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A MIXED OBSERVATIONAL SURVEY METHOD

Biographical Sketch

Gary M. Young received his B.S. degree in Mathematics from Virginia Polytechnic Institute in 1965. In the same year he joined the Triangulation Branch, Geodesy Division, National Ocean Survey (NOS), formerly the U.S. Coast and Geodetic Survey, where he analyzed and adjusted horizontal control data. Mr Young received his M.S. degree in Geodesy from Purdue University in 1970, and after completing further studies returned to the NOS in 1971. He is presently Chief, New Datum Section, Control Networks Division, National Geodetic Survey, NOS. He is a member of ACSM and AGU.

Abstract

For several years, geodesists have debated the proper role of electronic distances in modern geodetic surveys. The role must be defined on an individual basis as a function of the desired accuracy and ultimate purpose of the survey. This paper proposes a mixed mode of observations for the types of surveys currently being observed following conventional first–order triangulation techniques. The mixed observational procedure requires only a portion of the survey control stations be instrument–occupied, and directions and electro–optical distances be observed to the remainder of the stations. The method allows the substitution of truck – or trailer–mounted portable towers, equipped with targets and reflectors, for a portion of the Bilby towers which are presently required to provide theodolite observations from all primary control stations in a conventional triangulation network. For the example considered, the mixed observational method provided more accurate position determinations than were obtained using conventional triangulation.

Truck- and Trailer-Mounted NGS Portable Towers

Figure 1

A MIXED OBSERVATIONAL SURVEY METHOD

Introduction

This investigation was initiated by the Director, National Geodetic Survey, to explore possible avenues to implement the full field utilization of the NGS portable towers (Figure 1) recently developed by the Instrumentation and Equipment Branch, National Geodetic Survey. Specifically, can the NGS portable towers be used to reduce the number of horizontal control stations which would normally require the construction of Bilby steel towers ?

The construction of a typical Bilby tower requires one day's work by a building unit of five men. After all survey operations requiring the use of the tower have been completed, a four–man unit dismantles the tower in one–half day. Significant savings of time and effort could be realized if the NGS portable tower, which can be extended and lowered by one man in 30 minutes or less, were substituted for a percentage of the Bilby towers which would normally be required.

Field evaluations currently being completed show no significant difference in accuracy for observations to targets or reflectors shown from Bilby towers or NGS portable towers. Accordingly, this investigation seeks an alternate network configuration which does not require that all primary stations be theodolite–occupied so that the possibilities presented by the NGS portable tower are fully exploited. For purposes of this evaluation, NGS portable towers were assumed to have been utilized at unoccupied stations to support targets and reflectors shown to theodolite–occupied stations.

Field Data

The arc under consideration is located in a project in northeast Louisiana, and was observed by a National Geodetic Survey Party in the summer of 1970. The network evaluated consists of an arc of six braced quadrilaterals involving 14 stations, Directions were observed in accordance with First–order, Class I specifications ; distances were measured over all lines using electro–optical distance measuring instruments ; astronomic positions and azimuths were observed at both ends of the arc. Skew normal corrections and those for the deflection of the vertical and for the geodesic, while insignificant, were applied to all horizontal directions. Distances were reduced to the ellipsoid. The lines vary from 6,000 to 16,000 metres. Astronomic azimuths were converted to Laplace azimuths using conventional astro–geodetic procedures. The arc was originally observed so that triangulation versus trilateration comparisons could be made. The fact that all directions and distances were observed throughout the arc provided a ready–made set of field data for this investigation. Bilby towers, ranging in height from 31 metres to 38 metres, were required at 13 of the 14 stations involved. A 1.2 metre stand, located atop a grain elevator, was utilized at one station.

Adjustment scheme

Consider the observations that could be obtained if the seven stations along the northern border of the east–west arc were manned by theodolites and electro–optical instruments, and NGS portable towers equipped with targets and reflectors were centered over the seven stations along the southern edge. Adjustments of this framework, along with adjustments of more conventional configurations, should provide valid comparisons between the accuracy of station determinations provided by the different observational approaches. The data used in the following adjustments are actual field observations incorporating Bilby towers. Observations to NGS portable towers should be of comparable quality.

Six adjustments were performed. All six were identical in certain respects : the position of station number 1 was held fixed ; the weights for the direction observations were based on an assumed (a priori) standard error (m_D) of 0.4 seconds ; the electro–optical length observations, which are the mean of four separate measurements, were assigned an assumed standard error (m_L) of 5 mm + 1 ppm ; the astronomic azimuth observations were given an assumed standard error (m_A) of 1.0 second. From these assumed standard errors, the weighting scheme was computed as follows : The weights for the direction and azimuth observations were $1/m_D{}^2$ and $1/m_A{}^2$ respectively. The weights for the length observations were computed from the formula

$$1/m_L{}^2 = 1/(c^2 + ppm^2 + (5/3 \times \Delta h \times 10^{-5})^2)$$

where c is the instrument constant standard error ; ppm presents the distance–dependent standard error in parts per million of the measured distance ; and Δh is the elevation difference between the two stations involved. The $5/3 \times \Delta h \times 10^{-5}$ term incorporates the uncertainty in zenith distance observations which are used in the reduction of distances to the ellipsoid. Relative accuracy estimates, for both distance standard error accuracies and azimuth standard error accuracies, were computed between selected adjacent stations. Variances in latitude and longitude, and covariances in latitude–longitude were computed for all stations in each adjustment relative to the fixed position of station number 1 . The fixed position was assumed to be without error. From the variance–covariance matrix, error ellipses were obtained for each point. Ninety–five percent point error ellipses for Adjustments A , B , C , and D are shown in Figures 2, 3, 4, and 5. The adjustments were obtained utilizing a modified version of the NGS TRAVO5 least squares geodetic adjustment program on a National Oceanic and Atmospheric Administration CDC 6600 computer.

Adjustments A , B , and C

Adjustment A was a minimally constrained computation of all observed horizontal directions in the arc of triangulation. This adjustment examined the internal consistency of the direction observations, and indicated the accuracy of the position determinations which are a function of network geometry and the quantity and quality of observations. One electro–optically measured distance and one Laplace azimuth provided scale and orientation for the adjustment. The average of 24 triangle closures was 0".62 with a maximum closure of 1".71. The adjustment yielded an average correction to 62 directions of 0".19, a maximum correction to a direction of 0".62, and a maximum correction to an angle of 0".91.

Adjustment B utilized the same network in a trilateration mode; i.e., only electro–optical distance observations over all lines were included in the adjustment. One Laplace azimuth, identical to the one used in Adjustment A, oriented the network. The 31 distances received an average proportional part correction of 1:5,490,000, with the worst proportional part correction being 1:1,700,000.

Adjustment C was a computation of a "typical" first–order arc of triangulation. The adjustment included three electro–optically measured distances; one in every third quadrilateral. Additionally, two Laplace azimuths; one at each end of the arc provided orientation. In this adjustment, the corrections to the observed directions remained essentially the same as Adjustment A. The average proportional part correction to a distance was 1:4,670,000, with a maximum of 1:2,840,000.

These three adjustments were performed to evaluate traditional triangulation and trilateration techniques. To provide a basis of comparison for the three adjustments, one with another, and with the adjustments that follow, relative standard errors between selected adjacent stations were computed in each adjustment (see Table 3). To provide a good estimate of the overall relative accuracies in each adjustment, relative accuracies were computed along a diagonal of each quadrilateral. As expected, the trilateration network provided superior relative accuracies; the length standard error relative accuracies over the six diagonal lines averaged 1:990,000. Azimuth standard error relative accuracies averaged 0".59. Adjustment C, with three electro–optically measured distances and two Laplace azimuths, was next best; length accuracies averaged 1:480,000, and the average azimuth accuracy was 0".82. Adjustment A, with one electro–optical distance and one Laplace azimuth was least accurate: length accuracies averaged 1:200,000, and the average azimuth accuracy was 1".08; still well within First–order, Class I expectations.

Adjustments D, E, and F

Adjustment D consisted of a mixed observation configuration. Its comparison with the three previously mentioned adjustments provided the incentive for this study. The network included those observations which would be obtained if theodolites and electro–optical instruments (requiring the construction of Bilby towers) were utilized at each of the seven stations along the northern edge of the arc, and truck– or trailer–mounted portable towers equipped with targets and reflectors were centered over the seven stations along the southern edge. This would result in observations between the seven stations along the northern edge, and 19 directions to the unoccupied stations to the south. In addition, 25 electro–optically measured distances could be observed, utilizing reflectors atop the portable towers at the southern stations. These 25 distances were included in the adjustment. Two Laplace azimuths, one at each end of the arc, oriented the network.

The adjustment yielded an average correction to a direction of 0".22, with a maximum correction of 0".69. The maximum correction to an angle was 1".20. These are similar to the corrections obtained in Adjustments A and C. The 25 distances received an average proportional part correction of 2,800.000, with a maximum of 1 : 690,000.

Adjustments E and F determined the effect of poor or erroneous direction observations on the Adjustment D mixed model. This effect was secured by arbitrarily altering selected observed directions to determine the impact on the adjustments.

Adjustment E was identical to Adjustment D, except the direction from station 3 to station 6 was increased by three seconds ; and the direction from station 9 to station 8 was decreased by three seconds. Changes of such magnitude are beyond those which could be attributed to random error. The impact upon the adjustment was most pronounced. Unlike the previous four adjustments, whose variances of unit weight were within 95 percent chi–square confidence intervals, the variance of unit weight for Adjustment E was 3.56, far outside the acceptable range for 25 degrees of freedom of 0.52 to 1.63. Further indications were given by larger residuals and decreased relative accuracies (see Tables 2 and 3).

Adjustment F was computed to show that poor observational data are often obscured, if only direct adjustment results are considered. For Adjustment F, two other directions, perpendicular to the general direction of the arc, were altered by three seconds. The direction from station 3 to station 4 was increased by three seconds ; the direction from station 9 to station 10 was decreased by three seconds. The two directions that were altered in Adjustment E were returned to their observed values.

The results of Adjustment F were very different from those of Adjustment E. For example : the variance of unit weight decreased to 1.36 (within the acceptable range of 0.52 – 1.63). The average correction to a direction became 0".25, as opposed to 0".38 for Adjustment E ; and the proportional part corrections to observed distances, and the relative accuracy samples improved dramatically. The erroneous directions were well hidden if only the adjustment results are considered. The mixed observations procedure does not provide for the measurement of all angles in a triangle ; this precludes important field checks of observed data. Although not detailed in this paper, field checks, sufficient to verify observed values while at the station sites, are essential.

Mixed vs. Triangulation

As shown in Table 3, the mixed model (Adjustment D) exhibits much improved length accuracies over those obtained by conventional first–order triangulation (Adjustment C). The accuracies along the six diagonals average 1:820,000 vs. 1:480,000.

It seemed reasonable at the outset to expect any weakness of the mixed model to appear in the positional determinations of the stations along the southern edge, which in the mixed model were treated as unoccupied points. This was not the case. The length accuracies in the mixed model (Adjustment D) maintained the average accuracy factor of nearly two–to–one over triangulation (Adjustment C) : 1:560,000 vs. 1:310,000 . The azimuth accuracies were essentially the same in both adjustments.

Certain observational problems do exist when using the mixed model. An important by–product of most NGS surveys is the location by intersection of prominent physical objects, radio masts, water tanks, etc..., by theodolite observations. These are used by local surveyors as azimuth and position control for lower–order surveys. Many of the intersection stations visible atop Bilby towers are not visible from the ground. Careful reconnaissance should provide the determination of sufficient numbers of intersection stations for local surveyors.

Observations to azimuth marks and reference marks are most efficiently accomplished using a theodolite. These marks are visible from the ground ; therefore, theodolite observation could be made at ground level immediately before or after NGS portable towers have been set up. Theodolite observations to all intersection stations visible from the ground could also be made at this time. It might be necessary to observe astronomic azimuths at those stations where a suitable azimuth is not available,

The reduction (computation) of distances to a reference surface requires the determination of station elevations at both ends of the lines. These elevations are most easily obtained by trigonometric leveling (utilizing zenith distance observations). Observations sufficient to compute the elevations of all stations can be obtained at the stations requiring Bilby towers. The mixed model requires a greater number of distance observations than does triangulation ; hence, an increase in the amount of zenith distances.

In all aspects of the accuracy evaluations, the mixed model proved better than triangulation. This coupled with the fact that one–half to two–thirds of the Bilby towers normally constructed in a project could be replaced by NGS portable towers should more than offset the problems indicated in the preceding paragraphs. The mixed method is certainly worthy of further consideration.

Trilateration

One aspect has yet to be considered. Why continue with triangulation, or for that matter adopt a mixed method, when trilateration gave accuracies approaching 1 : 1,000,000 ?

As stated earlier, the determination of intersection stations is very important to local surveyors. Using current instrumentation, it is not feasible to determine intersection stations by trilateration. Ties to azimuth and reference marks are most easily obtained by theodolite.

Trigonometric leveling (by theodolite) requires zenith distance observations at one–half or more of the stations. In triangulation, and to a lesser extent, in the mixed model, theodolite observations are required as part of the normal observing procedure. In trilateration, current practices would necessitate theodolite observations, most of which are in addition to the distance observations required for the survey, at all stations.

Another problem encountered in trilateration is the small redundancy in conventional geodetic figures. The standard quadrilateral with all six distances measured yields only one degree of freedom. Hence, the test arc provided only six degrees of freedom, compared to 27 degrees of freedom for triangulation (Adjustment C), and 25 degrees of freedom for the mixed model. Many proponents of trilateration advocate the use of the hexagon with all stations intervisible (a pentagon is necessary to provide the same redundancy as a quadrilateral observed by triangulation) as the basic geodetic figure. The field reconnaissance to determine station sites with such intervisibilities, and the combining of hexagonal figures one with another to form a network, is very impractical if not impossible.

Except for special purpose surveys (e.g., crustal movement studies), conventional triangulation, or the mixed model supplies accuracies more than sufficient for the surveying community. It is felt that the increased time and effort required by trilateration is not warranted considering the large areas in the U.S. which have yet to be surveyed by any geodetic method or where monumentation is too sparsely spaced to meet current requirements.

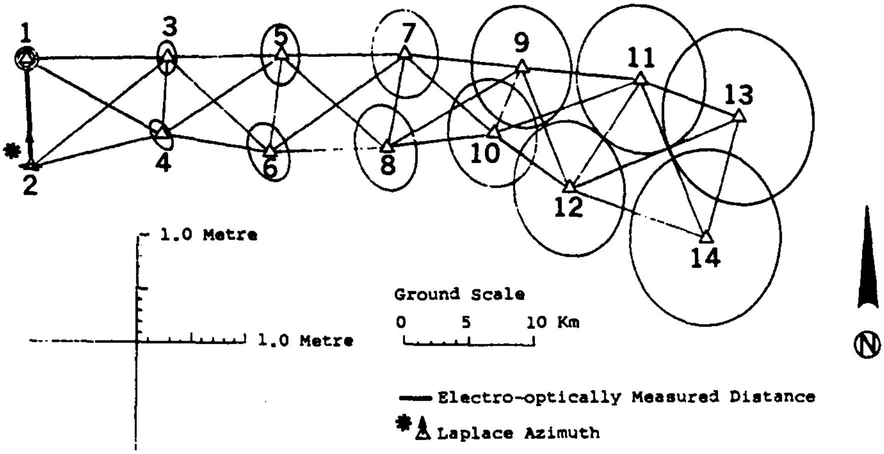
Comment

It could be argued by many, with some justification, that this exercise would have been more rigorous or elegant if viewed strictly from an error propagation standpoint. Others would contend that simulated observations reflecting best estimates of observational errors would show the worth of this method.

This evaluation was based on field observations because

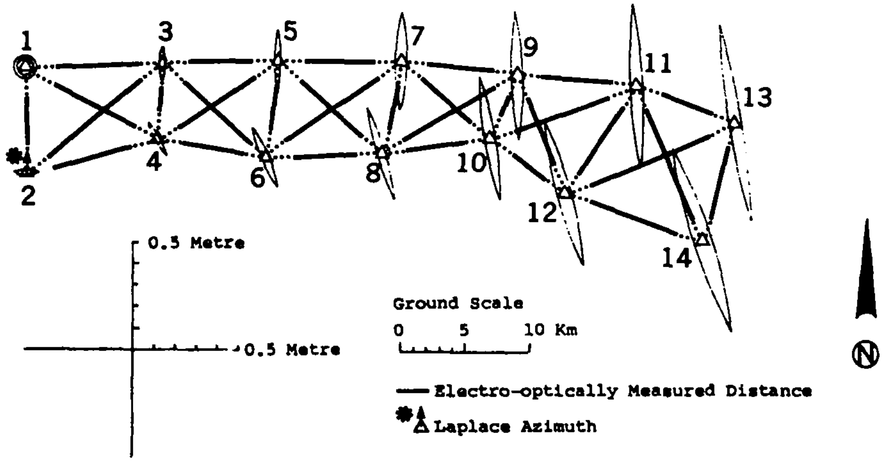
(1) observed data existed as a by–product of a previous evaluation

(2) no matter how carefully one simulates observations, the only *true* test is that under the actual field conditions encountered ; and using appropriate field instrumentation and specifications, the mixed model example provided better results than conventional triangulation.



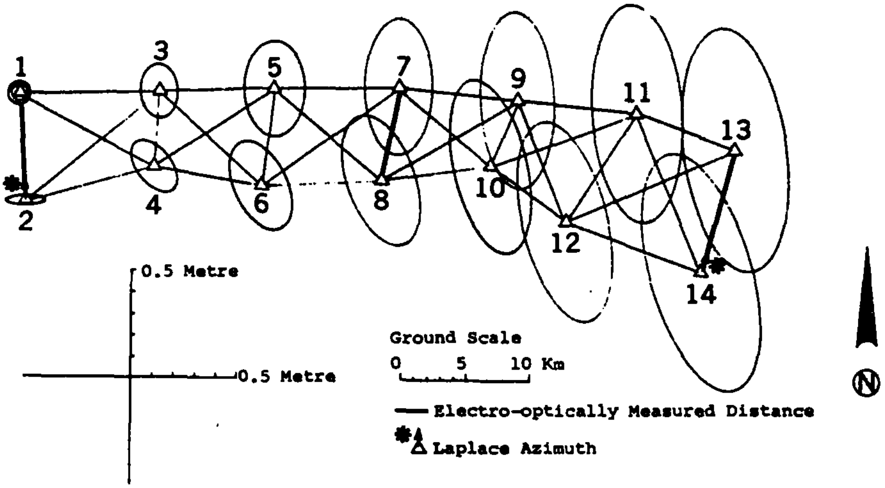
Scale for Ellipse Axes
(Adjustment A only)

Figure 2. Adjustment A - 95% Error Ellipses



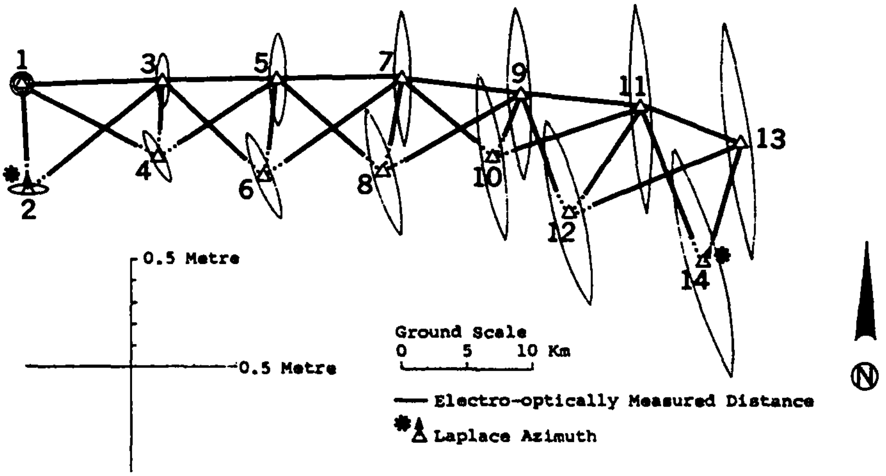
Scale for Ellipse Axes

Figure 3. Adjustment B - 95% Error Ellipses



Scale for Ellipse Axes

Figure 4. Adjustment C - 95% Error Ellipses



Scale for Ellipse Axes

Figure 5. Adjustment D - 95% Error Ellipses

Table 1

Explanation of Adjustments in Louisiana Test Arc

Adjustment A :	**Triangulation** : A minimally constrained adjustment of observed horizontal directions over all lines, with one electro–optical distance observation for scale, and one Laplace azimuth observation for orientation.
Adjustment B :	**Trilateration** : An adjustment of electro–optical distances over all lines, with one Laplace azimuth.
Adjustment C :	**Triangulation** : An adjustment of a first–order arc of triangulation including two Laplace azimuths, one at each end of the arc ; and three electro–optical distances, one every third quadrilateral.
Adjustment D :	**Mixed Model** : This adjustment included the directions and electro–optical distances which would be obtained if the seven stations along the northern edge of the arc were occupied using theodolites and electro–optical distance measuring instruments, and portable towers showing targets and reflectors were centered over the seven stations along the southern boundary. Two Laplace azimuths were included for orientation.
Adjustment E :	**Mixed Model** : An adjustment of the same observations as were adjusted in Adjustment D, except that two directions (direction from station 3 to station 6 and from station 9 to station 8) were arbitrarily altered by three seconds.
Adjustment F :	**Mixed Model** : An adjustment of the same observations as were adjusted in Adjustment D, except that two directions approximately perpendicular to the east–west arc of triangulation (direction from station 3 to station 4, and from station 9 to station 10) were arbitrarily altered by three seconds.

Table 2

Louisiana Test Arc

Summary of Adjustments

Adjustment	*A*	*B*	*C*	*D*	*E*	*F*
No. of Theodolite–occupied stations	14	0	14	7	7	7
No. of Electro–optical distances	1	31	3	25	25	25
No. of Laplace Azimuths	1	1	2	2	2	2
Average triangle closure	0".62		0".62			
Maximum triangle closure	1.71		1.71			
Average correction to direction	0.19		0.20	0".22	0".38	0".25
Maximum correction to direction	0.62		0.69	0.69	1.85	0.98
Maximum correction to angle	0.91		1.07	1.20	2.64	1.32
Average correction to Laplace azimuth			1.13	1.35	2.24	1.44
Average proportional part correction to a measured length (1 : part in)		5,490,000	4,670,000	2,800,000	1,600,000	2,240,000
Maximum proportional part correction to a measured length (1 : part in)		1,700,000	2,840,000	690,000	450,000	570,000

Table 3

Louisiana Test Arc

Summary of Adjustments

Adjustment		*A*	*B*	*C*	*D*	*E*	*F*
Degrees of freedom		24	6	27	25	25	25
Variance of unit weight		0.85	0.22	0.97	0.94	3.56	1.36
Acceptable Range (95 °/o Chi-square)		0.52–1.64	0.20–2.44	0.54–1.60	0.52–1.63	0.52–1.63	0.52–1.63
Length standard error proportional part relative accuracies (1 : part in)							
Along Diagonals*	Aver.	200,000	990,000	480,000	820,000	550,000	750,000
	Min.	390,000	1,000,000	470,000	850,000	590,000	780,000
	Max.	130,000	960,000	320,000	790,000	520,000	710,000
Along S. Edge**	Aver.	190,000	950,000	310,000	560,000	320,000	490,000
	Min.	350,000	960,000	360,000	620,000	360,000	540,000
	Max.	120,000	940,000	260,000	480,000	260,000	410,000
Azimuth standard error relative accuracies							
Along Diagonals*	Aver.	1”.08	0”.59	0”.82	0”.80	1”.56	0”.97
	Max.	1 .15	0 .67	0 .83	0 .82	1 .60	0 .99
	Min.	1 .00	0 .48	0 .81	0 .78	1 .52	0 .94
Along S. Edge**	Aver.	1 .08	0 .60	0 .82	0 .83	1 .62	1 .00
	Max.	1 .16	0 .68	0 .83	0 .85	1 .66	1 .03
	Min.	1 .01	0 .49	0 .82	0 .81	1 .57	0 .97

*– The accuracies along the diagonal lines (2–3, 3–6, 6–7, 7–10, 10–11, 11–14) provide a good estimate of the overall quality of the project.

**– The accuracies along the lines between the stations on the southern edge of the arc (2–4, 4–6, 6–8, 8–10, 10–12, 12–14) would exhibit any weakness that the mixed model would have in the determination of the "theodolite-unoccupied" stations along the southern edge of the arc.

Table 4

Louisiana Test Arc

Some Statistics based on Normalized Residuals

Adjustment	*A*	*B*	*C*	*D*	*E*	*F*
Number of Observations	64	32	67	58	58	58
Range	2.54	0.86	3.01	3.08	8.57	4.07
Minimum	−1.54	−0.55	−1.74	−1.35	−3.95	−1.62
Maximum	+1.00	+0.31	+1.27	+1.73	+4.62	+2.45
Mean	0.00	0.00	0.00	0.00	0.00	0.00
Median	+0.02	+0.01	−0.01	−0.03	0.00	−0.05
Average	+0.46	+0.16	+0.50	+0.50	+0.84	+0.59
Variance	0.32	0.04	0.39	0.40	1.54	0.59
Standard Error	0.56	0.20	0.62	0.64	1.24	0.77
Skewness	−0.37	−0.79	−0.35	+0.27	+0.32	+0.46
Kurtosis	+2.60	+3.25	+2.99	+3.06	+6.34	+3.69
Number of plus signs	33	18	33	28	30	28
Number of minus signs	31	14	34	30	28	30

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