Technical note

Monte Carlo simulation of a planar shoulder model

R. E. Hughes **K.-N. An**

Biomechanics Laboratory, Division of Orthopedic Research, 128 Guggenheim, Mayo Clinic and Mayo Foundation, Rochester, MN 55905 USA

Abstract—*Although variability of anthropometric measures within a population is a well established phenomenon, most biomechanical models are based on average parameter values. For example, optimisation models for predicting muscle forces from net joint reaction moments typically use average muscle moment arms. However, understanding the distribution of musculoskeletal morbidity within a population requires information about the variation of tissue loads within the population. This study investigated the use of Monte Carlo simulation techniques to predict the statistical distribution of deltoid and rotator cuff muscle forces during static arm elevation. Muscle moment arms were modelled either as independent random variables or jointly distributed random variables. Moment arm data was collected on 22 cadaver specimens. The results demonstrated the use of Monte Carlo techniques to describe the statistical distribution of muscle forces. Although assuming statistically independent moment arms did affect the statistical distribution shape, that assumption did not affect the median predicted forces. The standard deviations of muscle forces predicted using Monte Carlo techniques were similar to the standard deviation of muscle force predictions using the whole sample of specimens. It is concluded that Monte Carlo simulation techniques are a useful tool to analyse the interindividual variability of rotator cuff muscle forces.*

Keywords—*Variability, Shoulder, Rotator cuff, Modelling, Monte Carlo, Biomechanics, Muscle force, Computer simulation*

Med. Biol. Eng. Comput., 1997, **35**, 544–548

1 Introduction

ESTIMATES OF mechanical loads on muscle, tendons, ligaments, cartilage and bone are necessary for understanding the aetiology of musculoskeletal diseases and developing improved medical devices and treatments for upper extremity disorders. Many models have been developed for predicting internal joint loads from external forces acting on the body. These models are based on simple mechanical analyses of the body segments of interest, including the external forces acting on the body (contact forces and gravitational forces) and internal forces generated by active elements (muscles) and sometimes passive ones (ligaments and cartilage contact). Critical inputs to the models include parameters describing muscle physiology (physiological cross-sectional area) and musculoskeletal geometry (moment arms). Detailed models of upper extremity joints are typically statically indeterminate, and numerical optimisation is a popular method of selecting a single set of muscle forces for a given external loading condition (HÖGFORS *et al.*, 1995; VAN DER HELM, 1994; AN *et al.*, 1984*a*, 1989; BROOK *et al.*, 1995; COLLINS, 1995; CHOLEWICKI *et al.*, 1995; HERZOG, 1996). It is generally said that these models solve the 'general distribution problem' of partitioning net joint moment between individual muscles. These models use single estimates of parameters—typically the sample mean—in their computations.

However, there is great variability in anthropometric measures such as height and weight within the general population (SANDERS and MCCORMICK, 1987). Geometric and physiological parameters used in muscle force prediction models also vary widely within the population. For example, BASSETT *et al.* (1990) reported that the standard deviation of supraspinatus muscle moment arm measurements was 32% of the mean. Since moment arm magnitudes are critical to muscle force predictions, variability in moment arms leads to variability in muscle force estimates.

Knowing the distribution of muscle forces within a population is critical in understanding the range of biomechanical and biological responses to external loading of the body. Orthopaedic surgeons, for example, want to know why some total shoulder arthroplasties fail sooner in some people than in others. In terms of prevention, it is unclear why some people develop rotator cuff tendinitis while performing a physically demanding job and other people, who carry out exactly the same job, do not. It is unlikely that answers to these questions will come from analyses of average model parameter data: stochastic biomechanical models will be needed to investigate the well known variability in musculoskeletal morbidity within a population.

One approach to analysing the effect of variability on model predictions is to solve the biomechanical model for an entire

First received 6 September 1995 and final form 19 March 1997
Correspondence should be addressed to Kai-Nan An

sample of model parameters. This can be done when model parameters have been measured on a sample of cadaver specimens or live subjects and this data available to the modeller. However, often only summary statistics (typically, mean and standard deviation) are available when formulating a biomechanical model. Therefore, there is a need for a method to simulate the effect of population variability on model results when only descriptive statistics of model parameters are available in the literature. Monte Carlo methods (RUBENSTEIN, 1981) have been used to simulate stochastic processes in many fields, and would seem to be a logical method for analysing biomechanical models having random parameters. However, Monte Carlo techniques have been limited to modelling electrophysiological signals (MIRKA and MARRAS, 1993) in force prediction models.

The purpose of this study was to evaluate the use of Monte Carlo methods to model muscle moment arms as random variables in a planar analysis of the glenohumeral joint. The glenohumeral joint was selected because of the significant morbidity associated with rotator cuff pathology. A secondary objective was to compare the effect of modelling the muscle moment arms as multivariate random variables to that of modelling each as an independent univariate normal random variable.

2 Methods

2.1 *Mechanical model*

A static planar model of the shoulder was selected as the basis for analysis. Fig. 1 shows a free body diagram of the planar analysis of an arm elevated 60° in the plane of the scapula from the neutral hanging position. Gravity acts on the mass of the arm to generate a clockwise (adduction) moment. The arm is elevated by six muscles (anterior deltoid, middle deltoid, posterior deltoid, infraspinatus, subscapularis, and supraspinatus) in the model. The mechanical functions of these muscles are described by their elevation moment arms.

Let $\boldsymbol{R}$ be a six-dimensional vector containing the elevation moment arms of the six muscles, $\boldsymbol{R}=(R_1 \ldots R_6)$, and $\boldsymbol{F}$ be a corresponding six-dimensional vector of muscle force magnitudes, $\boldsymbol{F}=(F_1 \ldots F_6)$. Note that the actual force vector acting along the muscle's line of action is f_i, so that $F_i=|f_i|$. Then

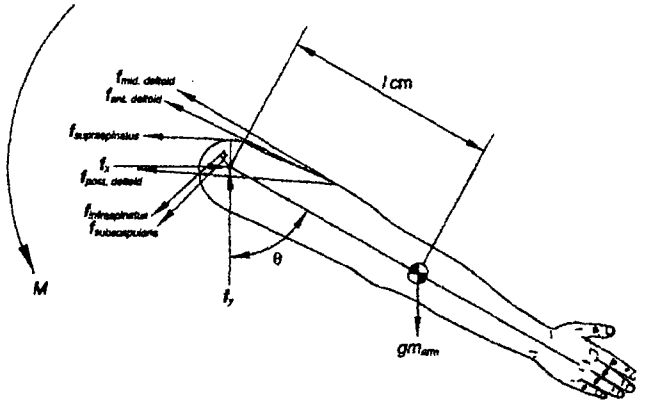

Fig. 1 *Free body diagram of the arm in an elevated posture. Θ is the angle from vertical, m_{arm} is the mass of the arm, l_{cm} is the distance from the glenohumeral joint center to the center of mass of the arm, g is the acceleration due to gravity (9·8 msec^{-2}), f_x and f_y are the intersegmental reaction forces, M is the net intersegmental moment, and f_i is the force vector of muscle i (note $F_i=|f_i|$).*

the moment equilibrium condition required to maintain moment equilibrium is

$$m_{arm}gl_{cm}\sin(\Theta)+\sum_{i=1}^{n}F_iR_i=0 \tag{1}$$

where g is the acceleration of gravity, m_{arm} is the mass of the arm, l_{cm}, is the distance from the glenohumeral joint to the centre of mass of the arm, and Θ is the elevation angle (from the vertical). Note that all muscle magnitudes must be non-negative, so the following inequalities are added to the model:

$$F_i \geq 0 \tag{2}$$

The mechanical system is statically indeterminate if $n>1$, e.g. there are an infinite number of muscle force combinations that satisfy eqns. 1 and 2 if there is more than one muscle. A standard method for rationally selecting one set of muscle forces is to assume that the central nervous system activates muscles to optimise some well-defined criterion function, $\Phi(\boldsymbol{F})$. A commonly used criterion for the shoulder (KARLSSON and PETERSON, 1992; VAN DER HELM, 1994) is the sum of squared muscles stresses, e.g.

$$\text{minimise } \Phi(\boldsymbol{F})=\sum_{i=1}^{n}\left(\frac{F_i}{a_i}\right)^2 \tag{3}$$

where a_i is the physiological cross-sectional area of the ith muscle. Then numerical optimisation routines can be used to solve the mathematical programming problem of choosing muscle force magnitudes to minimise eqn. 3 subject to the constraints of eqns. 1 and 2.

The anthropometry used in this study was that of a 63 kg person of 167 cm stature. Body segment masses and locations were scaled from total body mass and stature (CHAFFIN and ANDERSSON, 1984) to determine that the total shoulder adduction moment generated by the mass of the arm, $m_{arm}gl_{cm}\sin(\Theta)$, was 8.1 Nm at an arm elevation angle of 60°.

The results presented here are based on moment arm data collected from two studies that used the same protocol for measuring elevation moment arms of shoulder muscles (KUECHLE, 1994; LIU *et al.*, 1997). Combined, moment arms from 22 specimens were measured using the principle of virtual work (AN *et al.*, 1984*b*), where the tendon excursion-joint angle relationship was differentiated to obtain the moment arm about the instantaneous centre of rotation throughout the range of motion. Moment arms were selected for a glenohumeral angle of 40°, which corresponds to an arm elevation angle of 60° (MORREY and AN, 1990). Physiological cross-sectional areas of the subscapularis, infraspinatus, supraspinatus, anterior deltoid, middle deltoid, and posterior deltoid muscles were 13.51, 9.51, 5.21, 7.34, 9.63, and 8.92 cm^2, respectively (VEEGER *et al.*, 1991; KARLSSON and PETERSON, 1992). Table 1 summarises the moment arm values used. Table 2 gives the covariance matrix of the moment arms.

2.2 *Monte Carlo simulation*

In reality, muscle moment arms are typically measured on a collection of cadaveric specimens or medical images (CT and MRI scans), and the problem to be addressed in this paper is how to model the effect of the natural variability of these parameters on muscle force predictions. If there are measurements of moment arms on n human subjects are available, denote the vector of moment arms for the kth specimen as $\boldsymbol{R}^k=(R_1^k \ldots R_6^k)$. Likewise, denote the corresponding vector of muscle force magnitudes as $\boldsymbol{F}^k=(F_1^k \ldots F_6^k)$. Consider four methods of computing muscle forces from the collection of moment arms $\boldsymbol{R}^1 \ldots \boldsymbol{R}^n$ (for this data set, $n=22$):

Table 1 Means and standard deviations of muscle moment arms

Muscle	Mean, cm	Standard deviation, cm
Subscapularis	−0.32	1.25
Infraspinatus	−0.58	0.78
Supraspinatus	−2.07	1.07
Ant. deltoid	−1.66	0.77
Mid. deltoid	−2.46	0.56
Post. deltoid	0.97	1.38

Negative values represent abduction function; positive represent adduction. These values were computed from 22 shoulders harvested from cadavers obtained through anatomic donation programs in the upper midwest of the United States (mean age at death was 63, range 33–89)

Method 1: The 'average moment arm' method consists of two steps: (*a*) compute a vector of average moment arms from the n specimens, $\bar{\boldsymbol{R}} = (\Sigma \boldsymbol{R}^k)/n$ and (*b*) solve the muscle force prediction model of minimising eqns. 3 subject to eqns. 1 and 2 using $\bar{\boldsymbol{R}}$. This is the traditional method of solving the muscle force prediction problem using optimisation techniques.

Method 2: The 'whole sample' method consists of two steps: (*a*) solve the muscle force prediction model of minimising eqn. 3 subject to eqns. 1 and 2 for each $\boldsymbol{R}^1 \ldots \boldsymbol{R}^n$ separately and (*b*) average the resulting force vectors $\bar{\boldsymbol{F}} = (\Sigma \boldsymbol{F}^k)/n$.

Often complete data on a whole sample is unavailable to modellers; published reports of muscle moment arms provide means and standard deviations rather than tables of measurements on each specimen. Therefore, a method for estimating muscle force distributions using only summary information would be more practical than method 2.

Method 3: The 'univariate Monte Carlo' method has five steps: (*a*) compute the mean, $\bar{\boldsymbol{R}} = (\Sigma \boldsymbol{R}^k)/n$, and standard deviation, $\boldsymbol{S} = \Sigma(\boldsymbol{R}^k - \bar{\boldsymbol{R}})^2/(n-1)$, of muscle moment arms; (*b*) model the moment arm of each of the six muscles as a normally distributed random variable, e.g. $U_i \sim N(\bar{R}_i, S_i)$ (*c*) sample m samples from the normal distributions using a random number generator (JOHNSON, 1987) to obtain $(U_1^1 \ldots U_6^1) \ldots (U_1^m \ldots U_6^m)$; (*d*) solve the muscle force prediction model of minimising eqn. 3 subject to 1 and 2 for each vector $(U_1^i \ldots U_6^i)$ by substituting for $\boldsymbol{R}$; and (*e*) compute the mean and statistical distribution of the resulting muscle force predictions, $\boldsymbol{F}^1 \ldots \boldsymbol{F}^m$.

Method 3 assumes that moment arms of the muscles are uncorrelated. However, it is intuitively reasonable that specimens having a larger moment arm of one muscle may tend to have larger moment arms for other muscles. Thus, associations between model parameters should be included in the model.

Method 4: The 'multivariate Monte Carlo' method has five steps: (*a*) compute the means and covariance matrix of muscle moment arms; (*b*) jointly model the moment arms of the six-muscles as a six dimensional random vector, $(U_1^i \ldots U_6^i)$, that has a multivariate normal distribution described by the mean and covariance matrix determined in the first step; (*c*) sample m samples from the multivariate normal distribution using a random number generator (JOHNSON, 1987) to obtain $(U_1^1 \ldots U_6^1) \ldots (U_1^m \ldots U_6^m)$; (*d*) solve the muscle force prediction model of minimising eqn. 3 subject to eqns. 1 and 2 for each $(U_1^i \ldots U_6^i)$ by substituting for $\boldsymbol{R}$; and (*e*) compute the mean and statistical distribution of the resulting muscle force predictions, $\boldsymbol{F}^1 \ldots \boldsymbol{F}^m$.

The four solution methods were implemented in FORTRAN using the Numerical Algorithms Group software* for generating the random variables and solving the muscle force prediction model. Procedures E04NAF, G05DDF, and GOSEZF were used to solve the muscle force prediction optimisation model, sample from univariate normal distributions for Method 3, and sample from multivariate normal distributions for Method 4, respectively. Computations were performed on a VAX 8800 computer. Simulation of methods 3 and 4 used 1000 random vectors of muscle moment arms.

2.3 *Statistical analysis*

Wilcoxon signed rank non-parametric tests were used to determine whether the median predicted forces differed between models 2 to 4, because the predicted forces were not normally distributed. Chi-square goodness-of-fit tests were used to test the hypothesis that the cumulative distributions of the predictions from methods 2–4 were different. The number of degrees of freedom were reduced to account for tests of the means and variances. The muscle forces predicted by method I were compared to the 5th and 95th percentiles of the distributions predicted by methods 2 to 4. Statistical analyses were performed using SAS†.

3 Results

The primary advantage of methods 2–4 over the traditional method 1 is that they provide information about the statistical distribution of predicted muscle tensions. Fig. 2 shows the relative cumulative frequency distributions for methods 2–4 (the ordinate of the cumulative frequency distribution is the fraction of predicted forces that are less than or equal to each value on the abscissa). The infraspinatus, subscapularis, and posterior deltoid muscles all had a substantial cumulative relative frequency at 0 N of force, which indicates that those muscles were predicted to be inactive in many of the samples. All three of these muscles were anterior or posterior to the glenohumeral joint, and their moment arms were distributed over both abduction and adduction functions. The predictions of no activity in these muscles corresponded to samples in which the muscles had adduction moment arms, e.g. acting as antagonists to arm elevation.

The Chi-squared goodness-of-fit, tests indicated that the statistical distributions of forces determined by methods 3 and 4 were different at the $p < 0.05$ level of significance for all six muscles. Although the distributions differed between methods,

*Nag Inc., Downers Grove, Illinois, USA

†SAS Institute Inc., Cary, North Carolina, USA

Table 2 Covariance matrix of muscle moment arms used in method 4. The covariance matrix describes the associations of muscle moment arm magnitudes between muscles, and is necessary for specifying a multivariate normal distribution

	Subscapularis	Infraspinatus	Supraspinatus	Ant. deltoid	Mid. deltoid	Post. deltoid
Subscapularis	1.55	−0.03	0.24	0.04	0.09	−0.64
Infraspinatus	−0.03	0.62	0.53	−0.19	0.09	−0.25
Supraspinatus	0.24	0.53	1.14	−0.55	0.21	−0.98
Ant. deltoid	0.04	−0.19	−0.55	0.59	−0.21	0.57
Mid. deltoid	0.09	0.08	0.21	−0.21	0.32	−0.12
Post. deltoid	−0.64	−0.25	−0.98	0.57	−0.12	1.92

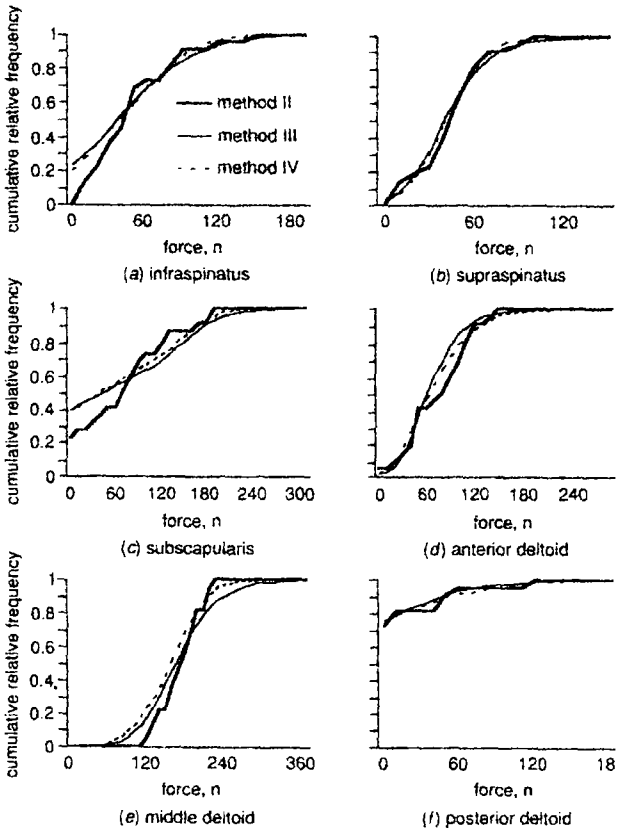

Fig. 2 *Cumulative relative frequency distributions of the predicted muscle forces for the six muscles in the model: (a) infraspinatus, (b) supraspinatus, (c) subscapularis, (d) anterior deltoid, (e) middle deltoid, and (f) posterior deltoid.*

the median predicted muscle forces for each muscle using methods 3 through 4 were not statistically different at the $p < 0.05$ level of significance. The muscle forces predicted by method 1 were within the middle 95% of the sample distributions of methods 2 to 4. Moreover, the standard deviations of methods 2–4 predictions were also similar (Fig. 3 gives the means and standard deviations of the predicted forces). The coefficients of variation (standard deviation represented as a percentage of the mean) of the supraspinatus were 55, 59, and 52% for methods 2–4, respectively. The largest coefficients of variation were for the posterior deltoid: 233, 238, and 234% for methods 3–4, respectively. Note that method 1 predicted zero posterior deltoid force.

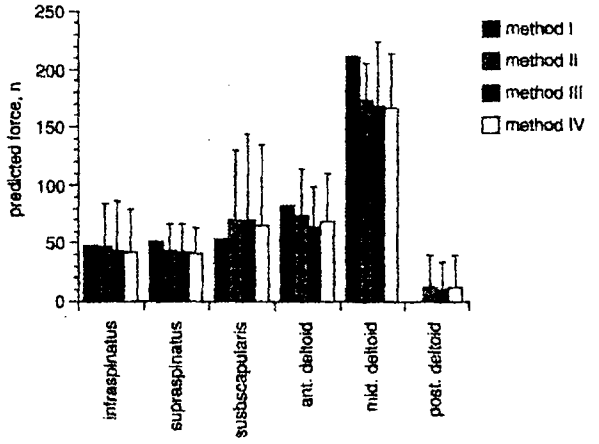

Fig. 3 *Mean predicted muscle forces using methods 1 through 4. Bars represent one standard deviation. Method 1 predicts zero posterior deltoid muscle force because the average moment arm for that muscle is positive, which represents an adduction function.*

4 Discussion

This study has demonstrated the utility of Monte Carlo simulation techniques in estimating the variability of biomechanical model outputs from variability in model input parameters. The statistical distributions of Monte Carlo predictions appeared similar to the statistical distribution of forces predicted by analysing the whole sample of anatomic specimens, although there was an insufficient number of specimens to test for differences in distributions. The simulations also illustrated that correlations between moment arms of different muscles did affect the statistical distributions of muscle force predictions. However, median predicted forces were not found to be affected. Modelling muscle moment arms as univariate normal distributions appeared to be sufficient for estimating the median and standard deviation of muscle forces, but the covariance of model parameters was needed for a full description of the statistical distribution of predicted forces. The results of these simulations are important, because they demonstrate that Monte Carlo techniques can be used to generate the statistical distributions of muscle forces without having to store and analyse the data of all anatomic specimens separately.

This initial effort to characterise variability in biomechanical model predictions suffer from four limitations common to analyses of the shoulder: it is only a static planar model of a three-dimensional system; the optimisation of model eqns. 1–3 assumes a specific central nervous system strategy of muscle activation; each muscle (or portion of the deltoid muscle) is modelled as a single force generating element with a single moment arm; and the sample of specimens used to measure moment arms may not have reflected a clinically relevant population. However, Monte Carlo techniques do not necessarily require any of these assumptions: the technique could be used to analyse the statistical distribution of forces from more sophisticated mechanical models such as those of NIEMINEN *et al.* (1995), VAN DER HELM (1994), and HÖGFORS *et al.* (1995). Likewise, the technique could be used to model the variation of moment arms in electromyograph (EMG)-driven models such as that of POPPEN and WALKER (1978). A simple model was selected for analysis to emphasise the stochastic aspects of the model.

Assumptions were also made about the stochastic properties of the model parameters. Specifically, it was assumed that only the muscle moment arms were random variables. In fact, there was variability in arm mass and arm length, which also affected net joint moment. There may be correlations between gross arm anthropometry and internal muscle moment arms. Moment arms and physiological cross-sectional areas may also be correlated. These correlations may be very important, but there is no published data available on correlation between shoulder muscle moment arms and other anatomic parameters.

Muscle moment arms were modelled using univariate (method 3) and multivariate (method 4) normal distributions. The small number of real samples of real moment arm data made it difficult to rigorously determine which distribution to use. Methods of distribution determination such as the Johnson's Translation System (JOHNSON, 1987) could be used if more data were available. The normal distribution assumption is most problematic for the supraspinatus and middle deltoid, because the normal distribution has a tail that stretches into negative values (e.g. adduction function). However, modelling the subscapularis, infraspinatus and posterior deltoid as a

normally distributed random variable works well, because the moment arm measurements on cadaveric specimens did produce both abduction and adduction values for these muscles. More research is needed to determine which probabilistic distributions are most appropriate for each muscle, and such investigations will require many more specimen measurements.

More research is needed on the application of Monte Carlo techniques to three-dimensional models, developing a large database of representative moment arm measurements, analysing the covariance structure of biomechanical parameters, and determining what analytical probability distributions best represent model parameters.

The predicted muscle forces were larger than those predicted by the three-dimensional finite element model of VAN DER HELM (1994). This difference may be due to different muscle moment arms used in the two models and different numbers of muscles in the models. In a paper describing model parameters, VAN DER HELM *et al.* (1992) question whether their finite element model should be based on average model parameters (such as method 1 here) or on every individual specimen's data (such as method 2). Data for a single 'median' cadaver is presented in VAN DER HELM *et al.* (1992), and appears to be the data used for generating the predictions in VAN DER HELM (1994). Thus, no comparisons of variability can be made between that study and results from this study.

Comparison of the predicted results with published electromyographic (EMG) studies shows patterns similar to these results. MCCANN *et al.* (1993) reported coefficients of variability of normalised EMG recordings during abduction for the supraspinatus and posterior deltoid to be 83 and 146%, respectively. Although some of the variability in the EMG data is due to noise in the signal, some is due to inter-subject differences. Note that the reported standard deviation was greater than the mean EMG for the posterior deltoid, which is consistent with the predictions of methods 2–4. However, method 1 predicted zero posterior deltoid force, which is inconsistent with EMG measurements (MCCANN *et al.* 1993; KRONBERG *et al.*, 1990).

In conclusion, this study illustrates a method for incorporating population variability into biomechanical modelling. The effect of correlation of model parameters with results indicates a need for experimental biomechanists to report data on the covariance of measured quantities in addition to the traditional mean and standard deviations. The availability of such data is necessary to model the population variability of tissue loadings derived from biomechanical models.

Acknowledgements—This study was supported by a grant from the Musculoskeletal Transplant Foundation and NIH grants AR41171 and HD07447. The contribution of the late Dr. Jain Liu is also gratefully remembered.

References

AN, K.N., KWAK, B.M., CHAO, E.Y. and MORREY, B.F. (1984*a*): 'Determination of muscle and joint forces: a new technique to solve the indeterminacy problem', *J. Biomech. Eng.*, **106**, pp. 364–367

AN, K.N., TAKAHASHI, K., HARRIGAN, T.P.I. and CHAO, E.Y. (1984*b*): 'Determination of muscle orientations and moment arms', *J. Biomech. Eng.*, **106**, pp. 280–282

AN, K.N., KAUFMAN, K.R. and CHAO, E.Y.S. (1989): 'Physiological considerations of muscle force through the elbow joint', *J. Biomechanics*, **22**, pp. 1249–1256

BASSETT, R.W., BROWNE, A.O., MORREY, B.F. and AN, K.N. (1990): 'Glenohumeral muscle force and moment mechanics in a position of shoulder instability', *J. Biomechanics*, **23**, pp. 405–415

BROOK, N., MIZRAHI, J., SHOHAM, M. and DAYAN, J. (1995): 'A biomechanical model of index finger dynamics', *Med. Eng. Phys.*, **17**, pp. 54–63

CHAFFIN, D.B. and ANDERSSON, G.B.J. (1984) 'Occupational biomechanics' (John Wiley, New York, USA)

CHOLEWICKI, J., MCGILL, S.M. and NORMAN, R.W. (1995): 'Comparison of muscle forces and joint load from an optimization and EMG assisted lumbar spine model: Towards development of a hybrid approach', *J. Biomechanics*, **28**, pp. 321–331

COLLINS, J.J. (1995): 'The redundant nature of locomotor optimization laws', *J. Biomechanics*, **28**, pp. 251 –267

HERZOG, W. (1996): 'Force-sharing among synergistic muscles: theoretical considerations and experimental approaches', *Exercise Sport Sci. Rev*, **24**, pp. 173–203

HÖGFORS, C., KARLSSON, D. and PETERSON, B. (1995): 'Structure and internal consistency of a shoulder model', *J. Biomechanics*, **28**, 767–777

JOHNSON, M.E. (1987): 'Multivariate statistical simulation' (John Wiley, New York, USA).

KARLSSON, D. and PETERSON, B. (1992): 'Towards a model for force predictions in the human shoulder', *J. Biomechanics*, **25**, 189–199

KRONBERG, M., NÉMETH, G. and BROSTRÖM, L.-A. (1990): 'Muscle activity and coordination in the normal shoulder', *Clin. Orthop.*, **257**, pp. 76–85

KUECHLE, D.K. (1994): 'Shoulder muscle moment arm during horizontal flexion, rotation, and elevation, MS Thesis, Department of Biomedical Sciences—Orthopedics, Mayo Graduate School, Rochester, MN, USA

LIU, J., HUGHES, R., SMUTZ, P., NEIBUR, G., AN, K.N. (1997): 'Roles of deltoid and rotator cuff muscles in shoulder elevation', *Clin. Biomech.*, **12**, pp. 32–38

MCCANN, P.D., WOOTTEN, M.E., KADABA, M.P. and BIGLIANI, L.U. (1993): 'A kinematic and electromyographic study of shoulder rehabilitation exercises', *Clin. Orthop.*, **288**, pp. 179–188

MIRKA, G.A. and MARRAS, W.S. (1993): 'A stochastic model of trunk muscle coactivation during trunk bending', *Spine*, **18**, pp. 1396–1409

MORREY, B.F. and AN, K.N. (1990): 'Biomechanics of the shoulder', *in* ROCKWOOD, C.A. and MATSEN, F.A. (Eds.): 'The Shoulder' (Saunders Co., Philadelphia, USA)

NIEMINEN, H., NIEMI, J., TAKALA, E.-P., VIIKARI-JUNTURA, E. (1995): 'Load sharing patterns in the shoulder during isometric flexion tasks', *J. Biomechanics*, **28**, pp. 555–566

POPPEN, N.K. and WALKER, P.S. (1978) 'Forces at the glenohumeral joint in abduction', *Clin. Orthop.*, **135**, pp. 165–170

RUBENSTEIN, R.Y. (1981): 'Simulation and the Monte Carlo method' (John Wiley, New York, USA)

SANDERS, M.S. and MCCORMICK, E.J. (1987) 'Human factors in engineering and design' (McGraw-Hill, New York, USA)

VAN DER HELM, F.C.T., VEEGER, H.E.J., PRONK, G.M., VAN DER WOUDE, L.H.V. and ROZENDAL, R.H. (1992): 'Geometry parameters for musculoskeletal modelling of the shoulder system', *J. Biomechanics*, **25**, pp. 129–144

VAN DER HELM, F.C.T. (1994): 'A finite element musculoskeletal model of the shoulder mechanism', *J. Biomechanics*, **27**, pp. 551–569

VEEGER, H.E.J., VAN DER HELM, F.C.T., VAN DER WOUDE, L.H.V., PRONK, G.M., ROZENDAL, R.H. (1991): 'Inertia and muscle contraction parameters for musculoskeletal modelling of the shoulder mechanism', *J. Biomechanics*, **24**, pp. 615–629