Benjamin W. REMONDI
National Geodetic Survey
Charting and Geodetic Services
National Ocean Services, NOAA
Rockville, MD 20852

GLOBAL POSITIONING SYSTEM CARRIER PHASE : DESCRIPTION AND USE

Abstract

After removing the modulation from the Global Positioning System (GPS) *signal* (L_1 *or* L_2) *a pure carrier signal remains. Suppose this carrier is continuously and precisely tracked by a* GPS *receiver. Furthermore, suppose the phase of the carrier is periodically measured and recorded (nearly simultaneously at two or more locations) with respect to receiver oscillators having the same nominal frequency as the* GPS *carrier. This paper first considers alternative modeling and processing approaches to these observational data for static operations. Then an approach to dynamic relative positioning using triple differences is presented. This approach should lend itself to performing centimeter accuracy relative surveys in seconds rather than hours. An approach to fixing cycle slips, automatically, is included.*

Introduction

The Department of Defense (DoD) has been developing the Navigation Satellite Timing and Ranging Global Positioning System (NAVSTAR GPS) since 1978, although it is actually an outgrowth of TIMATION and the Air Force's 621B Project (Easton 1978). DoD is now approaching the completion of its full-scale engineering development phase. An appreciation for GPS can be acquired from the Institute of Navigation's 1980 and 1984 monographs on GPS (Institute of Navigation 1980; 1984). It is anticipated that, between 1986 and 1989, 18 new GPS satellites will be placed in an orbital configuration such as to optimize spatial and temporal global coverage (Jorgensen 1984). Present plans call for placing three satellites (120° apart) in each of six evenly spaced orbital planes. These orbits will be nearly circular, inclined at 55°, and have 12–hour sidereal periods.

Each GPS satellite transmits unique navigational positioning and identification information centered on two L–band frequencies L_1 (1575.42 MHz) and L_2 (1227.6 MHz). The L_1 carrier signal is modulated with a precision code, known as the precise positioning service (PPS) code, and a coarse acquisition code, known as the standard positioning service (SPS) code. The L_2 carrier signal is currently modulated with only the PPS code. The Block II GPS satellites (circa 1987) will have the option to modulate L_2 with either the PPS code or the SPS code. These codes are pseudo-random

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noise codes : the PPS code has a chipping rate of 10.23 MHz and a repeat period of 37 weeks; the SPS code has a chipping rate of 1.023 MHz and a repeat period of 1 ms (Spilker 1978). Both signals, L_1 and L_2, are also modulated by a 50 bit per second (bps) message which includes, primarily, satellite orbit and timing information. The PPS and SPS codes are used for identifying the GPS satellites and, along with the 50 bps message, for satellite to receiver transit-time ranging and the synchronization of code receivers to GPS time. The PPS code is complex and difficult to acquire; a receiver may therefore first acquire the simpler SPS code and switch to the PPS code via the "handover word" in the 50 bps message (Van Dierendonck et al. 1978); if the a priori receiver location and orbital information are of high quality, immediate PPS acquisition is possible.

Thus, these L–band carriers are modulated by codes and message information. Receivers with knowledge of these codes have a number of advantages such as : (1) high signal noise suppression; (2) rapid and easy receiver clock synchronization to within 10 to 100 ns; (3) availability of GPS broadcast ephemeris data for real-time applications; (4) real-time availability of transit-time ranges for instantaneous point positioning to 10–30 m as well as several hour point positioning at the 1–meter level; (5) acquisition without a priori almanac preparation.

Measuring the carrier phase can be readily accomplished once the modulation is removed from the carrier. The code receivers use correlation methods to generate a modulation-free replica of the satellite carrier. Some receivers which do not possess knowledge of these codes strip the code and message from the signal (e.g., by squaring the signal); others can operate in spite of the phase discontinuities caused by the codes and the message data. (For example, the number of zero-crossings of a pure sinusoid will not change when subjected to random 180° phase reversals.) Some non-code GPS receivers, therefore, successfully treat the L–band signals, after signal processing, as signals having twice the nominal carrier frequency, and thus half the wavelength.

Although code receivers have some tremendous advantages from the point of view of real-time activities and simplicity (especially in the presence of dithering – intentionally perturbing the carrier), non-code receivers are useful as well, especially for relative positioning. Even real-time time interval transfer and real-time relative positioning can be accomplished with non-code receivers. In fact, it is theoretically possible to determine, in real time, the relative motion (trajectory) of one non-code GPS receiver with respect to another non-code receiver to the centimeter level.

In the following sections, carrier phase will be described, modeled and analyzed in terms of performing point positioning and relative positioning, although the emphasis will be decidedly on the latter. The discussion will begin with a description of carrier phase and the measurement of carrier phase. Then a model of the raw measurement will be developed. Some practical least-squares considerations will be included. Advantages and disadvantages of forming various linear combinations of the raw measurements will be discussed.

Emphasis will be placed on those combinations which have thus far proven to be most practical. Because loss of lock and cycle slips have proven to be nuisances, an approach to overcoming these problems automatically will be sketched. Finally, an approach to using GPS for the achievement of centimeter-level relative surveying in seconds (rather than hours) will be given. This same approach could be applied to centimeter-level relative trajectory determination.

A Simplified Description of Carrier Phase

The GPS satellite L_1 and L_2 carriers are, nominally, at 1575.42 MHz and 1227.6 MHz, respectively. Let f_s be the nominal frequency of one of these carriers. The phase of the carrier signal will now be described. *Figure 1* depicts the received carrier signal as function of time.

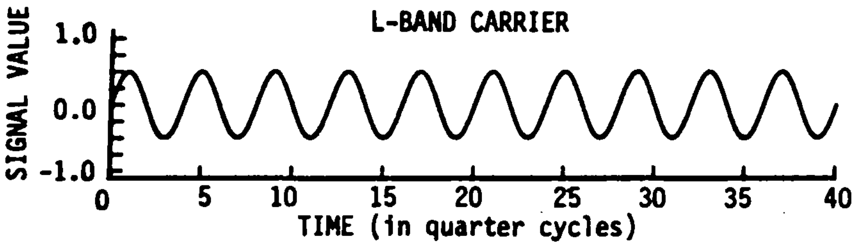

Fig. 1 — Received L–band carrier signal.

By the fractional phase of this carrier, $F(t)$, is meant the point in the cyclic oscillation at time t. For example, at $t = 1$ the phase is 0.25 cycles (or $90°$). One can plot the fractional phase as shown in *Figure 2.* If one describes phase as a monotonic function, $\phi(t)$, based on the number of cycles which have been received since an initial time t_0, the corresponding phase plot would be as depicted in *Figure 3.*

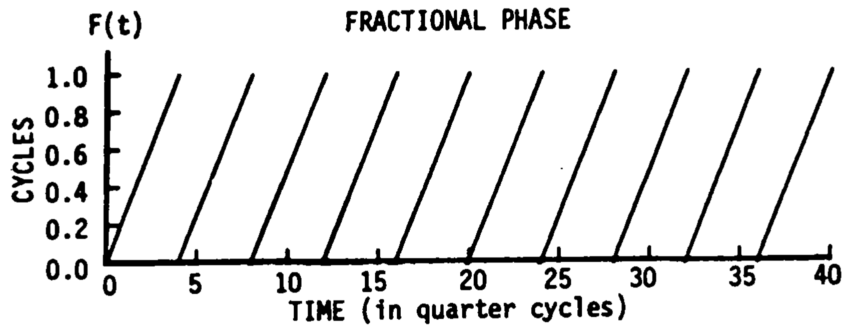

Fig. 2 — Fractional phase of the L–band carrier.

In this idealization the phase is shown as a linear function of time. The phase history of the received GPS carrier, however, would not be truly linear due to the Doppler effect (as well as numerous secondary effects such as oscillator drift, refraction effects, relativity, etc.). Consider the depiction in *Figure 4.* A GPS satellite transmits its L–band signal at very near the nominal frequency, f_s, and the oscillator of the GPS receiver outputs a sinusoidal signal at approximately the nominal frequency as well. The received satellite signal, however, has been shifted in frequency primarily due to range-rate (the Doppler effect). The depiction indicates that the signal is continuously transmitted and continuously received, but what is received was transmitted from a

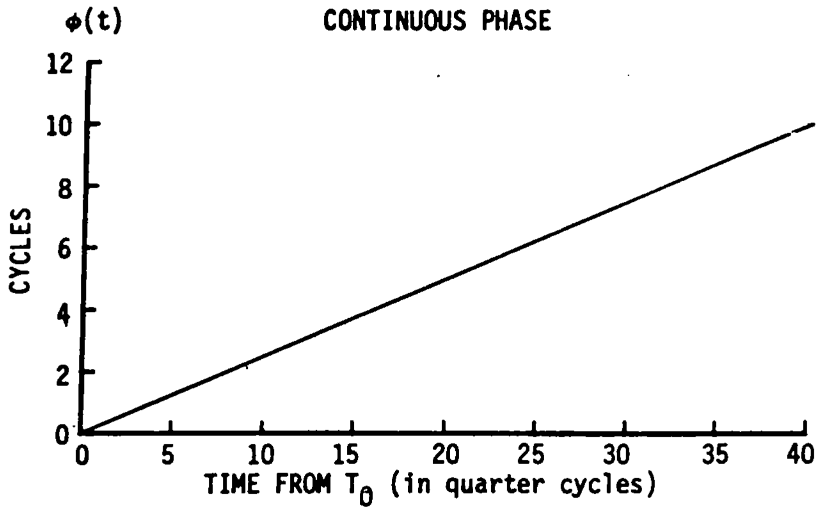

Fig. 3 — Continuous phase of the L–band carrier.

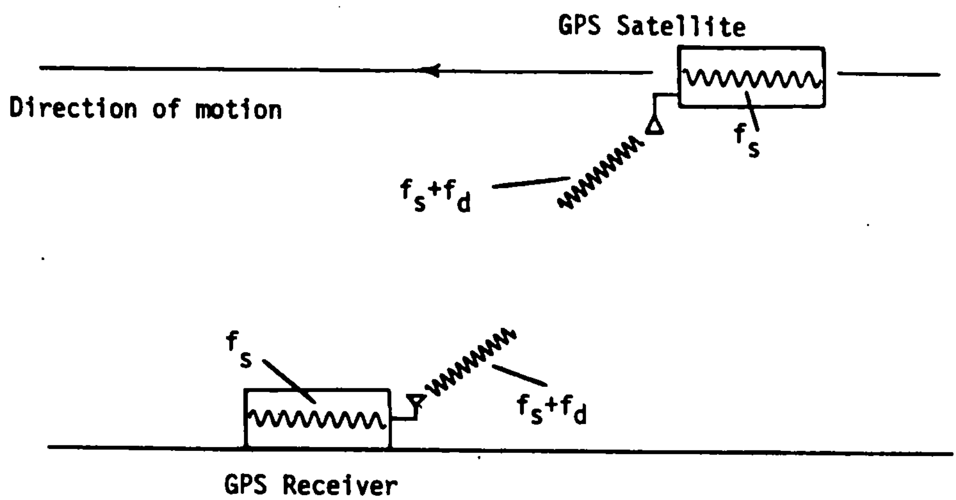

Fig. 4 — GPS satellite transmission and GPS receiver reception.

slightly earlier satellite location. Although the signal is transmitted continuously, and nominally received continuously, let us imagine only one cycle to be transmitted and later received. This "wave" can be viewed as traversing the medium much as an ocean wave traverses the sea. Because of the satellite-to-receiver motion, the satellite will move toward (or away from) the receiver during the transmission of this cycle; the effect of this is to change the wavelength (and therefore the frequency) of the propagating signal. This is the Doppler effect. Although range-rate changes the spacing of the peaks, the range-rate has no impact on the flight of a peak. The same argument holds for all phases.

Thus the phase, itself, is unaffected by motion, only the wavelength is. This demonstrates that one can treat an instantaneous carrier phase, observed at a receiver, as an event having a precise and unambiguous transmission time associated with it (i.e., the phase at actual receipt time is the same as the phase at actual transmission time).

Let us denote the phase of the carrier of the j–th GPS satellite at transmission time, t_T, as $\phi_s^j(t_T)$. A GPS receiver will receive this phase event at a time t_R according to its own time reckoning. In actuality, the receiver will be wrong and the true receipt time will be $t_R + \delta t_R$ (where δt_R represents how much the receiver's clock lags GPS ephemeris time). This is a crucial realization; this truth applies to the receipt of all phase events.

Conceptual Description of the Carrier Phase Measurement Process

One could count signal peaks or zero crossings as they were received, but the count would be meaningless unless it were made over a fixed time interval. This implies the receiver must have a stable clock or oscillator. One could time tag occasional zero crossings; this too requires a stable receiver oscillator. Since a stable oscillator is indispensable, why not measure the phase of the incoming signal with respect to the phase of the nominal GPS carrier signal generated by the receiver oscillator? Thus it makes sense to measure either $\phi_s^j - \phi_R$ or $\phi_R - \phi_s^j$. This is the approach used by the MACROMETER™ (Counselman and Gourevitch 1981; Counselman and Steinbrecher 1982) and the TI–4100 (Ward 1982). It is not the only approach, however. The SERIES approach (MacDoran et al. 1982; MacDoran et al. 1984a; MacDoran et al. 1984b) takes advantage of the fact that there is a rich spectrum emanating from GPS satellites. SERIES exploits the fact that there is a known phase relationship among the numerous frequencies (e.g., L_1, L_2, PPS code, SPS code, $L_1 - L_2$, etc.) and converts phase measurements, at many frequencies, to (biased) range measurements. SERIES uses the low frequency signals for ambiguity resolution, and the high frequency signals for range measurements precision. Other approaches are possible (e.g., interferometry). In this paper only the approach of measuring carrier phase relative to the phase of a GPS receiver's local oscillator will be considered in detail.

Consider the simplified depiction of a GPS receiver shown in ***Figure 5.*** Suppose it can track five GPS satellites simultaneously. We shall assume that there are no interchannel biases, or that one channel is used for all satellites in a multiplexing mode. Visualize the carriers of these GPS satellites passing through the receiver in the upper five panes while the receiver's own carrier (which is passing through the lower pane) performs two functions : First, the receiver time will be based on the number of cycles which pass the cross hair; Second, ***when*** the number of cycles that passes the cross hair corresponds to a prescheduled measurement time, say t_i, the five cross hair differences, $\phi_s^j - \phi_R$, $j = 1, \ldots, 5$, will be measured (e.g., within ± 0.01 cycles) and recorded. Let us realize, from the start, that although the receiver "believes" the time to be t_i in terms of GPS ephemeris time, the actual measurement time is $t_i + \delta t_i$, where δt_i is unknown (but very real and very important). Let us designate the transmission time associated with $t_i + \delta t_i$ by $t_T^j(i)$. (Since all the satellite phases are measured at the

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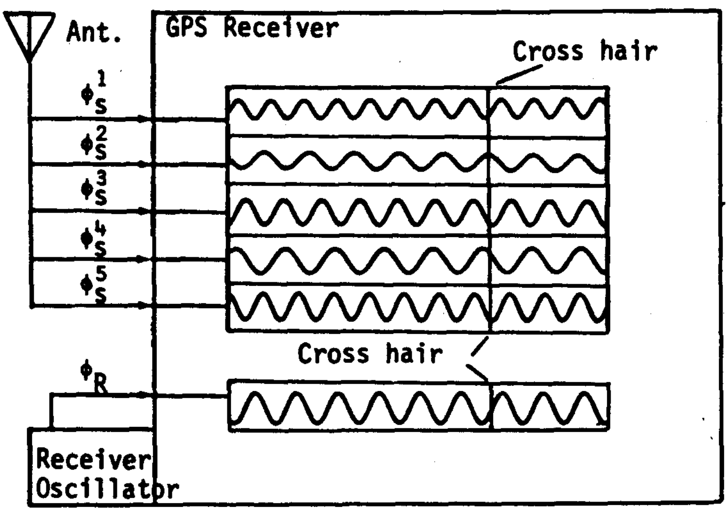

Fig. 5 — Conceptual depiction of a five-channel GPS receiver.

same time, the transmission times are satellite dependent.) It is not essential that our relative phase measurements be taken simultaneously among satellites or among receivers; however, it is convenient if that is so. Should they be taken at slightly different receiver times (within the stability period of the satellite oscillator), this could be dealt with satisfactorily. There may be a discrepancy between the receiver "realized" time t_i and the actual observation time. A constant discrepancy common to all channels would be indistinguishble from δt_i and would be harmless.

Random interchannel discrepancy differences at the few picosecond (ps) level would be consistent with phase measurements at the ± 0.01 cycle level. Large systematic variations (e.g., 100 ps/hr) are avoidable with careful design. It will be assumed here, without loss of generality, that the relative phase measurements are taken simultaneously. In summary, if $m^j(i)$ is our measurement, having time tag t_i, the measurement-model relationship (in its simplest form) is :

$$m^j(i) = \phi_s^j(t_T^j(i)) - \phi_R(t_i + \delta t_i) + \epsilon^j(i) \qquad (1)$$

where $\epsilon^j(i)$ represents measurement noise. Neither ϕ_s^j nor ϕ_R is restricted to being a fraction of a cycle; neither is m^j. They are continuous quantities. The theoretical time frame of which $t_i + \delta t_i$ is an epoch, is assumed to be in perfect phase alignment (at all times) with the theoretical time frame of which $t_T^j(i)$ is an epoch. That is, we assume that δt_i is the only correction term needed for the receiver clock to keep perfect GPS time, that any significant rate differences (drift) will be modeled by the time variations δt_i, and there are no unmodelled inter-satellite differences. The phase m^j has been tracked continuously since the first measurement, $m^j(1)$, was taken. $m^j(1)$ was made based on a phase alignment of the receiver clock with the incoming carrier signal but with *no* knowledge as to which cycle would represent perfect phase synchronization. Thus eq. 1 can be improved to acknowledge this integer cycle ambiguity :

$$m^j(i) = \phi_s^j(t_T^j(i)) - \phi_R(t_i + \delta t_i) + N^j(1) + \epsilon^j(i) \ . \tag{2}$$

So long as carrier tracking is maintained, only one integer unknown (per receiver, per satellite, for all times) appears in the model.

Developing the Model

Ignoring such factors as propagation delays, relativity, etc., the transmission time of the carrier phase [event] can be modeled as

$$t_T^j(i) = t_i + \delta t_i - \tau^j(t_i + \delta t_i) \ , \tag{3}$$

where $\tau^j \ (t_i + \delta t_i)$ is the actual signal transit time from the j–th GPS transmitter to the GPS receiver; τ^j contains the geometrical information of interest including the receiver coordinates. $\tau^j(t_i + \delta t_i)$ can be represented as $\tau^j(i)$ where :

$$\tau^j(i) \equiv \tau^j(t_i + \delta t_i) = \frac{1}{c}\rho^j\left[t_i + \delta t_i - \frac{1}{c}\rho^j\left(t_i + \delta t_i - \frac{1}{c}\rho^j(\ldots.)\right)\right], \tag{4}$$

c is the speed of light and ρ is an instantaneous geometric slant range. Notice that, in eq. 4, ρ^j is a *function* of all that follows it. These are not to be interpreted as multiplications. $\tau^j(i)$ is a series expansion of ρ^j/c about $t_i + \delta t_i$ requiring a priori values of the δt_i. In practice, $\tau^j(i)$ results from a subroutine call at $t_i + \delta t_i$. The starting ρ is computed at $t_i + \delta t_i$, whereas subsequent improved computations of ρ are evaluated at $t_i + \delta t_i - \rho/c$; this process continues until convergence is achieved. Under the assumption of an Earth–Centered Body-Fixed coordinate system, after each iteration the satellite position vector must be rotated by the amount of Earth rotation that occurs during signal transit.

ρ^j/c, for GPS, is always smaller than 100 ms, and since GPS atomic clocks are highly stable (e.g., $\Delta f = 10^{-10} \cdot f = 0.16$ Hz over 100 ms), one can substitute eq. 3 into eq. 2, realize that $\dot{\phi}_s^j(t_i) \doteq f_s^j$, and perform a Taylor expansion as follows :

$$m^j(i) = \phi_s^j(t_i) + f_s^j \cdot \left(\delta t_i - \tau^j(i)\right) - \phi_R(t_i + \delta t_i) + N^j(1) + \epsilon^j(i) \ . \tag{5}$$

A point to stress is that the (Taylor) expansion of ϕ_s^j is about $t = t_i$, the scheduled receipt time, and not the actual receipt time. t_i has the unique property that it is ***common*** to all receivers scheduled to take measurements at Epoch t_i. Thus t_i and $\phi_s^j(t_i)$ are, both, receiver independent.

Suppose there are n_s satellites, n_R receivers and measurements are scheduled to be made at $t_i = t_1 + (i-1) \cdot \Delta t$, where $i = 1, \ldots, n_E$ and Δt is the selected epoch interval. Suppose, for simplicity, all n_s satellites are "visible" over the scheduled period and there are no losses of lock. In such a case there would be $n_s \cdot n_R \cdot n_E$ measurements

$$m_M^j(i) = \phi_s^j(t_i) + f_s^j \cdot \delta_M t_i - f_s^j \cdot \tau_M^j(i) - \phi_R^M(t_i + \delta_M t_i) + N_M^j(1) + \epsilon_M^j(i) \quad (6)$$

where M represents the receiver and where the symbol $\delta_M t_i$ represents receiver M's clock error at t_i; it is not a product of quantities.

For a specific example, suppose $n_s = 6$, $n_R = 3$ and $n_E = 100$; there would be 1800 measurements. Assuming perfect orbits, no propagation medium delays and perfectly known satellite clock frequencies, f_s^j, etc., there would be 927 unknowns in the 1800 equations : 600 satellite clock parameters $\phi_s^j(t_i)$, 300 receiver clock parameters $\delta_M t_i$, 18 integer cycle ambiguities $N_M^j(1)$, and 9 coordinate unknowns (the receivers' coordinates are included in the $-f_s^j \cdot \tau_M^j(i)$ term).

This somewhat simplified discussion does not bring out that in the "real world" singularities must be addressed. Briefly, there is insufficient geometry to determine all the $\delta_M t_i$ and the $\phi_s^j - \phi_R^M$. This singular situation can be rectified by establishing references. Thus we can express $\delta_M t_i$ as $\delta_M t_i - \delta_M t_1 + \delta_M t_1$ and solve for n_R of the $\delta_M t_1$ and $(n_E - 1) \cdot n_R$ of the $\delta_M t_i - \delta_M t_1$. It would be seen that the $\delta_M t_1$ would be determined at the microsecond level whereas the $\delta_M t_i - \delta_M t_1$ would be determined at the subnanosecond level. The explanation is that $\delta_M t_1$ is determined from the satellite-station dynamics, whereas $\delta_M t_i - \delta_M t_1$ results from continuous phase tracking. Although slightly more involved, the same approach could be applied to $\phi_s^j - \phi_R^M$.

Another point to stress is the distinction between the nonlinear model given in eq. 6 and the linearized version to be discussed later. The nonlinear model should be used for the computational model and the linearized version of the model used for determining the analytic partial derivatives for the design matrix in the normal equations. A practical advantage is that the data reduction would still be precise should receivers inadvertently take measurements many seconds apart.

A More Complete Model

It is not the intent of this paper to be exhaustive, so only limited attention will be given to improving the above model. Relativity plays a role in point positioning and long base line relative positioning. The modeling equations to account for the relativistic effects can be found in a technical report by Gibson (1983). Another important aspect to precise positioning with GPS is the need for ultraprecise ephemerides. For orbit improvement, over a short arc (e.g., 3–hour), eq. 4 can be explicitly expanded in terms of initial orbital parameters along with the other terms mentioned previously. These can be either initial classical elements or, alternatively, cross-track, along-track and radial correction parameters (Anderle 1980; Beutler et al. 1984). Propagation delays caused by the ionosphere or troposphere can be measured, modeled, or estimated (Campbell, et al. 1984; Goad and Goodman 1974). Thus, for each satellite, j, and each receiver, M, the tropospheric delay, T, and the ionospheric delay, I, can be incorporated into the model as :

$$t^j_{T_M}(i) = t_i + \delta_M t_i - T^j_M(i) - I^j_M(i) - \tau^j_M(i) \ . \tag{7}$$

Equation 6 would then be updated to

$$m^j_M(i) = \phi^j_s(t_i) + f^j_s \cdot \delta_M t_i - f^j_s \cdot \left(T^j_M(i) + I^j_M(i) + \tau^j_M(i)\right) - \phi^M_R(t_i + \delta_M t_i) + \\ + N^j_M(1) + \epsilon^j_M(i) \ . \tag{8}$$

Forming Linear Combinations of Raw Observables

Depending on the objective, there are significant advantages and disadvantages to forming certain linear combinations of the basic phase observation $m^j_M(i)$. One-station combinations do not require common satellite visibility between satellite locations. When common satellite visibility is imposed (as in single differences), and the GPS receivers are part of a global network, unless one uses raw phase observables, one sometimes discards valuable data. Also, if the optimal estimation of the phase profiles of the GPS satellites is desired. one-station observables may be preferred.

On the other hand, the requirement of intervisibility is not overly restrictive due to the 20,000 km altitude of GPS satellites. When observations are formed using nearly simultaneous measurements from two stations, the $\phi^j_s(t_i)$ term drops out of the model; the behavior of the satellite clock only appears by way of the f^j_s factor over, at most, a fraction of a second. When observations are formed using nearly simultaneous measurements from two satellites, at one station, the station clock term $\delta_M t_i$ drops out of the model. The behavior of the receiver clock, δt_i, appears, therefore, in a relatively harmless role (see eq. 4) and the δt_i can be replaced by a linear model. Whereas hundreds and sometimes thousands of unknowns must be determined when modeling the raw phase observation, only a few need to be determined when two-station, two-satellite combinations are formed. One must account for the correlated nature of such observations, however, for precise applications. The three most useful one-station observables are $m^j_M(i), m^k_M(i) - m^j_M(i)$, and $m^j_M(i+1) - m^j_M(i)$, where M is the receiver, j and k are satellites, and $i+1$ is the epoch which follows epoch i. Although single-station combinations are advantageous for certain applications, only two-station applications will hereafter be considered.

Single Differences

Using eq. 6 as the fundamental model, define the single difference observation (Goad and Remondi 1984; Remondi 1984) using station 1 and station 2 as follows:

$$\begin{aligned} S(j,i) &= m^j_2(i) - m^j_1(i) \\ &= f^j_s \cdot \left[\left(\delta_2 t_i - \delta_1 t_i\right) - \left(\tau^j_2(i) - \tau^j_1(i)\right)\right] \\ &\quad - \left(\phi^2_R(t_i + \delta_2 t_i) - \phi^1_R(t_i + \delta_1 t_i)\right) + N^j(1) + \epsilon^j(i) \end{aligned} \tag{9}$$

where $N^j(1) = N^j_2(1) - N^j_1(1)$ and $\epsilon^j(i) = \epsilon^j_2(i) - \epsilon^j_1(i)$ (N and ϵ are not to be confused, here, with their earlier use in eqs. 2, 5, and 6). If the receiver oscillators are highly stable, $\phi_R(t_i + \delta t_i) = \phi_R(t_i) + f_R \delta t_i$. If the receiver oscillators have been tuned with respect to f_s, then

$$\phi_R^2(t_i) - \phi_R^1(t_i) + f_R^2 \cdot \delta_2 t_i - f_R^1 \cdot \delta_1 t_i = -f_s(\delta_2 t_i - \delta_1 t_i) + f_s(\delta_2 t_i - \delta_1 t_i) = 0 .$$

The minus sign stems from the convention established when δt was defined, (i.e., δt is negative when the clock runs fast because the measurement would be taken too soon). Thus the single difference model, under these assumptions, reduces to :

$$S(j,i) = f_s^j \cdot \left(\delta_2 t_i - \delta_1 t_i - \tau_2^j(i) + \tau_1^j(i)\right) + N^j(1) + \epsilon^j(i) . \tag{10}$$

This model is nonlinear in the δt_i in that the $\tau^j(i)$ will be evaluated at the true signal receipt time $t_i + \delta t_i$. To form the normal equations, one linearizes eq. 10 as follows (see eq. 4) :

$$\begin{aligned} S(j,i) \doteq\ & f_s^j(\delta_2 t_i - \delta_1 t_i) + N^j(1) + \epsilon^j(i) \\ & -\frac{1}{c} f_s^j \cdot \left[\rho_2^j(t_i) + \dot{\rho}_2^j(t_i)\left(\delta_2 t_i - \rho_2(t_i)/c\right)\right] \\ & +\frac{1}{c} f_s^j \cdot \left[\rho_1^j(t_i) + \dot{\rho}_1^j(t_i)\left(\delta_1 t_i - \rho_1(t_i)/c\right)\right] \end{aligned} \tag{11}$$

where $\dot{\rho}$ is range rate. The partials of $S(j,i)$ with respect to $\delta_M t_i$ are

$$f_s^j\left(1 - \frac{1}{c}\dot{\rho}_2^j(t_i)\right) \quad \text{and} \quad -f_s^j\left(1 - \frac{1}{c}\dot{\rho}_1^j(t_i)\right) ;$$

the partials of $S(j,i)$ with respect to the $N^j(1)$ are 1 for all j ; and the partials of $S(j,i)$ with respect to station location can be approximated by

$$\begin{aligned} -\frac{1}{c} f_s^j \cdot \left[\frac{\partial \rho_2^j}{\partial \vec{x}_2} - \frac{\partial \rho_1^j}{\partial \vec{x}_2}\right] &= -\frac{1}{c} f_s^j \cdot \frac{\partial \rho_2^j}{\partial \vec{x}_2} = +\frac{1}{c} f_s^j \cdot \left(\frac{\vec{\rho}_2^j}{\rho_2^j}\right)^T \\ -\frac{1}{c} f_s^j \cdot \left[\frac{\partial \rho_2^j}{\partial \vec{x}_1} - \frac{\partial \rho_1^j}{\partial \vec{x}_1}\right] &= +\frac{1}{c} f_s^j \cdot \frac{\partial \rho_1^j}{\partial \vec{x}_1} = -\frac{1}{c} f_s^j \cdot \left(\frac{\vec{\rho}_1^j}{\rho_1^j}\right)^T \end{aligned} \tag{12}$$

since the partials do not have to be perfect. Notice that the partial derivatives of the station components are components of the station to satellite unit vectors times a constant (f_s^j/c). In practice one avoids a singular solution by solving for $\delta_M t_i - \delta_M t_1$ (which comes from continuous phase tracking) and the $\delta_M t_1$ (which comes from satellite-station Doppler) rather than simply $\delta_M t_i$. It may also be useful to define a relative clock drift parameter $\delta_i = \delta_2 t_i - \delta_1 t_i$ and a common drift parameter $\xi_i = (\delta_2 t_i + \delta_1 t_i)/2$. The advantage of this is that ξ_i can be replaced with a constant or linear model thus reducing the number of unknowns. In such a case δ_1 and the $\delta_i - \delta_1$ would still be estimated. If the receiver clock differences are measured before and after data collection, then δ_1 can be computed a priori and need not be estimated. Simplifications to the data reduction can be found in Remondi (1984:49).

One aspect to single differences, not yet addressed, relates to the $N^j(1)$. In the least-squares process they are treated as real numbers. The coefficient of $N^j(1)$ is constant the coefficient of $\delta_2 t_1 - \delta_1 t_1$ is very nearly constant. Thus, there may be insufficient geometry to distinguish $N^j(1)$ from $\delta_2 t_1 - \delta_1 t_1$. Thus, the integer value of $N^j(1)$ cannot be isolated-even for short base lines. The integer difference ***between*** satellites ***will*** be apparent, however, because $f_s^j(\delta_2 t_1 - \delta_1 t_1)$ will substract out (reducing the effect of $\delta_2 t_1 - \delta_1 t_1$ by a factor of 10^5 to 10^6). Once the $N^k(1) - N^j(1)$ have been established they can now be fixed as integers in a subsequent reduction. Taking advantage of the integer nature of the $N^j(1)$ is important if one is to achieve millimeter accuracy--especially for short base lines. It should be pointed out that if tracking is interrupted, the integer number of cycles from that point will likely be wrong once tracking is resumed (the fractional phase measurement should be unaffected). One can account for the lost cycles during processing however. More will be said about this, later, in the triple difference discussion. In single difference mode, with $n_s = 6$, $n_R = 2$, $n_E = 100$, there would be 600 equations and 212 unknowns (assuming perfect orbits, no refraction unknowns, etc.). When more than two stations are involved, one should account for the correlated nature of the single differences, thus formed, by using a correlated weight matrix in the normal equations. Refer to Remondi (1984) for the techniques involved in forming the proper weight matrix for the normal equations. Although multiple base line examples are not included, the necessary techniques are included.

Delta Single Differences

Let us define the delta single difference as follows (Remondi 1984 : ch. 4) :

$$DS(j,i) = S(j,i+1) - S(j,i) \ . \tag{13}$$

The model and partials can be taken from subtracting those from the corresponding single differences. Notice, however, that there are no integer ambiguities $N^j(1)$ in this model. The advantage is the insensitivity to loss of lock; the disadvantage is that one cannot exploit the integer nature of integer ambiguities. Thus, for short base lines, the ultimate in accuracy may not be achievable. For many applications this is not a serious loss (e.g., industrial applications). Other disadvantages are the correlated nature of the observations (Remondi 1984 : 108-114) and the requirement to solve for the epoch time parameters (the $\delta_M t_i - \delta_1 t_i$) . An attractive feature of this formulation is that it becomes clear, from the residuals, how many cycles have been lost. This "knowledge" can be subsequently passed on to the single or double difference methods (see automatic cycle slip fixing, below). In the example given at the end of the section on single differences, there would be 594 equations and 206 unknowns. Introducing the relative and common clock parameters, δ_i and ξ_i, mentioned after eq. 12, would reduce this to approximately 107 unknowns since ξ_i can often be treated as a constant for all i.

Double Differences

The double difference observable is defined using satellites j and k at epoch i as follows (Bossler et al. 1980; Bock et al. 1984; Remondi 1984):

$$DD(j,k,i) = S(k,i) - S(j,i) \ . \tag{14}$$

The model and the partials are taken from corresponding differences of single differences. In this scheme one satellite, say j, becomes the reference satellite, and the integer unknowns are $N^k(1) - N^j(1)$. For short base lines these integers can be isolated since the contribution made by the clock drift terms, δt_i, has been reduced by the order of 10^6. Thus it is satisfactory, even for precision applications, to use a linear clock model for the station clocks. The double difference method is much like the single difference method. The slight disadvantage to using single differences is that one must determine the clock drift parameters (for time interval transfer this would be an advantage); a slight disadvantage of double differences is their correlated nature. In the example given at the end of the section on single differences, there would be 500 equations and 11 unknowns (if a [linear] relative clock drift and a common clock offset were to be estimated, rather than measured, there would be three additional unknowns).

Triple Differences

The triple difference observable, using satellites j and k, at epochs i and i + 1, can be defined (Remondi 1984) as either DS(k, i) – DS(j, i) or DD(j, k, i + 1) – DD(j, k, i). In either case:

$$\begin{aligned} T(j,k,i) = f_s^k \cdot \Big[\big(\tau_1^k(i+1) - \tau_2^k(i+1)\big) - \big(\tau_1^k(i) - \tau_2^k(i)\big)\Big] \\ - f_s^j \cdot \Big[\big(\tau_1^j(i+1) - \tau_2^j(i+1)\big) - \big(\tau_1^j(i) - \tau_2^j(i)\big)\Big] \ . \end{aligned} \tag{15}$$

The main advantage of the triple difference method is its robust nature. When a loss of lock is encountered only data at a single epoch will be edited, and processing will continue. In fact, numerous losses of lock can be handled with ease. For this reason, hundreds of base lines can be processed in (unattended) batch mode. If the receiver oscillators are synchronized and tuned, and if the station 1 coordinates are sufficiently well known, then as few as three parameters need to be estimated (namely, the coordinates of station 2). The main disadvantages of this scheme are: (1) the correlated weight matrix is more complicated; and (2) as with delta single differences, one cannot exploit the integer nature of the integer ambiguities. The first of these was easily implemented for the two-station case (Remondi 1984 : 140-147) and is similarly implemented for the many-station case. It has been shown (Remondi 1984 : ch. 7) that when one accounts for the correlated nature of triple differences, relative geodesy can be performed at the 1 ppm level when one does not, 5–10 ppm, or better, is achieved.

It is the author's experience that timewise correlations are more significant than geometrical correlations. Only six to nine parameters would need to be estimated, in the example cited above, using this method.

An Automatic Approach to Loss of Lock and Cycle Slips

When a satellite signal is obstructed, it can no longer be tracked. When the satellite reappears, tracking can resume. The fractional phase, then measured, would be the same as if tracking had been maintained; the integer number of cycles would be wrong, however. There are numerous possible approaches to dealing with this problem. A common approach is to hold the stations fixed and to edit the data manually. This has proven to work, but it can be tedious. Another approach is to model the data with piecewise continuous polynomials on a satellite dependent basis (Beutler et al. 1984). To implement this approach the data would have to be examined to find the breaks, which could be tedious. This would be followed with some manual editing at the few cycle level.

Although many approaches are possible, I shall herein sketch an ***automated*** approach which is easy to implement. Use the triple difference processing method (with or without a correlated weight matrix) to determine station location(s). Once convergence has been achieved, automatically search through the triple difference residuals to isolate "large" discontinuities in double differences, where the choice of what is deemed large is important. For example, 2 cycles or 10 times the root-mean-square (rms) of the residuals might be the criteria. The triple difference method is ideally suited for this task because (1) it is not confused by clock drift, and (2) it knows, based upon its own very good station solution, how many cycles to expect over any time interval. One would evaluate all such triple difference residuals over an epoch interval and determine which satellites had integer jumps and by how many cycles. (For example, if the SV–6 minus SV–8 residual was 10.02 cycles, and the SV–9 minus SV–8 residual was 12.97 cycles, one would remove 10 cycles from SV–6 and 13 cycles from SV–9 at all epochs from $i+1$ to the end. True, this might result in a common integer error for SVs 6, 8, and 9 at these epochs; it would drop out, however, in double difference mode.) Finally, the single – or double – difference method would be used to complete the processing. After convergence, a first difference approach could be used to isolate any 1–2 cycle discontinuities. With this approach, single or double difference processing would be as hardy as triple difference processing.

Using Triple Differences for Centimeter–Level Trajectory Determination and Performing Centimeter–Level Relative Positioning in Seconds

Let us consider the case where receiver 2 is moving. Suppose that receiver 1 is fixed at a known geodetic location and that the location of receiver 2 is known at time t_i. The objective, here, is to determine the geodetic location of receiver 2 at t_{i+1} regardless of the path taken by receiver 2 during the interval from t_i to t_{i+1}. First of all, a triple difference observation over the interval t_i to t_{i+1}, for satellites j and k, is equivalent to the satellite k delta single difference minus the satellite j delta single difference over the same interval. This is pointed out because a delta single difference is equivalent to a change in range. For one station (Remondi 1984 : 31) the continuous Doppler count can be integrated as follows :

$$\int_{t_i}^{t_{i+1}} f_d^j \, dt \doteq \frac{1}{c} f_s^j \cdot \left(\rho^j(t_{i+1}) - \rho^j(t_i)\right) + \text{ relativistic effects} . \tag{16}$$

For two nearby stations the relativity term is negligible and

$$\int_{t_i}^{t_{i+1}} (f_{d_2}^j - f_{d_1}^j)\, dt = \frac{1}{c} f_s^j \cdot \left(\rho_2^j(t_{i+1}) - \rho_2^j(t_i) - \rho_1^j(t_{i+1}) + \rho_1^j(t_i)\right). \tag{17}$$

The corresponding delta single difference observables have a large clock drift component, however. The triple difference observable, on the other hand, has only a small clock drift component which can (adequately) be modeled linearly. Notice that delta single differences (from t_i to t_{i+1}) are path independent and, consequently, so is the triple difference formed by their difference. We shall assume, herein, that the clock drift between the two receivers is accurately measured so that its estimation is not required. (This assumption is not needed, and the clock drift may also be estimated.) The triple difference, thus formed, based on satellites j and k, over t_i to t_{i+1}, with receiver 2 in motion, is

$$\begin{aligned} T(j,k,i) = {} & f_s^k \cdot \left[\left(\tau_1^k(i+1) - \tau_2^k(i+1)\right) - \left(\tau_1^k(i) - \tau_2^k(i)\right)\right] \\ & - f_s^j \cdot \left[\left(\tau_1^j(i+1) - \tau_2^j(i+1)\right) - \left(\tau_1^j(i) - \tau_2^j(i)\right)\right] \end{aligned} \tag{18}$$

where $\tau^j(i)$ was explicitly given in eq. 4. Only $\tau_2^j(i+1)$ and $\tau_2^k(i+1)$ are unknown since they depend on the location of receiver 2 at t_{i+1}. Moving the known terms to the left hand side (LHS) results in :

$$\begin{aligned} \text{LHS}(j,k) \equiv {} & T(j,k,i) - f_s^k \cdot \left(\tau_1^k(i+1) - \tau_1^k(i) + \tau_2^k(i)\right) \\ & + f_s^j \cdot \left(\tau_1^j(i+1) - \tau_1^j(i) + \tau_2^j(i)\right) . \end{aligned} \tag{19}$$

Ignoring measurement noise the nonlinear model would be :

$$\text{LHS}(j,k) = f_s^j \, \tau_2^j(i+1) - f_s^k \, \tau_2^k(i+1) \tag{20}$$

The corresponding linearization would be :

$$\begin{aligned} \text{LHS}(j,k) \doteq {} & \frac{1}{c} f_s^j \cdot \left[\rho_2^j(t_{i+1} + \delta t_{i+1}) + \dot{\rho}_2^j(t_{i+1} + \delta t_{i+1}) \left(-\frac{1}{c}\rho_2^j(t_{i+1} + \delta t_{i+1})\right)\right] \\ & - \frac{1}{c} f_s^k \cdot \left[\rho_2^k(t_{i+1} + \delta t_{i+1}) + \dot{\rho}_2^k(t_{i+1} + \delta t_{i+1}) \left(-\frac{1}{c}\rho_2^k(t_{i+1} + \delta t_{i+1})\right)\right] \end{aligned}$$

where $j = 1$ and $k = 2, 3, 4, 5, 6, \ldots$. (21)

At least three triple differences are required to determine all three station 2 coordinates at t_{i+1}. For *practical* applications three triple differences would often yield suboptimal results and four to six would be desirable. If RHS symbolizes the right-hand side of eq. 20, then

$$\mathrm{LHS}(j,k) = \mathrm{RHS}(j,k,\vec{x}_2(i+1))$$

$$\mathrm{LHS}(j,k) \doteq \mathrm{RHS}(j,k,\tilde{\vec{x}}_2(i+1)) + \frac{\partial\,\mathrm{RHS}(j,k,\tilde{\vec{x}}_2(i+1))}{\partial \vec{x}_2}\,\Delta\vec{x}_2 \quad (22)$$

where $\Delta\vec{x}_2 = \vec{x}_2(i+1) - \tilde{\vec{x}}_2(i+1)$, and where $\tilde{\vec{x}}_2(i+1)$ is the a priori value of $\vec{x}_2(i+1)$. In the first iteration, one could initialize $\tilde{\vec{x}}_2(i+1)$ to $\vec{x}_2(i)$-- although other schemes are possible. Let

$$y_k = \mathrm{LHS}(j,k) - \mathrm{RHS}(j,k,\tilde{\vec{x}}_2(i+1)),\ \text{where } j=1 \text{ and } k=2,3,4,5,6\ldots.$$

Then,

$$\Delta\vec{x}_2 = (A^T R^{-1} A)^{-1} A^T R^{-1}\vec{y} \quad (23)$$

where

$$A = (A_k) = \left[\frac{\partial\,\mathrm{RHS}(k)}{\partial \vec{x}_2}\right], \vec{y} = (y_k) \quad (24)$$

and R^{-1} is the correlated weight matrix between triple differences. The partial derivative computations given by

$$\frac{\partial\,\mathrm{RHS}}{\partial \vec{x}_2} \doteq -\frac{1}{c} f_s^j \cdot \frac{\vec{\rho}_2^{\,j}}{\rho_2^j} + \frac{1}{c} f_s^k \cdot \frac{\vec{\rho}_2^{\,k}}{\rho_2^k} \quad (25)$$

should suffice. Thus it should be straightforward to determine the new position of receiver 2 (to centimeter accuracy) at t_{i+1} provided : (1) lock is maintained on four or more satellites (six are recommended) ; (2) the geodetic location of station 1 is fixed and known ; (3) the geodetic location of station 2 at t_i is known relative to station 1 ; (4) the linear clock drift between receiver 1 and receiver 2 is known (e.g., to $1\,\mu s/hr$) ; (5) the satellite positions are approximately known ; and (6) redundant phase measurements are taken at a mark.

For the application where one transitions between survey marks, while maintaining carrier phase lock, one might take many measurements (e.g., 100) over a limited time interval (e.g., 10 seconds) before proceeding to the next mark. As suggested earlier, five or more satellites should be tracked in practical applications. Such enhancements would improve the noise characteristics considerably. For those applications where motion does not stop, one would also need to take frequent measurements and use either a hardware or a software smoother (or both) to reduce the effects of measurement noise and/or suboptimal satellite position geometry.

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