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INTRODUCTION TO PRACTICAL COMPUTATION OF ASTRONOMICAL REFRACTION

(2ème Partie) *

Refractive Index of Air

The refractive index of dry air has been determined by experiment to within a few parts in 10^8 under standard conditions. Its refractivity in ambient conditions, at pressure p and temperature T, can best be presented by the combination of the Lorenz–Lorentz formula according to which the expression

$$\frac{n^2-1}{n^2+2} = \frac{2}{3}(n-1)\left[1-\frac{1}{6}(n-1)+\frac{4}{9}(n-1)^2-\dots\right]$$

is directly proportional to density, and the equation of state $p = \rho CRT$, where C is a compressibility factor since dry air is not a perfect gas. It follows from the above that equation (7) previously employed for the refractivity of moist air can be more accurately written for the specific case of visible light ($c_w' = 0$),

$$n-1 = \frac{(n_0-1)T_0}{p_0}\left(1-\frac{n_0-1}{6}\right)\left(\frac{p}{T}\right)\left(\frac{C_0}{C}\right)\left[1+\frac{(n_0-1)T_0}{6p_0}\left(\frac{p}{T}\right)\right]-c_w(e/T)$$

For a standard wavelength of 0.574 microns adopted by Jordan–Kneissl [1970] and others for the average starlight, this gives according to the dispersion equations by Edlén [1953, 1966] :

$$(n-1)10^6 = 78.8828\left(\frac{p}{T}\right)\left(\frac{C_0}{C}\right)\left(1+0.00001315\,\frac{p}{T}\right)-12.28\left(\frac{e}{T}\right) \qquad (32)$$

where T is the absolute temperature in degrees Kelvin, p is the total pressure and e is the partial pressure of water vapour, both in millibars, C is the compressibility factor of dry air at temperature T and pressure p, and C_0 is the compressibility factor of standard air, i.e. dry air at 15°C and 760 torr (1013 mb).

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Numerically, product $(C_0/C)\left(1 + 0.00001315\frac{p}{T}\right)$ ranges from a minimum value of about 0.9998 in hot climates to a maximum of about 1.0010 in cold climates, and it may be replaced in equation (32) by a constant factor evaluated for the average local conditions.

Stars emit light in all the spectral colours, the dominant wavelength being dependent on the temperature of the luminous surface. Deviations from the standard wavelength employed in equation (32) can be taken into account by correction factors proportional to the dispersive constants of dry air. A compact equation for standard air is that by Edlén [1966] :

$$(\lambda 0.405 - 0.705\,\mu)\,; \qquad (n_0 - 1)\,10^6 = \frac{47230.0}{173.3 - 1/\lambda^2} \tag{33}$$

where λ is the wavelength of radiation expressed in microns. The correction factor applicable to the refractivity computed by equation (32) is accordingly $170.2649\,/\,(173.3 - 1\,/\,\lambda^2)$.

Reduction of Local Gravity to the Centroid of the Atmospheric Column

The value of g as required in the second term of equation (30) may be derived from the approximate gravity formula

$$g = 98.07\,(1 - 0.0026\cos 2\varphi - 0.00031\,\overline{H}) \times 10^1\ cm\ sec^{-2}$$

where φ is the latitude and $\overline{H}$ is the height, in kilometres above the sea level, of the centre of gravity in a vertical column of the atmosphere. Since atmospheric density is nearly proportional to refractivity $n - 1$, the height of the centre of gravity above ground level can be expressed by the ratio

$$\bar{r} - r_1 = \int_{r_1}^{r'} (n-1)(r - r_1)\,dr \Big/ \int_{r_1}^{r'} (n-1)\,dr$$

Integration by parts substituting $u = (r - r_1)^2$ and $v = n - 1$ gives

$$\int_{r_1}^{r'} (n-1)(r-r_1)\,dr = \frac{1}{2}\int_{1}^{n_1} (r - r_1)^2\,dn$$

and consequently, using equations (9) and (20), we obtain

$$\bar{r} - r_1 = \left(\frac{R}{g - R\beta}\right) T_1'\left[\,(p_1'/p_1) - (R\beta/g)\,(p^0/p_1)\,(T^0/T_1')\,\right] \tag{34}$$

where the primed quantities, as before, refer to values corrected for ground inversion.

Due to the poleward slope of the tropopause, and the seasonal variation of its height, there exists a correlation between ground temperature T_1' and pressure p^0 at the tropopause, high temperatures being accompanied by low pressures, and vice versa. This tends to smoothen out regional and seasonal variations in the height of the centre of gravity, and equation (34) evaluated for the average conditions,

$$\bar{H} = 7.3 + 0.9\,H$$

where H denotes the station height in kilometres above the sea level, appears to be accurate to within ± 0.4 km for all latitudes and all seasons. With this approximation, the expression for local gravity at the centroid of the atmospheric column becomes

$$g = 97.84\,(1 - 0.0026 \cos 2\varphi - 0.00028\,H) \times 10^1\ cm\ sec^{-2} \tag{35}$$

whereas the mean curvature radius of the earth

$$r_1 = 6378\,(1 - 0.0034 \cos 2\varphi + 0.00016\,H) \times 10^5\ cm \tag{36}$$

can be substituted for the radius vector of the base of the column,

Evaluation of Correction Term $\delta = \delta_1 - \delta_2 - \delta_3 + \delta_4$.

The effect on astronomical refraction of the curvature of atmospheric layers is almost totally absorbed into the second term of equation (30) , additional correction terms being required only at zenith distances greater than about 70 degrees. Accepting $\beta = -6.5\ ^\circ K\ km^{-1}$ as the tropospheric gradient of temperature for all latitudes and all seasons, and omitting insignificant terms in equations (31), we have numerically

$$\delta'' = \delta_1'' - \delta_2'' - \delta_3'' + \delta_4'' =$$

$$= 0''.000288\,(3 \tan^5 z_1 + 5 \tan^3 z_1)\,(p_1' T_1' + 0.190\, p^0\, T^0)\, 10^{-6} -$$

$$- 0''.013 \tan^5 z_1\, (p_1'^2 / T_1')\, 10^{-6} -$$

$$- 0''.014 \tan^7 z_1\, (p_1' T_1'^2 + 0.64\, p^0\, T^{0^2})\, 10^{-12} +$$

$$+ 0''.0003 \tan^9 z_1\, (p_1' T_1'^3 + 2\, p^0\, T^{0^3})\, 10^{-15} \tag{31a}$$

where $T_1' = T^0 + 6.5\,(H^0 - H)$, and $p_1' = p^0\,(T_1'/T^0)^{5.26}$. The units are here the same as before : the height of the tropopause above the ground level, $H^0 - H$, in kilometres, and the pressure in millibars.

As in the previous case of equation (34), any significant variation of the tropospheric contribution, whether of regional or seasonal origin, is largely compensated by the stratosphere in equation (31a). An average value of correction δ is therefore acceptable for all latitudes and all seasons.

Formulas and Tables

for the Computation of Astronomical Refraction

1. Standard Formula.

The correction for astronomical refraction under standard conditions is given by the formula

$$\Delta z_0'' = 16''.271 \tan z \left[1 + 0.0000394 \tan^2 z \left(\frac{p - 0.156\,e}{T}\right)\right]\left(\frac{p - 0.156\,e}{T}\right) -$$

$$- 0''.0749\,(\tan^3 z + \tan z)\left(\frac{p}{1000}\right) + \delta'' \qquad (30a)$$

where z is the apparent zenith distance, p is the total barometric pressure in millibars, e is the partial pressure of water vapour in millibars, T is the absolute temperature in degrees Kelvin ($T = 273.16 + t$ °C), and δ is a correction term obtained from Table III opposite.

Table III

Correction Term δ in Seconds of Arc

Apparent Zenith Distance	Station Height Above Sea Level						Apparent Zenith Distance
	0 km	1 km	2 km	3 km	4 km	5 km	
66°00'	+0.02	+0.01	+0.01	+0.01	+0.01	+0.01	66°00'
70°00'	+0.04	+0.04	+0.03	+0.03	+0.02	+0.02	70°00'
74°00'	+0.12	+0.11	+0.09	+0.08	+0.07	+0.06	74°00'
76°00'	+0.23	+0.20	+0.18	+0.16	+0.14	+0.12	76°00'
77°00'	+0.33	+0.29	+0.26	+0.23	+0.20	+0.17	77°00'
77°30'	+0.40	+0.35	+0.31	+0.27	+0.24	+0.21	77°30'
78°00'	+0.48	+0.43	+0.38	+0.33	+0.29	+0.25	78°00'
78°20'	+0.55	+0.49	+0.43	+0.38	+0.33	+0.29	78°20'
78°40'	+0.63	+0.56	+0.49	+0.44	+0.38	+0.33	78°40'
79°00'	+0.73	+0.64	+0.57	+0.50	+0.44	+0.38	79°00'
79°10'	+0.78	+0.69	+0.61	+0.54	+0.47	+0.41	79°10'
79°20'	+0.84	+0.75	+0.66	+0.58	+0.51	+0.45	79°20'
79°30'	+0.91	+0.80	+0.71	+0.63	+0.55	+0.48	79°30'
79°40'	+0.98	+0.87	+0.77	+0.68	+0.59	+0.52	79°40'
79°50'	+1.05	+0.94	+0.83	+0.73	+0.64	+0.56	79°50'
80°00'	+1.14	+1.01	+0.90	+0.79	+0.69	+0.61	80°00'

2. Local Modification of the Standard Formula.

The numerical coefficients in the standard formula are to some extent liable to local variation which will result in a slight loss of accuracy under extreme conditions – especially in very cold climates – unless they are modified to correspond to the average local conditions.

A locally corrected value for the numerical coefficient of the first term in the standard formula may be determined by evaluating the expression

$$16''.2707\,(C_0\,/\,C)\,[\,1 + 0.00001315\,(p\,/\,T\,)]$$

for the average local values of pressure and temperature. C refers here to the compressibility factor of dry air at pressure p and temperature T, and C_0 is the compressibility factor of standard air, i.e. dry air at 760 torr (1013 mb) and 15 °C, Numerical values for the compressibility factors can be obtained from Smithsonian Meteorological Tables [1951] .

The coefficient of the second term in the standard formula

$$0''.07485\,(1 + 0.0060\,\cos 2\,\varphi + 0.00012\,H)$$

may be determined as a function of latitude φ alone, the effect of station height H (in km) being totally negligible.

3. Correction for the Effective Wavelength of Starlight.

For an individual star, the astronomical refraction is obtained from standard refraction Δz_0 by the formula

$$\Delta z = C_\lambda\ \Delta z_0 \tag{37}$$

and

$$C_\lambda = 170.2649\,/\,(173.3 - 1\,/\,\lambda^2\,) \tag{38}$$

where λ is the effective wavelength in microns, i.e. the wavelength of the apparent colour of the star.

The most direct method of determining the apparent colours of the stars is by visual colorimetry, notably employed by H. Osthoff [1900] in his determination of the colours of 1009 fixed stars. In the Osthoff catalog, which still provides a chief source of information as far as refraction is concerned, stars have been classified according to colour into eleven main groups, ranging from 0^c (white) to 10^c (red). The whitest star cataloged is Sirius (0^c6), the reddest 19 Piscium (8^c8).

Table IV

Correction Factor C_λ in the

Osthoff (Schmidt) Colour Scale

Spectral Class	Apparent Colour	Wavelength * λ, mμ	Correction Factor C_λ
B	0^c	560	1.00090
A	1^c	563	1.00070
	2^c	566	1.00051
F	3^c	569	1.00031
	4^c	574	1.00000
G	5^c	582	0.99951
	6^c	591	0.99899
K	7^c	603	0.99833
	8^c	617	0.99761
M	9^c	633	0.99684
	10^c	650	0.99609

* – According to Jordan–Kneissl [1970].

The Henry Draper classification of stellar spectra into the types O, B, A, F, G, K and M corresponds to a sequence of decreasing surface temperatures, which gives some idea of the colour of the stars. It must be recognized, however, that the apparent colour as judged by the eye is not determined by the spectrum alone but also depends on the brightness of the source (Purkinje effect) ; hence in Table IV no definite limits can be set between the different spectral classes.

Atmospheric scattering of light causes varying degree of reddening of the stars, which further complicates the determination of the effective wavelength. A practical solution might be found in the use of selective light filters.

Accuracy of the Determination of Astronomical Refraction

The errors which affect the determination of astronomical refraction may be divided into the three principal groups of (a) the observational errors, (b) the errors in the refraction formula employed, and (c) the errors due to the departure of the atmosphere from its assumed state. Only the last–named group will be discussed, inasmuch as it sometimes may involve large errors in the determination of refraction.

The Effect of the Slope of Isopycnic Surfaces

The basic assumption that the atmosphere is made up of concentric spherical layers or shells (Fig. 1), holds only approximately even for the normal state of the troposphere. Since the refractivity of air at the sea level inevitably increases from the thermal equator toward the colder climates at the poles, the surfaces of constant refractivity must in the lower troposphere acquire a general slope toward the equator, unless this is prevented by some local disturbance. The systematic error as the result of the average inclination can be roughly estimated, as follows.

Consider the atmosphere resting upon a horizontal plane (AB in Fig. 2) and that initially, the isothermal and the isobaric surfaces are horizontal everywhere. The refractivity is assumed to be constant along the isopycnic surfaces, or surfaces of constant density, also represented initially by a set of horizontal planes. Now let the temperature be raised from B (north) toward A (south) at a constant rate dT / ds throughout the entire troposphere. An increase in temperature by dT will decrease the refractivity by

$$dn = -\frac{(n-1)\,dT}{T}$$

which near the ground level, where the pressure remains constant, corresponds to a vertical displacement of the isopycnic surfaces by

$$dr = \frac{T_1\,dn}{\beta m'(n_1 - 1)} = \frac{dT}{\beta m'}$$

as is evident from equation (18). The southward slope of the isopycnic surfaces at ground level is consequently

$$\psi_1 = \frac{dT/ds}{\beta m'} = -\frac{dT/ds}{g/R + \beta} \tag{39}$$

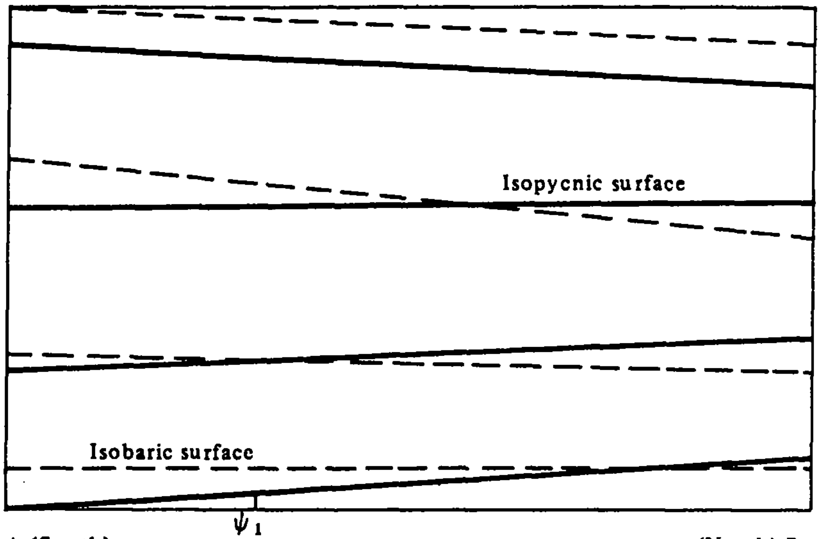

Figure 2 Meridional tilt of isopycnic surfaces. (Northern hemisphere).

which amounts to about 1' 52" for an assumed average value of $dT/ds = 0.015\ °K\ km^{-1}$ in the middle latitudes.

The vertical distance between any two isopycnic surfaces is contracted in the north and expanded in the south, so that eventually a height will be reached where the surfaces become horizontal. This occurs at a level where the temperature expansion is exactly balanced by the corresponding pressure contraction, that is, when

$$dp/p = dt/T$$

Differentiation of (16) with respect to T gives

$$dp/p = -\frac{g}{R\beta}\left(\frac{T_1 - T}{T_1 T}\right)dT$$

and equating the right sides of the above equations, we obtain the height of the level of constant density

$$h = (T - T_1)/\beta = RT_1/g \tag{40}$$

which is roughly 8.5 kilometres above the sea level throughout the world. At height h, $\psi = 0$. And since ψ is obviously a linear function of height, it follows that in the troposphere

$$\psi = \psi_1 \left[1 + \frac{g}{R\beta T_1} (T_1 - T) \right] \tag{41}$$

It will be noted that in the upper troposphere above the level of constant density, the slope of the isopycnic surfaces is reversed, and they become gradually more parallel to the isobaric surfaces which have their maximum slope near that same level.

Further up in the stratosphere where the temperature is assumed constant, the refractive index is a function of pressure alone and the isopycnic surfaces run parallel to the isobaric surfaces. From (16), the gradient of pressure along the tropopause is

$$dp^\circ / ds^\circ = \frac{gp^\circ}{R\beta T_1} (dT_1 / ds)$$

and since β is assumed constant, the tropopause slopes down in the north at a constant angle $\alpha = -(dT_1/ds)/\beta$. Differentiation of (10) further gives

$$dr^\circ = -\frac{RT^\circ}{g} (dp^\circ / p^\circ)$$

hence the isobaric surfaces cut the tropopause on the stratospheric side at an angle

$$dr^\circ / ds^\circ = -\frac{T^\circ}{\beta T_1} (dT_1 / ds)$$

along horizontal lines which run perpendicular to AB . In the stratosphere, then, under the above assumptions which do not greatly differ from the actual conditions, the slope of the isopycnic surfaces is approximately the same at all levels, and equal to

$$\psi_s = -\frac{dT_1 / ds}{\beta} \left(1 - \frac{T^\circ}{T_1} \right) \tag{42}$$

The direction of the slope is at right angles to the sea–level isotherms, and the surfaces slope down in the direction of decreasing temperatures.

We shall now go back to the differential equation of refraction, taking it in the form

$$\frac{d(\Delta z)}{dr} = \frac{dn}{dr} \tan z$$

and determine the effect of the tilt on the correction for refraction. In the above equation, slope angle ψ must obviously be taken into account as a correction

$$d(\tan z) = \sec^2 z\, dz = \psi \sec^2 z = \psi \tan^2 z + \psi$$

whereas its rate of change $d\psi / dr$ will affect dn/dr by an amount obtained from

$$\frac{dn}{dr + s\, d\psi} - \frac{dn}{dr} = \frac{dn}{dr}\left(1 - s\,\frac{d\psi}{dr}\right) - \frac{dn}{dr} = -\frac{dn}{dr} \tan z\,(\psi - \psi_1)$$

The combined effect on the correction for refraction is consequently

$$\delta(\Delta z) = \int_{n^0}^{n_1} \psi \tan^2 z\, dn + \psi_s \int_1^{n^0} \tan^2 z\, dn + \int_1^{n_1} \psi\, dn - \int_{n^0}^{n_1} \tan^2 z\,(\psi - \psi_1)\, dn$$

$$= \psi_1 \int_1^{n_1} \tan^2 z\, dn - (\psi_1 - \psi_s) \int_1^{n^0} \tan^2 z\, dn + \int_1^{n_1} \psi\, dn \simeq$$

$$\simeq \psi_1 \tan^2 z_1\,(n_1 - 1) - (\psi_1 - \psi_s) \tan^2 z_1\,(n^\circ - 1) + \psi_m\,(n_1 - 1)$$

where ψ_m stands for a certain mean value. Considering further that

$$\psi_1 - \psi_s = [\,1 + m'(T_1 - T^\circ)/T_1\,]\ \psi_1 \simeq 2\,\psi_1$$

and

$$\left|\psi_m (n_1 - 1)\right| << \left|\psi_1 (n_1 - 1)\right| \approx 0''.03$$

we have finally

$$\delta(\Delta z) = \psi_1 \tan^2 z_1\,(n_1 - 2n^\circ + 1) \simeq \psi_1 \tan^2 z_1 \times 10^{-4} \qquad (43)$$

For $z_1 = 80^\circ$ and $\psi_1 = 1'\,52''$, $\delta(\Delta z) \approx 0''.36$ which value has also been derived in Table V by numerical integration.

The derivation of equation (43) does not allow for the motion of air initiated due to the horizontal temperature gradient, nor for the consequent change

Table V

Meridional Tilt of Isopycnic Surfaces

(Atmospheric Model No. 2)

$z_1 = 80°$ $\quad$ $dT/ds = -0.015\ °K\ km^{-1}$

r, km 6380 +	ψ, rad	$\Delta\psi$, rad	K_1''	$-K_2''$	$K_1'' - K_2''$
0	0.000542	0.000000	0.101	0.000	0.101
1	0.000477	−0.000065	0.082	0.011	0.093
2	0.000412	−0.000130	0.065	0.020	0.085
3	0.000347	−0.000195	0.050	0.027	0.077
4	0.000282	−0.000259	0.037	0.033	0.070
5.6	0.000178	−0.000363	0.020	0.040	0.060
7.2	0.000075	−0.000467	0.007	0.044	0.051
8.8	−0.000029	−0.000571	−0.002	0.046	0.044
10.4	−0.000133	−0.000675	−0.009	0.046	0.037

Strat. −0.000547 Integrals ; 0 – 4 km : 0.340

$K_1 = -(dn/dr)\sec^2 z\,\psi$ 4 – 10.4 km : 0.331

$K_2 = -(dn/dr)\tan^2 z\,\Delta\psi$ *Strat. Effect;* $\rho''(n° - 1)\sec^2 z°\,\psi_s$: −0.304

$\Delta\psi = \psi - \psi_1$ Total Effect : 0".37

in the tilts of the isobaric and isopycnic surfaces. Nevertheless it is evident that the meridional tilt will cause the astronomical refraction to be somewhat different in different azimuths, the strongest refraction occurring in the direction of the maximum negative temperature gradient.

In the atmospheric layer next to the ground, the horizontal gradients of temperature may vary greatly according to local conditions. Isopycnic slopes in excess of a few degrees are not uncommon especially in coastal areas where the ground surface, strongly heated by insolation, is adjacent to the cooler surface of the sea. As in Figure 2, the isobaric surfaces will slope down toward the cooler air above the sea, the angle of slope increasing with elevation. The pressure gradient force, acting in the direction of decreasing pressures and at right angles to the isobaric surfaces, thus acquires a horizontal component which cannot be balanced by the force of gravity, and the air aloft is set in horizontal motion from land to sea. The weight of the atmosphere over the sea is increased at the expense of that over the land, consequently a pressure gradient is established at ground level and a returning current, the sea breeze, begins to move the surface air from sea to land. During the night, the land surface cools more rapidly than the sea, and the conditions reverse into what is known as the land breeze.

If h is the depth of the locally affected layer and ψ_1 is the isopycnic slope angle at ground level, then, as shown previously

$$\delta(\Delta z) = \int_{n_h}^{n_1} \psi \tan^2 z \, dn - \int_{n_h}^{n_1} \tan^2 z \, (\psi - \psi_1) \, dn + \int_{n_h}^{n_1} \psi \, dn$$

or approximately

$$\delta(\Delta z) = \psi_1 \left(\tan^2 z_1 + \frac{1}{2}\right)(n_1 - n_h) \tag{44}$$

Taking ψ_1 from (39), and neglecting the horizontal pressure gradient which is always small, we have for the average conditions in the middle latitudes

$$\delta(\Delta z)'' \simeq -0''.20\, h\, (dT/ds)\left(\tan^2 z_1 + \frac{1}{2}\right) \tag{44a}$$

where h is in kilometres, and dT/ds is in $°C/km$. Usually, in the case of the sea breeze, h is of the order of 500 metres at which altitude the wind ceases.

Horizontal temperature gradients in the surroundings of the observing station probably make one of the most important sources of error in the determination of refraction. They are particularly dangerous if strong enough to be accompanied by the characteristic wind, the sea breeze, the lake breeze, the land forest breeze.

In the vicinity of a weather front, which is the transition zone between two air masses of different origin, significant refraction errors may be caused by horizontal temperature gradients that occur at levels well above the surface layer. Cold air tends to push under the warm air like a wedge, and along its leading edge an inversion of temperature is produced in a sloping sheet of mixed air Most fronts are associated with inclement weather, but some are inactive and devoid of all condensation forms. It shall be noted that refraction errors will not be present in ordinary inversion layers which lie horizontal, such as the nocturnal ground inversions which are so typical of conditions best suited for astronomical observations.

Hydrostatic Equilibrium in the Atmosphere

The second term in the refraction formula,

$$-\frac{\rho''(n_0-1)RT_0}{r_1 p_0 g}(\tan^3 z_1+\tan z_1)p_1$$

is a "hydrostatic correction" which takes into account the effect of the spherical curvature of atmospheric layers. Standard refraction tables presently used in geodetic astronomy all rely in this respect on atmospheric models. The hydrostatic term was introduced in order to remove this uncertainty, however small it may be.

The hydrostatic equation is so fundamental in physical meteorology that it enters almost every important formula, and the meteorologist takes it for granted that equilibrium is being closely maintained in the actual atmosphere. Vertical accelerations other than due to the gravity surely exist in the atmosphere, but they are mostly small and always so localized that the departures from a true equilibrium have little consequence. Except in thunderstorms and other phenomena which need not be considered, vertical winds do not normally attain velocities anywhere near the speeds of horizontal winds. Nor does the barometric pressure fluctuate erratically, but stays remarkably steady as it should in a true state of equilibrium.

Small barometric fluctuations with amplitudes of usually less than 0.5 mb and periods of several minutes, such as appear like ripples on a barogram, are quite often observed in cold weather. They are caused by wave motion, analogous to water waves, at the interface of two distinct layers of air sliding past each other. Oscillation of isopycnic surfaces in such air billows has a harmful effect on refraction which can be reduced by taking the mean of several observations.

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All these different errors, and others less significant, grow rapidly in magnitude when the zenith distance increases. It would appear that refraction corrections for geodetic astronomy can be adequately attained up to zenith distances of 75 to 80 degrees Beyond that limit, the determination of refraction from simple meteorological observations seems to be impossible

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