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PREDICTION OF POLAR MOTION

Abstract

Based on an analysis of polar motion behavior, we found the possibility of predicting polar motion up to one year in advance. Comparing these predicted polar coordinates with the observed ones (smoothed), the rms of the differences is about 0["][02. The differences of the relative polar motion are much smaller. For any time *interval of 20 - 30 days throughout the whole year, the rms of the relative polar motion differences is about* $0''.01$ *. It appears that 80 - 90 % of the polar motion is composed of the stable, predictable Chandler and annual terms.*

1. The Behavior of Polar Motion and the Model for its Prediction

The most important components of polar motion are the Chandler and annual terms. Polar motion may have other components of a periodic (e.g., semiannual), secular or random nature. The Chandler and annual terms themselves may vary with time, particularly their amplitudes and phases and possibly their periods too. Nevertheless, if the Chandler and annual terms contribute to a large percentage of the total polar motion, and if they do not change rapidly with time, polar motion prediction is feasible since these terms can be modeled quite well. Acquiring a better procedure to make polar motion predictions, assessing their accuracy, and at the same time finding the quantitative contribution of the Chandler and annual terms are the purposes of this investigation.

Markowitz [1976, 1979] pointed out the resemblance between the 1969--1976 IPMS polar motion and a theoretical curve $(TC-2)$. We try to extend his idea in order to predict polar motion for practical applications.

1.1. Main Components of Polar Motion

1.1.1. Chandler motion

We extend the idea of Oesterwinter [1979] to determine the period of the Chandler motion, P_c , using longer time intervals of data Our estimate is $P_{C} = 1.187 \pm 0.001$ yr (433.5 days). Taking P_{C} as constant, we estimated the amplitudes and phases of the Chandler, annual and semiannual terms using ILS, IPMS, BIH and DMA 0.05-year data. The ILS, IPMS and BIH data sets used are from 1962-1979, and the DMA set from 1972-1979. The data was taken from the BIH and IPMS Annual Reports.

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The parameters of the Chandler motion are listed in *Table 1* in which A is the amplitude, ϕ the phase, ϵ the flattening of the Chandler ellipse, λ_0 is the (eastward) direction of the major axis of the Chandler ellipse, and σ is the standard deviation of the estimated parameter. In computing the phase, the adapted epoch is $t_0 = 1962.0$. To compute λ_0 , we used the following equation :

$$
\sin 2\lambda_0 = \frac{2\sin(\phi_x - \phi_y)(A_x \cdot A_y)}{A_x^2 + A_y^2}
$$
 (1)

Parameters of Chandler Motion

*** --** ILS is not included in the mean. Since DMA data carry a heavy weight in the BIH solutions, the mean values are somewhat overweighted in favor **of the** DMA data.

Larmor [1896] pointed out that the pole tide imposes a slight ellipticity on the path described by the pole of rotation. From theoretical consideration he calculated a value of 0.017 for the flattening ϵ , with the major axis pointing toward east longitude, $\lambda_0 = 6^\circ$. From the analysis of the unsmoothed latitude data 1899–1954, the flattening was estimated as $\epsilon = 0.01 \pm 0.05$ [Munk and MacDonald, 1960]. Our estimates are $\epsilon = 0.01 \pm 0.01$, $\lambda_0 = 3^\circ \pm 1^\circ$. Although the precision of the results is much improved, we still cannot identify the flattening of the Chandlerian motion with great certainty.

1.1.2. Annual motion

The parameters of annual motion are listed in *Table 2,* with the same notations and units as in *Table I.* From the table we can see that different polar motion services have significantly different estimates for amplitude and phase. The ILS has much larger systematic errors, its amplitude being about $20~\%$ smaller than the others.

Nevertheless, annual polar motion is elliptical without doubt. Our flattening estimate is 0.14 ± 0.014 . The major axis of the annual ellipse does not exactly lie on the x axis, but points to $\lambda_0 = 3^\circ \pm 1^\circ$, which nearly coincides with that of the

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Chandler. In any case, we can approximate the major axis of the annual ellipse with the x axis.

Table 2

Parameters of Annual Polar Motion *

* -- Here for the convenience of comparing it with the other results, the annual motion is described as $x = A \cos(2\pi t + \phi)$. Later in eq. (2) we use $x = A \sin(2\pi t + \phi)$, so the phase may differ by 90 ~ .

o _ ILS is not included in **the mean**

1.1.3. Semiannual term

The parameters of the semiannual term may be found in *Table 3.* Such a term might exist in polar motion. If it exists, the amplitude must be smaller than $0''\hspace{-1pt}.01$; thus with the observation accuracies now available, it can hardly be determined.

Table 3

1.2. Model for Polar Motion Prediction

From the above it is clear that we can neglect the semiannual term in the model for polar motion prediction but must take the annual motion as an ellipse. The data also shows a linear trend, therefore it is better to include it in the model. The final model we use for prediction is

$$
x = x_0 + K_x(t - t_0) + b_x \sin 2\pi (t - t_0) + c_x \cos 2\pi (t - t_0) +
$$

+
$$
B_x \sin \frac{2\pi (t - t_0)}{P_C} + C_x \cos \frac{2\pi (t - t_0)}{P_C}
$$

$$
y = y_0 + K_y(t - t_0) + b_y \cos 2\pi (t - t_0) - c_y \sin 2\pi (t - t_0) +
$$

+
$$
B_y \cos \frac{2\pi (t - t_0)}{P_C} - C_y \sin \frac{2\pi (t - t_0)}{P_C}
$$
 (2)

in which P_C is a constant (1.187 yr). The parameters to be estimated are K_x , b_x , $c_x, K_y, b_y, c_y, B_x, C_x, B_y$ and C_y . We use six years of polar coordinates to estimate these parameters, then use the parameters to calculate the following year's predicted polar coordinates. Other time spans of data were also tried ; a six- to sevenyear data span seems to be the best.

2. The Results of Prediction

2.1. Prediction of the BIH Polar Coordinates

The prediction errors (rms of prediction minus observed, from BIH Circular D) are given in *Table 4.* Data used for the prediction are from the BIH Annual Reports given at 0.05-year intervals, but in practice one could use five years of Annual Report data plus Circular D data for the last year or so. The estimated parameters of the model (in eq. (2)) are also given in **Table 4.** Because of the change of epoch the parameters of Chandler motion, B and C , change from year to year. For comparison, we give the amplitudes of the estimated Chandler motion in the last column. The overall averages of the prediction errors are $\sigma_{0.05-\text{yr}} = 0''.020$, $\sigma_{5-\text{day}} = 0''.022$.

2.2. Prediction of the IPLS and DMA Polar Coordinates

The results when using IPMS 0.05-year smoothed data (provided by the IPMS Annual Reports) and using DMA 0.05-year smoothed data (provided by the BIH Annual Reports) are given in *Table 5*. Only the annual mean values of σ_x and σ_y are given. For comparison, we also list the BIH's σ in the last column. The average prediction errors for IPMS and DMA are also about 0'.02.

2.3. Relative Polar Motion

In some cases we are interested not in the polar coordinate itself but in the change of polar coordinates over a certain time interval. We call this the relative polar motion within the time interval. Using previously predicted polar coordinates, we form the relative polar motion of any given time interval and compare it with the Circular D

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Table 4

Prediction of the BIH Polar Coordinates (unit 0['].001)

 $* - \sigma_{0.05-\gamma r}$ is estimated from (pred. -0.05 yr smoothed normal value)

** $-\sigma$ _{5-day} is estimated from (pred. -5-day Circular D value)

Table 5

Prediction Errors for IPMS and DMA (unit 0".001)

- * Used DMA bi-daily solutions for making prediction and compared with BIH Circular D.
- **- Used BIH global solution for prediction and compared with Circular D.

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polar motion difference during the same interval. The rms results are presented in *Table* 6.

Table 6

Prediction Errors for Relative Polar Motion (0'.001)

The polar motion prediction error is plotted in *Fig. 1.* For the time interval of 20-30 days, the relative polar motion prediction error is only about $0''.01$ (30 cm), which for any present application is more than sufficient.

Since the sigma of predicted polar motion change within five days is $0''$.0023, the average (smoothed) polar motion rate per day could be predicted with the $\sigma = 0''.0005$.

2.4. Comparison with VLBI Observations

In *Table* 7we list the DSN VLBI polar motion data SOLV 71809 F provided by Fanselow [private communication, 1981] as well as the predicted polar coordinates of the same date. Predictions are the same as in Section 2.1 For comparison, we also give the

Fig. 1 - Polar motion prediction error. (time =0 denotes the time of last data available).

Table 7

Pole Coordinates of DSN VLBI, Prediction and BIH (0["].001)

 $* - AH$ the x coordinates of the BIH are in the BIH 1979 system.

BIH Circular D polar coordinates (interpolated to the same day). We computed the rms of (VLBI - BIH), of (VLBI - pred.), and of $(BIH - pred.)$. The results are $0''.009$, 0".013 and 0".013 respectively.

The prediction errors of relative polar motion (for all time intervals of less than two months) are

relative $(VLBI - pred.) = 0''.008$ relative $(VLBI-BIH) = 0''.011$ relative $(BIH - pred.) = 0''.009$

It appears that both VLBI and BIH data agree better with the predictions than with each other.

2.5. Comparison of Different Methods of Prediction

The main difference in our method with respect to the BIIl rapid service or that of the U.S. Naval Observatory is in the length of the data used. Using long time interval data, the observing errors will be greatly reduced, but the parameters are not instantaneous- they are mean values over the long time intervals. On the other hand, using short time interval data, the estimates are nearly instantaneous, but they are much more influenced by observing errors. Using only $3-6$ months: data, sometimes the estimated Chandler (or annual) amplitudes were less than $0''$.02 which is unreasonably small. This implies that the estimates must not be the real instantaneous parameters, but the contaminated ones, which means that the observing error is more serious than the real change in the Chandler and annual parameters. Therefore, we propose to use longer intervals of data. Other methods using a short time span of data still can make fairly good predictions for short time periods although the estimated parameters seem to be meaningless. The reason is the correlation compensation. Once the prediction time is extended, the correlation is decreased, and the prediction error increases quickly.

2.6. Prediction for Several Years in Advance

Tables 8 and 9 list the two-year prediction error. Comparing *Table~5* with 8 and 6 with 9, we see that they are nearly the same as the one-year prediction.

Table 8

Two--year Prediction Errors for IPMS and BIH (0".001)

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Fig. 2 gives the long-term (four-year) prediction results. The data used are BIH polar motion from 1972.0 to 1977.95 ; the predictions are extended from 1978.0 to

Fig. 2 - Long-term (4-yr) predicted pole path versus BIH pole path (1978.0 - 1981.9).

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1981.9. Compare the predicted pole path with the BIH results. The BIH pole path $78.0 -$ 81.0 is from BIH "Annual Report for 1980"; the path from 81.0 to 8].6 is drawn according to BIH Circular D data. From the curve we see that the predicted polar motion generally follows the trend of the observed polar motion quite well. Although the real polar motion path might have some complicated changeable details, the main trend is regular and predictable.

3. Error Analysis

Differentiating equation (2), and averaging over a year, we obtain

$$
\sigma_{x}^{2} \approx \sigma_{x_{0}}^{2} + \sigma_{x_{K}}^{2} \overline{(t-t_{0})^{2}} + \frac{1}{2} (\sigma_{b_{x}}^{2} + \sigma_{c_{x}}^{2} + \sigma_{B_{x}}^{2} + \sigma_{c_{x}}^{2}) + \sigma_{\delta_{x}}^{2}
$$
\n
$$
\sigma_{y}^{2} \approx \sigma_{y_{0}}^{2} + \sigma_{y_{K}}^{2} \overline{(t-t_{0})^{2}} + \frac{1}{2} (\sigma_{b_{y}}^{2} + \sigma_{c_{y}}^{2} + \sigma_{B_{y}}^{2} + \sigma_{c_{y}}^{2}) + \sigma_{\delta_{y}}^{2}
$$
\n(3)

Each of the above coefficients may contain modeling errors as well as observing error ingredients. The error in P_C is included in δx and δy ; these two terms may also include other modeling errors, random errors, etc. We try to estimate how much might be ascribed to modeling errors and how much to observing errors.

3.1. Systematic Errors in the BIB and IPMS Results

It is known that the BIH 1968 system has systematic errors, mostly in x (and $UT1 - UTC$) (see BIH Annual Report for 1979, p. D-77, and [Feissel, 1980 ; Robertson et al., 1980]). We refer to these as the stable part of systematic errors. Besides this, there remains a changing part. Writing the difference of $IPMS - BIH$ results in the form

difference = A + B sin
$$
2\pi
$$
 T + C cos 2π T + D sin 4π T + E cos 4π T (4)

Table 10 is extracted from BIH Annual Report for 1979, page D-98. From this table we can see that the coefficients of A, B, C, D, E not only have a non-zero mean value (stable part), but also change from year to year, To the extent that stable systematic errors of the BIH are modelable by eq. (2), they will be contained in the prediction. The changing part of the A , B , C , D , E coefficients will effect the prediction error. The sigma values of these changing parts are given at the bottom of *Table 10.*

Since all these values are calculated from (IPMS - BIH), σ_A , σ_B , etc. only reflect the observing systematic error, σ_{A} corresponds to σ_{x_0} (or σ_{y_0}) in equation (3) ; similarly $\sigma_{\bf B}$ corresponds to $\sigma_{\bf b}$ (or $\sigma_{\bf b}$), $\sigma_{\bf C}$ to $\sigma_{\bf c}$ (or $\sigma_{\bf c_{\bf y}}$) . $\sigma_{\bf D}$, $\sigma_{\bf E}$ will go into $\sigma_{\delta_\mathbf{x}}$ (or $\sigma_{\delta_\mathbf{y}}$). Suppose that $\sigma_\mathbf{A}$, etc, are half due to IPMS and half due to BIH, then by simple calculation we can see that these changing systematic errors above will cause prediction errors of $0''.009-0''.013$. Including the random error (see [Capitaine and Feissel, 1974]), the prediction error might reach about 0".015 for the BIH and IPMS.

3.2. Systematic Errors in DMA Polar Motion

The polar motion determined by DMA is more precise than by optical instruments. Nevertheless, Doppler polar motion also suffers from systematic errors as

Table 10

Comparison to the BIH Global Solution (IPMS - BIH)

can be seen below by comparing polar motion results determined from different satellites.

In *Fig. 3* we plot the differences of Satellites 34-67, 92-67, and 81-67. In 1980, the differences between Satellite 92 and 67 have approximately the same feature as in 1979, especially the y component. For comparison we put the two--year differences together in *Fig. 4* (the scale is different from that in *Fig. 3).* The systematic feature is prominent. According to [Anderle, 1981], the bias between Satellites 92 and 67 may be caused by the modeling error (mostly the gravity field model).

Over the years, DMA has changed satellites several times, and the A, B, C, D , E coefficients have changed from satellite to satellite. This situation would cause significant prediction error. This might be the reason why the prediction using DMA data did not show much improvement over that using BIH data.

The systematic error between different satellite implies that every new technique might in some way suffer from unexpected systematic errors. Before we use any new technique for constructing a polar motion service, we must have certain time intervals of simultaneous observations and must analyze them carefully for possible systematic errors.

Fig. 3 - Polar motion differences between different Transit satellites.

Fig. 4 - Polar motion differences between Satellites 92 and 67.

3.3. Modeling Error

Fig. 5 shows the prediction error of BIH, IPMS and DMA in 1979. Though the BIH and DMA data are not independent, the similarity of the three curves still reflects the influence of modeling errors. In the common part, we find an annual term with a semi-amplitude of $0''015$. Because the time interval of the above calculations is only one year, we could not determine whether the modeling error is due to the Chandler motion change or annual change, or both.

The modeling error depends on the excitation and dissipation of the polar motion so it varies with time. Since the overall prediction error is about 0".02 and the part produced by the observing error is about $0''.015$, the average modeling error in these years is also about $0''015$. Comparing the polar motion amplitude with the modeling error, we conclude that $80 - 90 %$ of polar motion is composed of the stable (at least stable within several years) Chandler and annual motions, and so it is predictable.

3.4. The Problem of ILS Data

As in Section 3.1, we used the BIH Annual Report for 1979 (page D-97) to get the changing part of the systematic error in $(BIH - ILS)$. The results are listed in Table 11. In this case we think the systematic difference is mostly due to the ILS. The systematic errors together with the 1 m random error (see [Capitaine and Feissel, 1974]) may cause a $0''.04-0''.05$ prediction error. In several instances there occur large variations in the coefficients. Take the x coordinates of 1976 and 1979 as an example :

Fig. 5 - (Predicted - observed) in 1979.

$$
B_{x_{1979}} = -0''.098, \qquad B_{x_{1976}} = 0''.072, \qquad \Delta B_{x_{1976-79}} = 0''.170
$$

$$
C_{x_{1979}} = -0''.037, \qquad C_{x_{1976}} = 0''.063, \qquad \Delta C_{x_{1976-79}} = 0''.100
$$

$$
\sqrt{\Delta B_x^2 + \Delta C_x^2} \approx 0''.2
$$

The changing part of the systematic error itself is an annual term with a semi-amplitude of $0''\!\!.2$.

Units : 0.001 arcsecond X and Y A B C D E A B C D E Ω 22 34 22 11 10 12 17 14 9 8

Comparison to the BIH Global Solution (ILS - BIH)

We used $1962 - 1979$ ILS data to make predictions (during this time period polar motion was supposed to be stable); the prediction error is $0''.05$. Using 1949 $-$ 1962 [LS data (polar motion parameters were supposed to vary rapidly) we also get $0''.05$. This implies that the poor prediction accuracy of the ILS may be due mostly to observing errors in the ILS. The same situation exists for BIH predictions. In *Table* 5we have listed only the results using the data from 1968. Using the BIH 1962-1968 data, the rms increases to $0''.03$ because of the relatively poor quality at that time. Therefore, we assume that in the past if high quality observations had been provided, high accuracy predictions could also have been made.

4. Conclusion

Based on the analysis of the polar motion behavior, we found the possibility of predicting polar motion about one year in advance. Comparing these predicted polar coordinates with the smoothed observed ones, the rms of the differences is about $0''\ll0$. This accuracy is better than that of the other now available methods. It is sufficient for some practical applications where real time polar motion data is needed, such as the polar motion correction for field, astronomy (geodetic) observations. In high accuracy techniques such as VLBI and laser ranging, this prediction can be used When real time approximate polar motion values are needed. Also from the model, the time derivatives of polar motion can easily be obtained if needed in VLBI and satellite laser ranging data analyses.

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