N° 5 to 8) serve to record the star transit on the printing chronograph of Belin.

4. During the transit of the star in front of N° 9 and 10 slots the level tube bubble centre is moved in the opposite direction to the tilt, which was made previously, so that its centre is distant about 5 intervals from the level tube centre. This movement is made by revolving the setting screw (and reading out a scale on the screw head). The recording of star transits is continued all the time without interruptions.

5. After the star transit in front of the N° 11 to 14 slots is recorded, the registering device is put off.

6. Both bubble ends are read out.

7. The registering device is put on and western N° 2 to 5 slots are observed. The position of instrument and of level tube does not change.

8. While the star passes in front of N° 6 and 7 slots, the level bubble is moved to its initial position (see para. 1).

9. The star transit in front of N° 8 to 11 slots is recorded, then the registering device is put off and both bubble ends are read out.

10. The instrument position shall be changed before the observations of a next star will start.

Below are shown the results of transit recording of the star 1638 W/E in front of slots while the value of one interval of a suspended level tube graduation of a Zeiss transit instrument was determined; to each slot correspond two impulses : the first one signals the moment of apparition of the star in the slot, the second one - the moment of disappearance.

| | | | | | |
|---|---|---|---|---|---|
| Eastern slots | 5 | $34^m$ | $12^s$, 306 | | |
| | | 34 | 15 , 936 | | |
| Level tube readings : | 6 | 34 | 19 , 497 | | |
| | | 34 | 23 , 174 | | First group of four slots for first tilt |
| E : 6,4 W : 44,7 | 7 | 34 | 26 , 728 | | |
| | | 34 | 30 , 209 | | |
| | 8 | 34 | 33 , 881 | | |
| | | 34 | 37 , 408 | | |
| | 9 | 34 | 40 , 929 | | |
| | | 34 | 43 , 316 | | |
| | 10 | 34 | 46 , 857 | | Movement of the level bubble |
| | | 34 | 50 , 389 | | |
| | 11 | 34 | 54 , 022 | | |
| | | 34 | 57 , 644 | | |
| | 12 | 35 | 01 , 154 | | |
| E : 16,1 W : 44,4 | | 35 | 04 , 787 | | Second group of four slots for second tilt |
| | 13 | 35 | 08 , 206 | | |
| | | 35 | 11 , 914 | | |
| | 14 | 35 | 15 , 354 | | |
| | | 35 | 19 , 033 | | |
| | 15 | 35 | 22 , 483 | | |
| | | 35 | 26 , 024 | | |
| | | 35 | 39 , 843 | 1 | Western slots |
| | | 35 | 43 , 351 | | |
| | | 36 | 46 , 870 | 2 | |
| | | 36 | 50 , 516 | | |
| | | 36 | 54 , 036 | 3 | |
| E : 16,1 W : 54,4 | | 36 | 57 , 586 | | Third group of four slots for second tilt |
| | | 37 | 01 , 144 | 4 | |
| | | 37 | 04 , 811 | | |
| | | 37 | 08 , 231 | 5 | |
| | | 37 | 11 , 896 | | |
| | | 37 | 15 , 455 | 6 | |
| | | 37 | 20 , 181 | | Movement of the level bubble |
| | | 37 | 23 , 729 | 7 | |
| | | 37 | 27 , 217 | | |
| | | 37 | 30 , 753 | 8 | |
| | | 37 | 34 , 224 | | |
| | | 37 | 37 , 840 | 9 | |
| | | 37 | 41 , 409 | | |
| | | 37 | 45 , 027 | 10 | |
| E : 8,2 W : 46,6 | | 37 | 48 , 521 | | Fourth group of four slots for third tilt |
| | | 37 | 52 , 151 | 11 | |
| | | 37 | 55 , 701 | | |

## Computation of observations

Let designate : $S_1$, $S_2$, $S_3$ and $S_4$ - the mean values of the moments of star passage in front of the slots of N° 1, 2, 3 and 4 groups;

$d_E$ , $d_W$ : angular distances between the eastern groups (N° 12,5 - 6,5 slots) and the western ones (N° 9,5 - 3,5 slots ), reduced to the equator;

$\delta$ : declination of a star;

$\Delta t$ : time changes due to the tilt changes;

$\Delta p$ : movements of the level tube bubble centre due to the revolutions of the setting screw;

$I$ : tilt coefficient of the Mayer's formula :

$$I = \frac{\cos(\varphi - \delta)}{\cos \delta}$$

then we shall obtain :

$$S_2 - S_1 = d_E \sec \delta + t_E \qquad t_E = (S_2 - S_1) - d_E \sec \delta$$

$$S_4 - S_3 = d_W \sec \delta + t_W \qquad t_W = (S_4 - S_3) - d_W \sec \delta$$

and the value of one level tube interval shall be :

$$\tau = \frac{\Delta t_E}{\Delta p_E \cdot I} = \frac{\Delta t_W}{\Delta p_W \cdot I}$$

or

$$\frac{\dfrac{S_1 + S_4}{2} - \dfrac{S_2 + S_3}{2} \pm \dfrac{d_W - d_E}{2} \sec \delta}{\dfrac{\Delta p_E + \Delta p_W}{2} \cdot I}$$

We see that in course of using the observations with transporting of telescope in its sockets in midway of observations the last term in the numerator will disappear, because it shall get following value :

$$\frac{d_E - d_E}{2} \sec \delta \quad \text{or} \quad \frac{d_W - d_W}{2} \sec \delta$$

In the example showed below following values for d were admitted; they were received from a greater number of normal observations for determination of a time correction.

$$d_E = 24^s,4585$$

$$d_W = 24^s,4628$$

V

Final remarks. As can be seen from the example - precision of results obtained by this way are not worse than the precision, which can be obtained with most precise level triers; besides, the results are free from the systematic errors due to the trier constants and factors mentioned above.

Computation example : Observations from May, 13 1963

| N° | δ | sec δ | d. sec δ | d. sec i +Δt | Δt | 2. Δp | I | 2. Δp. I | t / 4. Δp. I |
|---|---|---|---|---|---|---|---|---|---|
| 483 W | +56° 09',62 | 1,7957 | 43ˢ,9201 | 42ˢ,3806 | 1ˢ,5395 | 25,2 | 1,7920 | 45,1584 | 0ˢ,01705 |
| | | | 43,9278 | 45,0714 | 1,1436 | 19,4 | | 34,7648 | 0,01645 |
| 1630 E | +49 52,57 | 1,5518 | 37,9614 | 36,7931 | 1,1683 | 22,4 | 1,5501 | 34,7222 | 0,01682 |
| | | | 37,9547 | 38,9907 | 1,0360 | 20,0 | | 31,0020 | 0,01671 |
| 1638 W | +55 10,82 | 1,7513 | 42,8342 | 41,6217 | 1,2125 | 19,4 | 1,7494 | 33,9384 | 0,01786 |
| | | | 42,8417 | 43,8170 | 0,9753 | 15,7 | | 27,4656 | 0,01775 |
| 1665 E | +53 06,45 | 1,6658 | 40,7501 | 39,6350 | 1,1151 | 18,9 | 1,6657 | 31,4817 | 0,01771 |
| | | | 40,7430 | 41,7201 | 0,9771 | 17,2 | | 28,6500 | 0,01705 |
| 509 W | +49 29,84 | 1,5397 | 37,6588 | 36,6090 | 1,0498 | 19,5 | 1,5324 | 29,8818 | 0,01757 |
| | | | 37,6654 | 38,5278 | 0,8624 | 16,3 | | 24,9781 | 0,01726 |
| 1688 E | +53 54,60 | 1,6976 | 41,5280 | 40,4101 | 1,1179 | 18,0 | 1,6971 | 30,5478 | 0,01830 |
| | | | 41,5207 | 42,5300 | 1,0093 | 16,6 | | 28,1819 | 0,01791 |
| 1700 W | +51 08,95 | 1,5942 | 38,9917 | 38,0060 | 0,9857 | 17,0 | 1,5937 | 27,0929 | 0,01819 |
| | | | 38,9986 | 39,8866 | 0,8880 | 16,0 | | 25,4992 | 0,01741 |
| 1714 E | +51 57,73 | 1,6229 | 39,7007 | 38,6598 | 1,0409 | 17,7 | 1,6228 | 28,7236 | 0,01812 |
| | | | 39,6937 | 40,6887 | 0,9950 | 18,3 | | 29,6972 | 0,01675 |
| 531 W | +52 01,27 | 1,6251 | 39,7475 | 38,6210 | 1,1265 | 19,6 | 1,6250 | 31,8500 | 0,01768 |
| | | | 39,7545 | 40,6851 | 0,9306 | 16,7 | | 27,1375 | 0,01715 |
| 1750 E | +49 31,73 | 1,5407 | 37,6898 | 36,6660 | 1,0238 | 19,2 | 1,5387 | 29,5430 | 0,01733 |
| | | | 37,6832 | 38,7311 | 1,0479 | 20,2 | | 31,0817 | 0,01686 |

Mean result $\frac{1}{4}\tau^{s} = 0^{s},01740 \pm 0^{s},00012$